CHAPTER - II

REFINEMENT IN THE HHOB APPROXIMATION

INTRODUCTION :

In recent years numerous calculations, corresponding to nearly as many theoretical description, have been made of amplitude of high -energy collisions of charged particles with atomic targets. Collision process between electron and atom were studied particularly at intermediate and high energy region. The theoretical methods employed in the calculation of the differential cross section are reviewed by Burke and Wiliams (1977),Moiseiwitsch (1977), Bransden and Mc Dowell (1977-1978), Callaway (1980) and so many. These methods were broadly classified as:

(a) expansion methods, (b) methods based on the construction of Optical potential, (c) the Born approximation and its extensions, (d) the distorted wave methods, (e) semiclassical methods, and (f) many body theory.

Theoretically the scattering problem was basicallly attempted with mainly two ideas:

(i) nature of the interaction (direct, static, exchange, polarization),
(ii) validity (angular and energy regions) of the methods for cross section calculations.

Despite lot of work reported for the calculation of the differential cross section using various methods and numerical techniques for the computational work, it is observed that there still exists a wide gap between the theory and experiment results. Recent developments in the experimentation have made the availability of the diffrential data of electron and positron scattering by a variety of target atoms like hydrogen, helium, lithium, neon and argon and so on.

The advent of the latest electronic computers and their subsequent use in the calculation required in the theoretical techniques have made the testing more easier and faster. The combination of these powerful techniques with that of possible new experimentals checks have caused the continual activity in this area.

Having an experimental and theoretical impetus in the data, it is an appropriate time to extend the calculation by modifying or by refining the approximation in the method used.

There are several approximate methods described, but because of enormous complexity in describing and predicting the results of associated experiments, most of the cited works have had as their objective the determinations of accurate and computationally feasible theoretical procedures. Included among the more successful methods are variations of traditional impact parameter studies (D.P.Dewangan (1975)), the simplified Born approximation (A.R. Holt, B.L. Moisewitsch (1968); A.R. Holt (1972); A.R. Holt, B. Santoso (1973); M.J. Woolings (1972)), Glauber (E. Gerjuoy, B.K. Thomas (1974); T.T. Glen (1976); J.E.- Golden, J.H. McGuire (1976)) and modified Glauber (L. Hambo, J.C.Y. Chen, T. Ishihara (1973); J.C.Y. Chen, C.J. Joachain, K.M. Watson (1972); W. Williamson, Jr. and G. Foster (1975); M.R. Flannery, K.J. McCann (1974); C.J. Joachain, R.- Vanderpoortan (1973)) approaches, calculations (S. Geltman, M.B. Hidalgo (1971); A.D. Stauffer, L.A. Morgan (1975)); and the eikonal-Born series approach (F.W. Byron, Jr. and C.J. Joachain (1973); F.W. Byron, Jr. and K.J. Latour (1976); F.W. Byron, Jr. and C.J. Joachain (1977)); C.J. Joachain, K.H. Winters, L. Cartiaux, R.M. Mentezmorenoc (1977); F.W. Byron, Jr. and C. J. Joachain (1973)).

These reviews are concerned with the current state of the theory of scattering of electrons by atoms. It emphasizes the intermediate and high energy region, begining at the first excitation threshold and extending upward. The upper boundary of the intermediate energy region is not clearly defined; it is generally taken few times the ionization threshold. The energy range just above the first excitation threshold has been considerable recent interest to both theorists and experimenters. It is well known that the differential cross section (DCS), for elastic scattering of electrons by most of the atoms possess deep minima at one oor more then one electron impact energy and scattering angle (Kollath and Lucas).

The elastic differential cross section for hydrogen and helium exhibits a pronounced peak through out the intermediate energy region. Several approaches yields such a peak, but in order to describe it properly, it is essential to account for the polaraizability of the target atom.

The present study is aimed at suggesting yet another description of high -energy collisions. Prompted by the work -and success -of Byron and Joachain in their eikonal -Born series approach to medium -to high energy electron -atom collisions, and constitutes an extension of the earlier work of Yates (A.C. Yates (1974)). The primary purpose of the current analysis is to develop an alternative high -energy expansion of differential cross section in terms of the reciprocal powers of k_i (where $\hbar k_i$ is the momentum of the incident particle), through O (k_i^{-2}) , which is computationally tractable, yet derived from analogously treated second and third Born terms. Also the suggestion of anamalous behavior of the small -angle high -energy differential cross section in electron atom collisions (C.B.O. Mohr (1969)) can also be described. In the present work we have given complete description of the high energy approximation given by Yates (1974,1977,1979).

Electron hydrogen atom collision is one of the most fundamental problem of the atomic physics and has been studied numerous time both theoretically and experimentally. One of the most important aspects involved is that agreements between experiments and even the most sophisticated theories are not found fully satisfactory. Motivated by the above and considering the simplicity of the approach and from the observation presented in this chapter, one can conclude that the present approach would be more useful in studying many aspects of the scattering involving different atoms. We first made an attempt to study the refinement applied to the HHOB approximations to study the elastic scattering of electrons by hydrogen atom as a test case. Having obtain success for the hydrogen atom as a test case, we further extend it for the atomic target like helium and lithium atom also.

Thus, the thrust of this chapter is aimed as to outline briefly this method, but more importantly to describe a new way to obtain accurate scattering information at intermediate and high energies. We first describe the HHOB approximation in detail than we study the refinement of the approximation. Having obtain the analytical integral equation, we apply it to study the different case described above. At the end of the chapter we discuss the results obtain through the refinement in the HHOB approximations. We have made adequate comparison in the form of graphs and tables. We compare the results with the various experimental and theoretical results available.

HIGH ENERGY HIGHER ORDER BORN APPROXIMATION (HHOB THEORY) :

The HHOB (High Energy Higher Order Born) approximation has been shown to be a successful theoretical approach for analysing electron, positron scattering at intermediate and highenergies. There exists a vast literature which testifies the success of the HHOB theory. A review of the approximation, method used and the method of successful solution is given by A. C. Yates (1974, 1977, 1979).

In arriving the final expression of the free particle scattering amplitude in the HHOB approximation, a well known generalized Born series description is introduced and then transformed into a more convenient form in the analysis of the HHOB theory.

To begin with, let us confined to the specific case of an electron scattering with a neutral N- atom. Atomic units are used throught, except otherwise will be stated. Let $\vec{k}_i$, $\vec{k}_f$ and $q = \vec{k}_i - \vec{k}_f$ will denote respectively , the initial and final momenta of the scattered electron, and the momentum transfer to the target as a result of the collision. We consider the non -relativistic collision. Since we are interested in the intermediate and high energy regions, we shall first neglect the effects of the Pauli principle between the incident and target electrons, we may ignore the spin of the projectile. The indices i and f therefore label the momentum of the projectile together with the internal quantum numbers of the target. Corrections due to exchange is treated seperately.

We begin with the Born series, which is obtained if one solves the Lippmann -Schwinger equation by the perturbation technique. Hence Born series is a perturbative series in powers of the interaction potential. Consider the generalized Born series for the scattering amplitude which describes the collision of an electron with N -electron atom with the initial and final atomic states and energies given by (ψ_i, E_i) and (ψ_f, E_f) respectively is written. One can write the scattering amplitude from the Born series as,

$$f_{i\dashrightarrow f} = \sum_{n=1}^{\infty} f^{(n)}_{i\dashrightarrow f} \qquad (1),$$

where,

$$f^{(1)}_{i\dashrightarrow f} = -\frac{1}{2\pi} \int d\mathbf{r}_0 \, e^{i\mathbf{q}\cdot \mathbf{r}_0} V_{fi}(\mathbf{r}_0) \qquad (2),$$

where $\vec{r}_0$ is the coordinate of the projectile electron and r_i (i= 1, ... , z) is the position coordinate of the atomic electron.

The interaction potential between the incident electron and the target atom for N -electron is given by,

$$V = \sum_{i=1}^{z} \frac{1}{r_{i0}} - \frac{z}{r_0} \qquad (3),$$

with $r_{i0} = | \mathbf{r}_i - \mathbf{r}_0 |$.

The most convenient form for the second Born term i.e. $f^{(2)}_{i\to f}$ can be obtained as follows.

Consider the three terms of the Born series from equation (1.27) and the interaction potential V or V_d from (3) and the Green's function from equation (1.18) fro the evaluation of the second Born term of the Born series. A more convenient form can be obtained by transforming the interaction variables ($\vec{r}_0$,$\vec{r}_0'$) to the set ($\vec{r}_0$,Y), with Y = $\vec{r}_0 - \vec{r}_0'$ in the matrix elements of second and third Born terms, and replacing $\mathbf{y}$ by $\vec{r}_0'$, then the terms reducein the following form

$$f^{(2)}_{i\to f} = -\frac{1}{\pi} \sum_n \int d\mathbf{r}_0 \; e^{i\mathbf{q}\cdot\mathbf{r}_0} \; V_{fn}(\mathbf{r}_0)\, I_n \qquad (4),$$

where,

$$I_n = \int d\mathbf{r}_0' \; e^{i\,\mathbf{k}_i\cdot\mathbf{r}_0} \; V_{ni}(\mathbf{r}_0 - \mathbf{r}_0') \; G_n(\vec{r}_0) \qquad (5),$$

where $G_n(\vec{r}_0')$ is the Green's operator. The basic assumption involved are introduced by the transformation of variable $\vec{S} = \vec{k}' - \vec{k}_n$ in I_n where,

$$I_n = -\frac{1}{(2\pi)^3} \int d\mathbf{r}_0 \; e^{i(\mathbf{k}_i - \mathbf{k}_n)\cdot\mathbf{r}_0} \; V_{ni}(\mathbf{r}_0 - \mathbf{r}_0') \int \frac{d\mathbf{s}\; e^{i\mathbf{s}\cdot\mathbf{r}_0}}{(s^2 + 2\,\mathbf{s}\cdot\mathbf{k}_n - i\epsilon)} \qquad (6).$$

Since the difference of $\vec{k}'$ and $\vec{k}_n$ is very small, one can assume that V_{ni} is a slowly varying function over the distance of a wavelength of the scattering amplitude i.e. $k_n a >> 1$. Where " a "is the range of the potential V_{ni} . Further one can expand $(s^2 + 2\,\mathbf{s}\,\mathbf{k}_n - i\epsilon)^{-1}$ in powers of s^2, the d$\mathbf{s}$ integral of above equation can be obtained as

$$\int \frac{d\vec{s}\; e^{i\mathbf{s}\cdot\mathbf{r}_0'}}{s^2 + 2\mathbf{s}\cdot\mathbf{k}_n - i\epsilon} = \int \frac{d\mathbf{s}}{2\mathbf{s}\cdot\mathbf{k}_n - i\epsilon} \left[1 + \frac{D^2_{\vec{r}_0'}}{2\mathbf{s}\cdot\mathbf{k}_n - i\epsilon} \right] e^{i\vec{s}\cdot\vec{r}_0'} ,$$

where D is the differential operator with respect to $\vec{r}_0'$, and d$\mathbf{s}$= $ds_x\, ds_y ds_z$. In the above d$\mathbf{s}$ integral, $ds_x ds_y$ integrals can be evaluated by the use of definitions of delta functions and the ds_z integral can be evaluated using the contour integral techniques for the first and second poles (Boas,1966). Then the closed form of the d$\mathbf{s}$ integral can be written as

$$I_n = \frac{i}{2k_n}\int d\vec{r}_0'\; e^{-i(\vec{k}_i - \vec{k}_n)\circ\,\vec{r}_0}\; V_{ni}(\vec{r}_0 - \vec{r}_0') \left[1 + i\, D^2_{\mathbf{r}_0'} \frac{z_0'}{2k_n} \right]$$

$$\delta(\vec{b}_0')\; H(z_0')$$

$$= \frac{i}{2k_n} [I_{n1} + I_{n2}] \qquad (7),$$

where, $H(z_0')$ is the heavyside function. D is thedifferential operator with respect to $\vec{r}_0'$. d$\mathbf{s}$ integration is performed in a cylinderical polar coordinate system by choosing $\mathbf{k}_n$ as a polar axis and writing,$\vec{r}_0' = \vec{b}_0' + z_0'\hat{\mathbf{k}}_n$. The above equation can be further simplified by using the property of the the delta function,

$$I_{n1} = \int_{-\infty}^{+\infty} e^{-i(\mathbf{k}_i - \mathbf{k}_n)\cdot \hat{\mathbf{k}}_n z'_0} H(z'_0)\, V_{ni}(\mathbf{r}_0 - \mathbf{r}'_0)\, dz'_0 \Big|_{\mathbf{b}'_0} = 0$$

now considering the second part i.e. I_{n2}

$$I_{n2} = \frac{1}{2k_n} \int d\mathbf{r}_0\, e^{i(\mathbf{k}_i - \mathbf{k}_n)\, \mathbf{r}_0'}\, V_{ni}(\mathbf{r}_0 - \mathbf{r}_0')\, D^2_{\mathbf{r}}\, \delta(\mathbf{b}_0')\, H(z_0')$$

integrating by parts yields into,

$$= -\frac{1}{2k} \{\int d\mathbf{r}_0'\, D_{\mathbf{r}'} [\delta(\mathbf{b}_0)\, z_0' H(z_0')]\, e^{i(\mathbf{k}_i - \mathbf{k}_n)\cdot \mathbf{r}_0'} V_{ni}(\mathbf{r}_0 - \mathbf{r}_0')$$

$$- \int d\mathbf{r}_0'\, D_{\mathbf{r}_0'} [\delta(\mathbf{b}_0')\, z_0'\, H(z_0')]\, [e^{i(\mathbf{k}_i - \mathbf{k}_n)\cdot \mathbf{r}_0}\, V_{ni}(\mathbf{r}_0 - \mathbf{r}_0')]\}$$

again integrating by parts in the second term of the above expression. We have ,

$$= -\frac{1}{2k} \Big\{\int d\mathbf{r}_0' D_{\mathbf{r}_0'} [\, \delta(\mathbf{b}_o')\, z_0'\, H(z_0')\,]\, e^{i(\mathbf{k}_i - \mathbf{k}_n)\cdot \mathbf{r}_0'}\, V_{ni}(\mathbf{r}_0 - \mathbf{r}_0') -$$

$$[\int d\mathbf{r}_0' \delta(\mathbf{b}_0')\, z_0' H(z_0')\, D\mathbf{r}\, e^{i(\mathbf{k}_i - \mathbf{k}_n)\cdot \mathbf{r}_0'}\, V_{ni}(\mathbf{r}_0 - \mathbf{r}_0') - \int d\mathbf{r}_0'$$

$$\delta(\mathbf{b}_0')\, z_0'\, H(z_0') D^2_{\mathbf{r}_0'}\, e^{i(\mathbf{k}_i - \mathbf{k}_n)\cdot \mathbf{r}_0}\, V_{ni}(\mathbf{r}_0 - \mathbf{r}_0')\,]\, \}$$

$$= -\frac{1}{2k} \int d\mathbf{r}_0'\, \delta(\mathbf{b}_0')\, z_0' H(z_0')\, D^2_{\mathbf{r}_0'}\, e^{i(\mathbf{k}_i - \mathbf{k}_n)\cdot \mathbf{r}_0}\, V_{ni}(\mathbf{r}_0 - \mathbf{r}_0')$$

after the $D^2_{\mathbf{r}_0'}$ operation , the corresponding term of the I_{n2} and I_{n1} yields in I_n .

$$I_n = \frac{i}{2k} \int_{-\infty}^{+\infty} dz'_0 \; e^{i(k_i - k_n)\hat{k}_n \cdot z'_0} \; H(z'_0) \; \left[\left(1 + \frac{1}{2} \frac{z'_0}{k} \; O \right) \right.$$

$$\left. V_{ni}(r_0 - r'_0) \right] \Big|_{b'_0 = 0}$$

The basic approximations which we shall make is to replace $k_i - k_n$ by an average excitation energy difference Δ. We have

$$\begin{aligned} (k_i - k_n) \cdot \hat{k}_n &= k_i \cos\theta_{in} - k_n \\ &= k_i - k_n + O\,(k_i \theta_{in}^2) \\ &= k_i - k_n \end{aligned}$$

hence,

$$I_n = \frac{i}{2 k_n} \int_{-\infty}^{+\infty} dz'_o \;\; e^{\,i\beta_{in} z'_0} \;\; H(z'_0) \; [1 + i \frac{z'_0}{2k_n} \; D^2 r'_0]$$

$$V_{ni}(r_0 - r'_0) \Big|_{b'_0 = 0} \qquad (8),$$

where $\beta_{in} = k_i - k_n = \frac{\Delta E}{k}$, on using the energy conservation condition, the above expression embodies the central approximation.

The evaluation of the scattering amplitude can be performed in the cylinderical coordinate system by choosing in such a way that the z - axis is always perpendicular to $\underline{q}$. Thus $\underline{q}$ is two dimensional , and the position coordinate of the z + 1 electron can be written as $\vec{r}_i = \vec{b}_i + \vec{z}_i \; \hat{y}$, $i = 0, 1 \ldots, z$. Where $\hat{y}$ is a unit vector in the z -direction. Let X denotes the target coordinates. In order to simplify the 2nd order Born term it is necessary to take the fourier transform of interaction potential (3).

$$V(r_0,..,r_N) = \int d\underline{p}\; e^{i\underline{p}\cdot\underline{b}_0} \int_{-\infty}^{+\infty} dp_z\; e^{-ip_z\cdot z_0}\; \bar{V}(\vec{p}+p_z\hat{y}\,,\; \vec{r}_1\cdots\vec{r}_N) \qquad (9),$$

where,

$$V(\vec{p}+p_z\hat{y}\,,\, r_1,\ldots,r_N) = \frac{1}{2\pi^2(p^2+p_z^2)} \sum_{j=1}^{N} (e^{i\vec{p}\cdot\vec{b}_j + ip_z z_j} - 1) \qquad (10).$$

The general form of the $V_{ni}(\vec{r}_0')$ in the above expression has been defined as,

$$V_{nm}(r_0) = \langle \psi_n(x) \mid V(x) \mid \psi_m(x) \rangle \qquad (11).$$

Substituting (9,8) in the 2nd Born term (4), the coresponding second Born term can be written as

$$f^{(2)}_{HEA} = \frac{i}{2\pi k} \sum \int dr_0\; e^{i\underline{q}\cdot r_0}\; V_{fn}(r_0) \int_{-\infty}^{+\infty} dz_0'\; H(z_0')\; \bar{e}^{i\beta_{in} z_0'}$$

$$\left[V_{ni}(r_0 - z_0'\hat{y}) + \frac{iz_0'}{2k} D^2_{r_0'} V_{ni}(r_0 - r_0') \Big|_{\underline{b}_0'=0} \right] \qquad (12).$$

The infinite summation over atomic states can be treated, with varying degree of accuracy, by any one of the several approximate methods (Woolings and McDowell 1972; Byron and Joachain,1977; A. C. Yates,1978).

$$\sum_n V_{fn}(r_0)\, V_{ni}(r_0 - r_0') = V_f(r_0)\, V_i(r_0 - r_0') \qquad (13),$$

Yates (1978) repleced $\beta_{in} = \frac{\Delta E}{k}$, where ΔE is the average energy transferred to intermediate atomic states during the course of collision. The summation in the above equation runs over all the target states, except the ground state. ΔE is taken as the average excitation energy because the energy region to be deal is only the intermediate and high energy region. Hence, according to Yates(1974 -79) 2nd Born term can be defined as,

$$f^{(2)}_{HEA} = -\frac{i}{2\pi k_i} \int d\mathbf{r}_0 \, e^{i\mathbf{q}\cdot\mathbf{r}_0} \quad \langle \psi_f \,|\, V(\mathbf{r}_0 , \ldots \mathbf{r}_z)$$

$$\int dz'_0 \, H(z'_0) \; e^{-i\beta_i z'_0} \; [\, V(\mathbf{r}_0 - z'_0 \hat{y} , \vec{r}'_0, \ldots, \vec{r}'_z) \; + $$

$$\frac{iz'_0}{2k} \; D^2_{\mathbf{r}_0} V(\mathbf{r}_0 - \mathbf{r}'_0 , \mathbf{r}_1, \ldots , \mathbf{r}_z) \,|_{\mathbf{b}'_0 = 0} \;]\; | \psi_i \rangle \qquad (14).$$

On using equation (9) and carrying out $D^2_{\mathbf{r}'_0}$ operation, the preceding result can be rewritten as,

$$f^{(2)}_{HEA} = \frac{i}{2\pi k} \int d\mathbf{p} \int_{-\infty}^{+\infty} dp_z \int d\mathbf{p}' \int_{-\infty}^{+\infty} dp'_z \;\langle \psi_f \,|\, \bar{V}(\mathbf{p} + p_z \hat{y} \,; \mathbf{r}_1 \cdots \mathbf{r}_N)$$

$$\bar{V}(\mathbf{p}' + p'\hat{y} \,; \mathbf{r} \ldots \mathbf{r}) \,|\, \psi \rangle \int d\mathbf{p} \;\; e^{i(\mathbf{q} - \mathbf{p} - \mathbf{p}')\mathbf{b}_0}$$

$$\int_{-\infty}^{+\infty} dp_z \;\; e^{i(p_z - p'_z) z'_0} \quad \left(1 + \frac{p'^2 + p_z^2}{2k} \right) \;\; \frac{\partial}{\partial \beta_i} \int_{-\infty}^{+\infty} dz'_0 \; H(z'_0)$$

$$e^{-i(p'_z - \beta_i)\, z'_0}$$

$$= \frac{4\pi^3}{k} \left[\, i \int d\mathbf{p} \left(1 + \frac{1}{2k} \frac{\partial}{\partial \beta_i} \left(p^2 + \beta_i^2 \right) \right) U^{(2)}_{fi}(\mathbf{q} - \mathbf{p} - \beta \hat{y} \;;\; \mathbf{p} + \beta_i \hat{y})\right.$$

$$- \frac{1}{\Pi} \mathcal{P} \int d\underline{p} \int_{-\infty}^{+\infty} dP_z \left(1 + \frac{\underline{P}^2 + P_z^2}{2k} \frac{\partial}{\partial \beta_i} \right) \frac{1}{P_z - \beta_i}$$

$$U_{fi}^{(2)} (\underline{q} - \underline{p} - p_z \hat{y} \; ; \; \underline{p} + p_z \hat{y})] \qquad (15),$$

where $\mathcal{P}$ is the principal integral value. The general form of $U_{fi}^{(2)}$ (.......;........) can be written as

$$U_{fi}^{(2)} (\underline{p} + p_z \hat{y} \; ; \; \underline{p}' + p_z' \hat{y}) =$$
$$\langle \psi_f | \bar{V} (\underline{p} + p_z \hat{y} ; \; \underline{r}_1 \ldots \underline{r}_N) \bar{V} (\underline{p}' + p_z' \hat{y} \; ; \; \underline{r}_1 \ldots \underline{r}_N) \, | \psi_i \rangle$$

In arriving at the final form of equation (15), it has been necessary to use the usual integral representations of the one -and two -dimensional δ functions, and the additional result of B. Friedman (1969)

$$\int_{-\infty}^{-\infty} dx \, e^{-i\alpha x} H(x) = \Pi \, \delta \, (\alpha) - i \, \mathcal{P} \left(\frac{1}{\alpha} \right).$$

High energy approximation to the differential cross section is valid through $O(k_i^{-2})$, is sought. Since the first Born approximation term is real and of zeroth order in K_i^{-1}, the imaginary part of the scattering amplitude is required only through $O(k_i^{-1})$, whereas the real part is needed through $O(k_i^{-2})$. Further, it should be apparant from equation (8) and definition of β_i that the leading k_i dependence of the various terms of equation (15) will be of order no lower than that explicitly given. Also, except for a possible complex phase factor common to all terms of the Born series, $U_{fi}^{(2)}$ is effectively real. Consistents with these comments, the real and imaginary parts of $f_{HEA}^{(2)}$ can be written as,

$$\mathrm{Re}\,1\; f_{HEA}^{(2)} = \frac{4\pi^2}{k_i} \mathcal{P} \int d\underline{p} \int_{-\infty}^{+\infty} dp_z \,(P_z - \beta_i)^{-1}\, U_{fi}^{(2)}(\underline{q} - \underline{p} - \underline{p}\hat{y}\,; p + p_z\hat{y}) \qquad (16),$$

$$\mathrm{Re}\,2\; f_{HEA}^{(2)} = - \frac{2\pi^2}{k_i^2} \frac{\partial}{\partial \beta_i} \mathcal{P} \int d\underline{p} \int_{-\infty}^{+\infty} dp_z (p_z - \beta_i)^{-1} \,(P^2 + P_z{}^2)$$

$$U_{fi}^{(2)}\; (\underline{q} - \underline{p} - p_z\hat{y}\,;\, \underline{p} + p_z\hat{y}\,) \qquad (17),$$

$$\mathrm{Im}\; f_{HEA}^{(2)} = \frac{4\pi^3}{k_i} \int d\underline{p}\;\; U_{fi}^{(2)}\; (\underline{q} - \underline{p} - \beta_i\hat{y}\; ;\; \underline{p} + \beta_i\hat{y}\;) \qquad (18).$$

Equations (16,17,18) constitute the HHOB approximation to the second Born term. If β_i is set equal to zero in equations (16) vanishes, and the leading term of real part of $f_{HEA}^{(2)}$ is then propertional to $O(k_i^{-2})$. Similarly, the imaginary part of $f_{HEA}^{(2)}$ identically becomes Glauber's estimate of the second Born term Yates (1973,1974). Where $\mathcal{P}$ is the principal value.

Similarly evaluation of the third Born term can also be performed. the differential cross section (DCS) for the direct scattering can be written from the expression of the scattering amplitude. The scattering amplitude in the HHOB theory is given as,

$$f_{HHOB}^{d} = f_{i\dashrightarrow f}^{(1)} + \mathrm{Rel1}\; f_{HEA}^{(2)} + \mathrm{Rel2}\; f_{HEA}^{(2)} + \mathrm{Rel3}\; f_{HEA}^{(3)} + i\; \mathrm{Im}\; f_{HEA}^{(2)} \qquad (20),$$

where the meanings of the symbols used in the above expression are given above and in Yates (1979). Which represents a wholly consistent approximation, treating all terms equivalently. Since the direct evaluation of the third Born term is extremely difficult, and in the light of the the above discussion concerning the properties of the Born term at all angles by the corresponding Glauber third term.

Hence we write the expression of the scattering amplitude as,

$$f^{d}_{HHOB} = f^{(1)}_{i-->f} + \mathrm{Re1}\, f^{(2)}_{HEA} + \mathrm{Re2}\, f^{(2)}_{HEA} + i\, \mathrm{Im}\, f^{(2)}_{HEA} + f^{(3)}_{GES} \qquad (21).$$

The differential cross section (DCS) for the elastic scattering through $O(K_i^{-2})$ for fixed q follows

$$\frac{d\sigma}{d\Omega} = \frac{K_f}{K_i} \mid F_{HHOB} \mid^2 \qquad (22).$$

The total collisional cross sections are obtained using the Optical theorem (Taylor , 1972). The TCS's expressed as,

$$\sigma^{tot} = \frac{4\Pi}{k^2} \; \mathrm{Im}\, f(\theta = 0) \qquad (23).$$

The assumption in high energy approximation (HEA) were made along with the small angle approximation of Glauber (1959). HEA concerned with the elucidation of character of second and third Born terms for short wavelength (k a >> 1) and for small momentum transfers (small angles). The partial expansion of equation (6) was necessitated by a desire to include a plausible and reasonably accurate discription of virtual excitations (target polarization). It was also shown for elastic scattering of electrons by hydrogen atom, that when q --> 0 for large k_i the real and imaginary parts of the second Born term approaches to the corresponding terms of the simplified Born approximation (byron and Joachain, 1977). The difference between the HEA and simplified Born approximation is the term of order k_i^{-2} when q = 0 in HEA. It was also concluded that the HEA provides an accurate discription of these terms for small q.

REFINEMENT IN THE HIGH ENERGY HIGHER ORDER BORN APPROXIMATION :

Inspired by the success of the HHOB theory applied to study the elastic scattering of electrons by a target atoms like hydrogen, helium and lithium (Rao and Desai, 1982 ; Suja and Desai, 1988). In this section of we extend the same process but after applying certain refinement to the second Born term. We discuss the HHOB approximation by applying certain refinement in the expression of the second Born term in the HHOB theory. As a part of the refinement of the above procedure we include certain number of low lying energy states in the sum on n appearing in equation (12), and then we perform the sum of the remaining states according to Yates (1979). The proposed refinement is tested for the hydrogen, helium and lithium. We find that there has been a considerable amount of improvement in the cross sections when compared with the other theoratical and experimental results. Since we are considering the non -relativistic collision, we can neglect the Pauli principle as well as, we can neglect the spin of the projectile electron. The effect of the exchange is treated separately using the Ochkur approximation (1963). First we derive the terms of the HHOB theory using the refinement. Then we study the effect of such refinement on the cross section. We first discuss the results obtain for the case of hydrogen atom and then we take up the helium problem in the similar way. At the end we discuss the results of the cross sections obtain for the case of lithium atom, where we treat the lithium atom as a three electron system. We have made an adequate comparison in the form of graphs and tables. As a part of the refinement by means of including certain low lying energy states in the calculation we rewrite the equation (12) as,

$$f^{(2)}_{HEA} = \frac{i}{2\pi k_i} \int d\mathbf{r}_0 \, e^{i\mathbf{q}\cdot\mathbf{r}_0} \, V_{f0}(\mathbf{r}_0) \int_{-\infty}^{+\infty} dz'_0 \, H(z'_0) \, e^{i/\beta \, i z'_0} \, V_{0i}(\mathbf{r}_0 - z'_0\hat{y})$$

$$+ \frac{i}{2\pi k_i} \sum_{n=0} \int d\mathbf{r}_0 \, e^{i\mathbf{q}\cdot\mathbf{r}_0} \, V_{fn}(\mathbf{r}_0) \int_{-\infty}^{+\infty} dz'_0 \, H(z'_0) \, e^{i/\beta \, in z'_0} \, V_{ni}(\mathbf{r}_0 - z'_0\hat{y}) \qquad (25),$$

$$= \frac{i}{2\pi k_i} \int d\mathbf{r}_0 \, e^{i\mathbf{q}\cdot\mathbf{r}_0} \, V_{f0}(\mathbf{r}_0) \int_{-\infty}^{+\infty} dz'_0 \, H(z'_0) \, e^{i/\beta \, i0 z'_0} \, V_{0i}(\mathbf{r}_0 - z'_0\hat{y})$$

$$+ \frac{i}{2\pi k_i} \sum_{n=0} \int d\mathbf{r}_0 \, e^{i\mathbf{q}\cdot\mathbf{r}_0} \, V_{fn}(\mathbf{r}_0) \int_{-\infty}^{+\infty} dz'_0 \, H(z'_0) \, e^{i/\beta \, in z_0'} \, V_{ni}(\mathbf{r}_0 - z'_0\hat{y})$$

$$- \frac{i}{2\pi k_i} \int d\mathbf{r}_0 \, e^{i\mathbf{q}\cdot\mathbf{r}_0} \, V_{f0}(\mathbf{r}_0) \int_{-\infty}^{+\infty} dz'_0 \, H(z'_0) \, e^{i/\beta \, in z_0'} \, V_{0i}(\mathbf{r}_0 - z_0'\hat{y})$$

It can be written further as,

$$= \frac{i}{2\pi k_i} \int d\mathbf{r}_0 \quad e^{i\mathbf{q}\cdot\mathbf{r}_0}\, V_{f0}(\mathbf{r}_0) \int_{-\infty}^{+\infty} dz_0'\, H(z_0')\, e^{i/\beta\, i0^{z_0'}} \quad V_{0i}(\mathbf{r}_0 - z_0'\hat{y})$$

$$+ \frac{i}{2\pi k_i} \int d\mathbf{r}_0 \quad e^{i\mathbf{q}\cdot\mathbf{r}_0}\, V_f(\mathbf{r}_0) \int_{-\infty}^{+\infty} e^{i/\beta\, in^{z_0'}}\, H(z_0')\, dz_0' \quad V_{0i}(\mathbf{r}_0 - z_0'\hat{y})$$

$$- \frac{i}{2\pi k_i} \int d\mathbf{r}_0\, e^{i\mathbf{q}\cdot\mathbf{r}_0} \quad V_{f0}(\mathbf{r}_0) \int_{-\infty}^{+\infty} dz_0'\, H(z_0')\, e^{i/\beta\, in^{z_0'}} \quad V_{0i}(\mathbf{r}_0 - z_0'\hat{y})$$

$$= \frac{i}{2\pi k_i} \int d\mathbf{r}_0\, e^{i\mathbf{q}\cdot\mathbf{r}_0} < \psi_f \mid V(\mathbf{r}_0, \ldots, \mathbf{r}_z) \mid \psi_f > \int_{-\infty}^{+\infty} dz_0' \quad H(z_0')$$

$$< \psi_i \mid V(\mathbf{r}_0 - z_0'\hat{y};\, \mathbf{r}_1, \ldots, \mathbf{r}_z) \mid \psi_i > \; + \; \frac{i}{2\pi k_i} \int d\mathbf{r}_0\, e^{i\mathbf{q}\mathbf{r}_0} \quad < \quad \psi_f \quad \mid$$

$$V(\mathbf{r}_0, \ldots \mathbf{r}_z) \int_{-\infty}^{+\infty} dz_0'\; e^{-i/\beta\, in^{z_0'}} < \psi_i \mid V(\mathbf{r}_0 - z_0'\hat{y};\, \mathbf{r}_1, \ldots, \mathbf{r}_z) \mid \psi_i >$$

$$- \frac{i}{2\pi k_i} \int d\mathbf{r}_0\, e^{i\mathbf{q}\cdot\mathbf{r}_0} < \psi_f(\mathbf{r}_0, \ldots \mathbf{r}_z) \mid V \mid \psi_f(\mathbf{r}_0, \ldots, \mathbf{r}_z) > \int_{-\infty}^{+\infty} H(z_0')$$

$$dz_0'\; e^{-i/\beta\, in^{z_0'}} \quad < \psi_i \mid V \mid \psi_i >$$

further,

$$f^{(2)}_{HEA} = \frac{i}{2\pi k_i} \int d\mathbf{r}_0\, e^{i\mathbf{q}\cdot\mathbf{r}_0}\, V_{00}(\mathbf{r}_0, \ldots, \mathbf{r}_N) \int_{-\infty}^{+\infty} dz_0'\, V_{00}(\mathbf{r}_0 - z_0'\hat{y}) + \frac{i}{2\pi k_i}$$

$$\int d\mathbf{r}_0\, e^{i\mathbf{q}\cdot\mathbf{r}_0} < \psi_0 \mid V(\mathbf{r}_0, \ldots, \mathbf{r}_N) \int_{-\infty}^{+\infty} dz_0'\, H(z_0') \quad e^{-i/\beta\, i^{z_0'}} \quad V(\mathbf{r}_0 \; - z_0'\hat{y};$$

$$\mathbf{r}_1, \ldots, \mathbf{r}_N) \mid \psi_0 > \; - \; \frac{i}{2\pi k_i} \int \; d\mathbf{r}_0 \quad e^{i\mathbf{q}\cdot\mathbf{r}_0} \quad V_{00}(\mathbf{r}_0, \ldots, \mathbf{r}_N) \int_{-\infty}^{+\infty} dz_0'$$

$$H(z_0')\, e^{-i/\beta\, i^{z_0'}} \quad V_{00}(\mathbf{r}_0 - z_0'\hat{y};\, \mathbf{r}_1, \ldots, \mathbf{r}_N)$$

$$= \frac{i}{2\pi k_i}\int d\underline{r}_0\, e^{i\underline{q}\cdot\underline{r}_0}\, V_{00}(\underline{r}_0,\ldots,\underline{r}_N) \int_{-\infty}^{+\infty} dz_0'\, V_{0i}(\underline{r}_0 - z_0'\hat{y}) + \frac{i}{2\pi k_i}$$

$$\int d\underline{r}_0\, e^{i\underline{q}\cdot\underline{r}_0} < \psi_0 \mid V(\underline{r}_0,\ldots\underline{r}_N) \int_{-\infty}^{+\infty} dz_0'\, H(z_0')\, e^{-i\beta_i z_0'}\ V(\underline{r}_0 - z_0'\hat{y};$$

$$\underline{r}_1,\ldots\underline{r}_N) \mid \psi_0 > - \frac{i}{2\pi k_i}\int d\underline{r}_0\, e^{i\underline{q}\cdot\underline{r}_0}\, V_{00}(\underline{r}_0,\ldots,\underline{r}_N) \int_{-\infty}^{+\infty} dz_0'\, H(z_0')$$

$$e^{-i\beta_i z_0'}\ dz_0'$$

now the first term of the above equation reduces to a form,

$$= \frac{i}{2\pi k_i}\int d\underline{r}_0\, e^{i\underline{q}\cdot\underline{r}_0} < 0 \mid V \mid 0 > \int_{-\infty}^{+\infty} dz_0' < 0 \mid V \mid 0 >$$

further it can be written as

$$= \frac{i}{2\pi k_i}\int \int_{-\infty}^{+\infty} d\underline{b}_0\ dz_0'\ e^{i\underline{q}\cdot\underline{b}_0 + iq_z z_0} \ \langle 0| \int d\underline{p}\ e^{-i\underline{p}\cdot\underline{b}_0} \int_{-\infty}^{+\infty} dp_z$$

$$e^{-ip_z z_0}\ \bar{V}(\underline{p} + p_z\hat{y};\ \underline{r}_1,\ldots\underline{r}_N)\ |0\rangle \int_{-\infty}^{+\infty} dz_0' \ \langle 0| \int d\underline{p}'$$

$$e^{-i\underline{p}'\underline{b}_0} \int_{-\infty}^{+\infty} dp_z'\ e^{-ip_z'(z_0 - z_0')}\ \bar{V}(\underline{p}' + p_z\hat{y};\ \underline{r}_1,\ldots\underline{r}_N) \mid 0 >$$

$$= \frac{i}{2\pi k_i}\int d\underline{p} \int d\underline{p}' \int d\underline{b}_0\ e^{i(\underline{q}-\underline{p}-\underline{p}')\underline{b}_0}\ V_{00}(\underline{p} + p_z\hat{y}) \int_{-\infty}^{+\infty} dz_0'$$

$$\int_{-\infty}^{+\infty} dp_z \int_{-\infty}^{+\infty} dp_z'\ e^{-i(p_z + p_z')z_0}\ e^{ip_z' z_0'}\ H(z_0')$$

using the properties of the Dirac delta functions properties, we have

$$= \frac{i}{2\pi k_i}\int d\underline{p} \int d\underline{p}'\ (2\pi)^2\ V_{00}(\underline{p} + p_z\hat{y})\ \delta\ (\underline{q} - \underline{p} - \underline{p}') \int_{-\infty}^{+\infty} dz_0'$$

$$V_{00}(\underline{p}' + p_z\hat{y}) \int_{-\infty}^{+\infty} dz_0'\ (2\pi)^2\ \delta(z_0')\ \delta\ (z_0' - z_0)$$

$$= \frac{i}{2\pi k_i} \int d\underline{p}\; (2\pi)^2\, \bar{V}(\underline{p} + p_z\hat{y}) \int d\underline{p}'\, \delta(\underline{q} - \underline{p} - \underline{p}') \int_{-\infty}^{+\infty} dz_0' \int_{-\infty}^{+\infty} dp_z$$

$$\int_{-\infty}^{+\infty} dp_z'\; \bar{V}(\underline{p}' + p_z\hat{y})\; (2\pi)\; \delta(p_z)\; dz_0'\;\; e^{i(p_z - p_z')\, z_0'}\; H(z_0')$$

This will only survive when $\underline{p}' = 0$ or $\underline{p}' = \underline{p}$, i.e. property of the Dirac delta function. Hence,

$$= \frac{i}{2\pi k_i}(2\pi)^3 \int \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \bar{V}_{00}(\underline{q} - \underline{p},\; p_z\hat{y})\;\; e^{i(p_z' - p_z)z_0'}\;\; H(z_0')\;\; d\underline{p}$$

$$dp_z'\; dz_0'$$

further,

$$= \frac{i}{2\pi k_i} \int d\underline{p} \int d\underline{p}'\;\; V_{00}(\underline{p} + p_z\hat{y}) \int_{-\infty}^{+\infty} dz_0\;\; \bar{V}_{00}(\underline{p}' + p_z'\hat{y})\;\;\; e^{ip_z z_0'}$$

$$e^{-i(p_z + p_z')\;\; z_0'} \int_{-\infty}^{+\infty} dz_0'\;\; H(z_0') \int_{-\infty}^{+\infty} dp_z \int_{-\infty}^{+\infty} dp_z' \int d\underline{b}_0$$

$$e^{i(\underline{q} - \underline{p} - \underline{p}')\cdot\underline{b}_0}$$

$$= \frac{(2\pi)^2 i}{2\pi k_i} \int d\underline{p} \int d\underline{p}'\; \bar{V}_{00}(\underline{p} + p_z\hat{y}) \int_{-\infty}^{+\infty} dz_0\; \bar{V}_{00}(\underline{p}' + p_z\hat{y}) \int_{-\infty}^{+\infty} dz_0'$$

$$H(z_0') \int_{-\infty}^{+\infty} dp_z \int_{-\infty}^{+\infty} dp_z'\;\; \delta(\underline{q} - \underline{p} - \underline{p}')\; e^{ip_z}\;\; e^{i(p_z + p_z'\hat{y})\; z_0'}$$

$$= \frac{(2\pi)^2 i}{2\pi k_i} \int d\underline{p}\; \bar{V}_{00}(\underline{p} + p_z\hat{y}) \int_{-\infty}^{+\infty} dz_0 \int_{-\infty}^{+\infty} dz_0'\; H(z_0') \int_{-\infty}^{+\infty} dp_z\; e^{-ip_z z_0}$$

$$\int dp_z'\; e^{-i(p_z'(z_0 - z_0')}\;\; \bar{V}_{00}(\underline{q} - \underline{p} + p_z'\,\hat{y})$$

$$= \frac{(2\pi)^2 i}{2\pi k_i} \int d\underline{p}\; \bar{V}_{00}(\underline{p} + p_z\hat{y}) \int_{-\infty}^{+\infty} dz_0\; e^{-ip_z z_0 - ip_z'} \int_{-\infty}^{+\infty} dz_0'\; H(z_0')$$

$$\int_{-\infty}^{+\infty} dp_z'\;\; e^{ip_z' z_0'}\;\; \bar{V}_{00}(\underline{q} - \underline{p} + p_z'\hat{y}) \int_{-\infty}^{+\infty} dp_z$$

$$= \frac{(2\pi)^2 2\pi i}{2\pi k_i} \int d\underline{p} \; \overline{V}_{00}(\underline{p} + p_z\hat{y}) \; \delta(p_z + p_z') \int_{-\infty}^{+\infty} dz_0' \; H(z_0') \int_{-\infty}^{+\infty} dp_z$$

$$\int_{-\infty}^{+\infty} dp_z' \; e^{ip_z'z_0'} \; \overline{V}\,(\underline{q} - \underline{p} + p_z'\hat{y})$$

$$= \frac{(2\pi)^2 2\pi i}{2\pi k_i} \int d\underline{p} \; V_{00}(\underline{p} - p_z\hat{y}) \int_{-\infty}^{+\infty} dz_0' \; H(z_0') \int_{-\infty}^{+\infty} dp_z' \; e^{ip_z'z_0'}$$

$$\overline{V}(\underline{q} - \underline{p},\, p_z'\hat{y})$$

further,

$$= \frac{(2\pi)^2 2\pi i}{2\pi k_i} \int d\underline{p} \; V_{00}(\underline{p} - p_z'\hat{y}) \int dp_z' \; \overline{V}(\underline{q} - \underline{p},\, p_z\hat{y}) \int_{-\infty} dz_0'$$

$$H(z_0') \; e^{ip_z'z_0'}$$

Hence using the property of the heaviside function and Dirac delta function. We can write the above as,

$$= \frac{(2\pi)^2 2\pi i}{2\pi k_i} \int d\underline{p} \; V_{00}(\underline{p} - p_z'\hat{y}) \int dp_z' \; \overline{V}(\underline{q} - \underline{p}, p_z'\hat{y}) \; [\pi\,\delta(-p_z') - i\,\mathcal{P}$$

$$(\frac{1}{-p_z'})\,]$$

We can simplify it further and we can write the above further as,

$$= (2\pi)^3 \int\int d\underline{p}\, dp_z,\; \overline{V}(\underline{\Delta} - \underline{p} + p_z, \hat{y}) \; \overline{V}(\underline{p} - p_z, \hat{y}) \; [\; \pi\,\delta(p_z' - \beta_i) - i\,\mathcal{P}$$

$$(\frac{1}{-p_z'})\,]$$

$$= (2\pi)^3 \int\int d\underline{p}\, dp_z,\; \overline{V}(\underline{\Delta} - \underline{p} + p_z, \hat{y}) \; \overline{V}(\underline{p} - p_z, \hat{y}) \; \delta(p_z' - \beta_i) \; -$$

$$(2\pi)^3 \int\int d\underline{p}\, dp_z,\; \overline{V}(\underline{\Delta} - \underline{p} + p_z, \hat{y}) \; \overline{V}(p - p_z, \hat{y}) \; i \; \mathcal{P}\,(\frac{1}{p_z - \beta_i})$$

$$= 8\pi^4 \int\int d\underline{p}\, dp_{z'}\, \bar{V}(\Delta - \underline{p} + p_z,\hat{y})\, \bar{V}(\underline{p} - p_z,\hat{y})\, \delta(p_{z'} - \beta_i) - 8\pi^3 i\, \mathcal{P}$$

$$\int dp_{z'} (p_z' - \beta_i)^{-1} \int d\underline{p}\, \bar{V}(\Delta - \underline{p} + p_z,\hat{y})\, \bar{V}(p - p_z,\hat{y})$$

This will only survive for $\beta_i = 0$. Hence,

$$= \frac{(2\pi)^2 2\pi i}{2\pi k_i} \int d\underline{p}\, V_{00}(p)\, \bar{V}(\underline{q} - \underline{p}) + \frac{(2\pi)^2 (2\pi)}{2\pi k_i}\, \mathcal{P} \int \int d\underline{p}\, dp_{z'}\, \frac{\bar{V}_{00}(\underline{p} - p_z,\hat{y})\, \bar{V}_{00}(\underline{q} - \underline{p}, p_z,\hat{y})}{-p_z'}$$

The contribution of the second term must be zero. Since cosine varies from -1 to +1. Hence the second term reduces to zero. Finally,

$$f_{HEA}^{(2)} = \frac{i}{2\pi k_i} \int d\underline{r}_0\, e^{i\underline{q}\underline{r}_0}\, V_{00}(\underline{r}_0,\ldots,\underline{r}_N) \int_0^{\infty} dz_0'\, V_{00}(\underline{r}_0 - z_0'\hat{y}) +$$

$$\frac{i}{2\pi k_i} \int d\underline{r}_0\, e^{i\underline{q}\underline{r}_0} < \psi_0 \mid V(\underline{r}_0,\ldots,\underline{r}_N) \int_{-\infty}^{+\infty} dz_0'\, H(z_0')\, e^{-i\beta_i z_0'}$$

$$V(\underline{r}_0 - z_0'\hat{y}, \underline{r}_1,\ldots,\underline{r}_N) \mid \psi_0 > - \frac{i}{2\pi k_i} \int d\underline{r}_0\, e^{i\underline{q}\underline{r}_0}\, V_{00}(\underline{r}_0,\ldots \underline{r}_N)$$

$$\int_{-\infty}^{+\infty} e^{-i\beta_i z_0'}\, H(z_0')\, V_{00}(\underline{r}_0 - z_0'\hat{y}, \underline{r}_1,\ldots,\underline{r}_N)\, dz_0'$$

Finally the above equation can be simplified further in the following form.

$$f_{HEA}^{(2)} = \frac{4\pi^3}{k_i} i \int U_{st}^{(2)}(\underline{q} - \underline{p}, \underline{p}')\, d\underline{p} + \frac{4\pi^3}{k_i} i \int d\underline{p}$$

$$U_{00}^{(2)}(\underline{q} - \underline{p} - \beta_i \hat{y} ; \underline{p} + \beta_i \hat{y}) - \frac{4\pi^3}{k_i} i \int d\underline{p}$$

$$U_{st}^{(2)}(\underline{q} - \underline{p} - \beta_i \hat{y} ; \underline{p} + \beta_i \hat{y}) - \frac{4\pi^2}{k_i} \mathcal{P} \int d\underline{p}$$

$$\int_{-\infty}^{+\infty} \frac{dp_z}{p_z - \beta_i} U_{00}^{(2)} (\underline{q} - \underline{p} - p_z\hat{y} ; \underline{p} + p_z \hat{y}) + \frac{4\pi^2}{k_i}$$

$$\mathcal{P} \int d\underline{p} \int_{-\infty}^{+\infty} \frac{dp_z}{p_z - \beta_i} U_{st}^{(2)} (\underline{q} - \underline{p} - p_z\hat{y} ; \underline{p} + p_z\hat{y})$$

$$- \frac{2\pi^2}{k_i^2} \frac{\partial}{\partial \beta_i} \mathcal{P} \int d\underline{p} \int_{-\infty}^{+\infty} \frac{dp_z}{p_z - \beta_i} (p^2 + p_z^2)$$

$$U_{00}^{(2)} (\underline{q} - \underline{p} - p_z\hat{y} ; \underline{p} + p_z \hat{y}) + \frac{2\pi^2}{k_i^2} \frac{\partial}{\partial \beta_i} \mathcal{P} \int d\underline{p}$$

$$\int \frac{dp_z}{p_z - \beta_i} U_{st}^{(2)} (\underline{q} - \underline{p} - p_z \hat{y} ; \underline{p} + p_z \hat{y}).$$

We use this equation and the form of the potential to get the terms in the scattering amplitude through $O(k_i^{-2})$ for the calculation of the differential cross sections (DCS). We use the form of $U_{fi}^{(2)}(\ldots\ldots; \ldots\ldots)$ given below,

$$U_{fi}^{(2)} (x, X ; y, Y) = \langle \psi_f \mid \bar{V} (x, X) \bar{V}(y, Y) \mid \psi_i \rangle$$

We now use the form of interaction potential (3) and the corresponding wave function for the target atom to evaluate the above terms of the HHOB theory i.e., set of equations (2, 16, 17, 18,). The above equation is then written in the form of the integral terms such as $I_1(\ldots\ldots)$, $I_1'(\ldots\ldots)$, $I_2(\ldots)$ and so on. The evaluation of this integral is explained in detail in the appendix.

$$= - \frac{i}{2\Pi k_i} \sum_{i,j=1}^{N} \gamma_i \gamma_j \; I_1(q^2, \lambda_i^2, \lambda_j^2) + \frac{4}{k_i \Pi^2} \frac{\partial}{\partial \lambda}$$

$$[- \frac{1}{\lambda^2} (2 I_2(\beta_i, \lambda^2) - \frac{q^2}{q^2+\lambda^2} I_2(\beta_i, 0))] + \frac{1}{2 k_i} \frac{\partial}{\partial \beta_i}$$

$$(\frac{1}{q^2+\lambda^2} I_3 (\beta_i, 0) + \frac{1}{\lambda^2} I_3 (\beta_i, \lambda_i^2) - I_2(\beta_i, \lambda^2))$$

$$- \frac{4i}{\Pi k_i} \frac{\partial}{\partial \lambda} \frac{1}{\lambda^2} (2 I_2 (\beta_i^2 . \lambda^2) - \frac{q^2}{q^2 + \lambda^2} I_1(\beta_i^2 . 0))$$

$$+ \frac{i}{\Pi k_i} \sum_{i,j=1}^{N} \gamma_i \gamma_j \; I_1(q^2, \beta_i^2, \lambda_i^2, \lambda_j^2) - \frac{1}{k_i \Pi^2}$$

$$\sum_{i,j=1}^{N} \gamma_i \gamma_j \; [I_4(\beta_i^2, \lambda_i^2, \lambda_j^2) + \frac{D'}{2 k_i} (I_3(\beta_i, \lambda^2)$$

$$- \lambda_i^2 \; I_4 (\beta_i^2, \lambda_i^2, \lambda_j^2))] \qquad (26),$$

where D' is the differential operator with respect to λ's. The λ's and γ's are the constants and they are defined for various target atoms in each section the values are also given. We now apply this refinement first to the hydrogen atom as a test case. Contribution of the imaginary term will be due to the term corresponding to $\beta = 0$, static part plus the term corresponds to finite β and also corresponds to direct interaction potential V_d or V after the refinement is given to the HHOB approximation. Similarly the real part will also have the contribution due to finite β value, $\beta = 0$ and due to the interaction potential V or V_d. Here the term corresponds to $\beta = 0$ will have the contribution corresponds to the static part. We use the Cox -Bonhamm parameters for the static part. We calculate the terms individually and then we perform the final sum. We employ certain numerical techniques for the evaluation of the integrals and to do the computational work. The value of β is taken accurate measured experimentally.

Elastic scattering of electrons by the ground (1S) state of hydrogen atom :

Electron scattering from the atomic hydrogen (H; z = 1) is one of the most basic problems in atomic physics. The wave function of the hydrogen is known exactly, so there can be no uncertainty in the amplitude arising from the use of bound state wave functions. In the present study we study the elastic scattering of electrons by hydrogen atom, at the incident energies ranging from 100 to 700 eV. We use the refined HHOB approximation. The exchange effects are included in the DCS calculation. We use the Ochkur approximation to calculate the exchange scattering amplitude. We use eqution (26) to calculate the differential cross section (DCS) and total cross section (TCS) in energy range mentioned above. First we consider the ground state wave function of the hydrogen atom as

$$\begin{aligned} \psi_{1s}(r_1) &= \frac{1}{(\Pi)}^{1/2} \text{ Exp } (- r_1) \\ &= A \text{ Exp } (- r_1) \qquad (27), \end{aligned}$$

the product of the initial and final wave function is written as,

$$\psi_i(r_1)\,\psi_f^*(r_1) = A^2\ \mathrm{Exp}\,(-2\,r_1)$$

$$= -\,A^2\ D^n(y)\ \frac{e^{(-y\,r_1)}}{r_1}$$

$$= -\,A^2\ \frac{\partial^n}{\partial\,y^n}\ \frac{1}{r_1}\ e^{(-yr_1)}\ \Big|\ \begin{matrix} n = 1 \\ y = 2 \end{matrix}$$

$D^n(y)$ represent the derivative with respect to y and n stands for the order of the corresponding derivatives. The interaction potential for the electron and target hydrogen atom can be written as,

$$V_d = -\frac{1}{r_0} + \frac{1}{|\,r_1 - r_0|} \qquad (28),$$

where r_0 and r_1 are the position vectors for the incident electron and the target hydrogen atom. The first Born term for the hydrogen atom can be written as,

$$f^{(1)}_{i\to f} = -\frac{1}{2\Pi}\int dr_0\ e^{iq\cdot r_0}\ V_{fi}(r_0)$$

$$= -\frac{1}{2\Pi}\int\int dr_0\,dr_1\ e^{iq\cdot r_0}\,V_d\ \psi_f^*(r_1)\psi_i(r_1)$$

$$= A^2\ 2\Pi\ \frac{(q^2 + 2\,y^2)}{(q^2 + y^2)^2} \quad \text{substituting value of } y$$

$$= 2\ \frac{(q^2 + 8)}{(q^2 + 4\ )^2} \qquad (29).$$

This expression is the first Born approximation for hydrogen atom. Now the imaginary part and the real part of the second Born term is written in the form of,

$$\text{Im } f^{(2)}_{HKA} = \frac{4\Pi^3}{k} \int d\underline{p}\, U^{(2)}_{fi} (\underline{q} - \underline{p} - \beta_i \hat{y} \; ; \underline{p} + \beta_i \hat{y})$$

$$= \frac{4\Pi^3}{k} \int d\underline{p} < \psi^*_f(\underline{r}_1) \,|\bar{V}(\underline{q} - \underline{p} - \beta_i \hat{y}, \underline{r}_1) \,\bar{V} (\underline{p} + \beta_i \hat{y} ,\underline{r}_1)|\, \psi_i(\underline{r}_1)>$$

using the fourier transform of the $\bar{V}(.......)$ given by equation (10) in the above matrix element, the above equation reduces to

$$= (\Pi k_i)^{-1} \int d\underline{p} \int d\underline{r}_1 \,[\exp (i(\underline{q} - \underline{p})\cdot \underline{b}_1 - i\beta_i z_1) - 1] \,[\exp (i\underline{p}\cdot \underline{b}_1 +$$

$$i\beta_i z_1) - 1] \;\psi_i(\underline{r}_1)\, \psi^*_f(\underline{r}_1) \,[(|\underline{q} - \underline{p}|^2 + \beta_i^2) \;(p^2 + \beta_i^2)]^{-1}$$

$$= - A^2(\Pi k_i)^{-1} D(Y) \int d\underline{p} \,[(|\underline{q} - \underline{p}|^2 + \beta_i^2) \;(p^2 + \beta_i^2)]^{-1} \int d\underline{r}_1$$

$$e^{(-y\underline{r}_1)} \,[\exp(i(\underline{q} - \underline{p})\cdot \underline{b}_1 - i\beta_i z_1) - 1] \,[\exp(i\underline{p}\cdot \underline{b}_1 + i \beta_i z_1) - 1] \qquad (30).$$

The typical solution of the $d\underline{r}_1$ integral is given as,

$$\int d\underline{r}_1 \; e^{(-y\underline{r}_1)} \; e^{i\underline{q}\cdot \underline{r}_1} = \frac{4\Pi}{(q^2 + y^2)}$$

using this the above $d\underline{r}_1$ integral can be evaluated easily. Further, using the partial fraction techniques and after the canlcellation of the integrand having opposite sign and evaluation of the d$\underline{p}$ integral given in appendix, the closed form for the imaginary term can be written as,

$$\text{Im } f^{(2)}_{HEA} = - \frac{4A^2}{k} D(y) \frac{1}{y}2 \,[2I_2(\beta_i^2, y^2) - q^2(q^2 + y^2)^{-1} I_1{}'(\beta_i, 0)] \qquad (31).$$

This integral terms appeared in the equation (31) are discussed in detail in the appendix. Equation (31) through refinement will have two more term one with the finite value of β and with $\beta = 0$ term. Where we use the Cox -Bonham parameters for the evaluation of the term. Hence we will have the correction due to this two terms i.e. finite β value and for $\beta = 0$ value as mentioned in equation (25). Contribution of real part of order $O(k_i^{-1})$ is same as the results obtained for imaginary term. Hence after evaluating the integrals dp and dp_z (given in appendix), the closed form of this scattering amplitude is written as,

$$\text{Rel 1 } f_{HKA}^{(2)} = 4 A^2 (k_i^{-1}) D(y) \frac{1}{y}2 \{ 2 I_2(\beta_i^2 , y^2) - q^2 (q^2 + y^2)^{-1} I_2' (\beta_i^2 , 0) \} \quad (32),$$

Now to evaluate the 2nd term of the real part of the order $O(k_i^{-2})$ can be written in a similar way to the Rel 1 part of order $O(k_i^{-1})$. Here, while evaluating the term $(p^2 + p_z^2)$ get cancelled with the same type of the denominator term. Following the same techniques after evaluating the dp and dp_z integral (given in appendix) and using the partial fraction techniques we can write the the 2nd term of the scattering amplitude through $O(k_i^{-2})$ can be written as,

$$\text{Rel 2 } f_{HEA}^{(2)} = \frac{2}{A^2 \pi k_i^2} D(y) D' [I_3(\beta_i^2 , 0) + \frac{1}{y}2 I_3(\beta_i , y) - I_2(\beta_i^2, y^2)] \quad (33),$$

where D' is the differential operator with respect to β_i. where the integrals I' s are defined in the appendix. The terms corresponding to $\beta = 0$ (static part) is treated using the Cox -Bonham parameter and the solution corresponds to $\beta = 0$ are given in equation (26) in the form of integral term $I_1(\ldots.)$. We treat the calculation of the third term of GES i.e. $f^{(3)}_{GES}$ and the calculation of the exchange separately.

Calculation of the third term of GES ($f^{(3)}_{GES}$) for hydrogen atom :

The third term of the GES is written using the expression of Yates(1974). It is written as follows :

$$f^{(3)}_{GES} = \frac{1}{8} k_i^{-2} T^{-3} \left(\frac{\partial}{\partial T} \right) \frac{T^4}{1 + T^2} \left[4 \left| \ln \left(\frac{1 + T^2}{T} \right) \right|^2 + \frac{\Pi^2}{3} - A(T) \right]$$

$$\text{where, } A(T) = 2 (\log T)^2 + \frac{\Pi^2}{6} \sum_{n=1}^{\infty} \frac{(-T^2)^n}{n^2} \qquad T \leq 1 \quad \text{and}$$

$$= - \sum_{n=1}^{\infty} \frac{(-1/T^2)^n}{n^2} \qquad T > 1 \; ; \; T = q/2. \qquad (34),$$

which is a dimensionless vector. This expression is obtained after the cancellation of the diverging integrals given by Yates(1974). This is then reformulated in a convenient form for the present study. This term is a differential operator acting on a dimensionless vector T (= q/y). We introduce D(y) operator using the partial differentiation techniques, instead of T differentiation. The modified form of the above equation is written as,

$$= - \frac{y^4 \Pi}{16}\frac{A^2}{\Pi} (k_i^{-2})\ D(y)\ (q^2 + y^2)^{-1}\ (\ 4\ \{\ \log\ (\frac{q^2 + y^2}{qy})\ \}^2 + \frac{\Pi^2}{3}$$

$$- 2\ A(q,\ y^2)\ |_{y=2}$$

$$= - \frac{y^4}{16} A^2 \Pi\ (k_i^{-2})\ D(y)\ F(q,y) \qquad (35),$$

further,

$$A(q,y^2) \ = 2\ (\log(\frac{q}{y}))^2 + \frac{\Pi^2}{6}\ \Sigma\ \frac{(-q^2/y^2)^n}{n^2} \qquad ;\ q/y > 1.$$

Hence, finally we get the consistent picture of DCS through $O(k_i^{-2})$.

Calculation of exchange amplitude for hydrogen atom :

We include the Ist term of the exchange amplitude equation (1.39) using Ochkur approximation (Joachain, 1975). The exchange amplitude is written as,

$$g_{och} \ = - \frac{2}{k}\ 2 \int dr_1\ e^{iq\cdot r_1}\ \psi_i(\ r_1)\ \psi_f^*\ (r_1)$$

$$= \frac{8\Pi A^2}{k^2}\ D(y)\ \frac{1}{q^2 + y^2}$$

finally we get,

$$g_{och} \ = - \frac{32}{k^2} \quad \frac{1}{q^2 + y^2} \qquad (36).$$

It is obevious from the above equations that a consistent calculation of small -angle electron -atom elastic differential cross section through $O(k_i^{-2})$ requires the inclusion of the term g_{och}. Hence for the elastic scttering of electrons by atomic hydrogen we write differential cross section (DCS) as,

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} \left| f^{d}_{HHOB} + g_{och} \right|^2 + \frac{3}{4} \left| f^{d}_{HHOB} - g_{och} \right|^2 \qquad (37),$$

where the direct amplitude f^{d}_{HHOB} is obtained from equation (21) and g_{och} is given by (36). For electron -helium atom elastic sctttering we have,

$$\frac{d\sigma}{d\Omega} = \left| f^{d}_{HHOB} - g_{och} \right|^2 \qquad (38).$$

RESULTS AND DISCUSSION :

We choose the data both theoretically and experimentally for the comparison with our results in such a manner that the data's are useful for the wide energy range and for sufficienlty large angular region. First we discuss our results with the experimental results than with the other theoretical methods. Thus the method throws the light on the improvements in the results obtained through the refinement in the HHOB theory. This method of our comparison makes explanation more simpler. Although there are some comparison where the data are not given in complete range. This is due to either inability of the particular method used or the data are not available for the comparison.

We have used equations (26 ,28 ,31 to 37) of the scattering amplitude to calculate the DCS for the hydrogen atom at incident energies 100 to 700 eV in the angular range $\theta \leq 140^\circ$. We use equation (21) i.e. expression of scattering amplitude in the HHOB theory. We used the new value of the imaginary term, Ist term of the real part through $O(K_i^{-1})$, 2nd part of the real part through $O(K_i^{-2})$ after the refinement is applied to the HHOB theory.

We have used the value of the excitation energy DE = 0.05556 a.u. as calculated by Joachain et al (1977 b) in the calculation of the 2nd Born term after the refinement is applied.

We compared our results in the tabular form as well as in a graphical form with the other theoretical and experimental results available. We compare our results with , EBS (Eikonal Born Series) method of Joachain et al (1977 b), simplified 2nd Born approximation of Joachain et al (1977 a), TPE - two potential Eikonal approximation results of Pundir et al. Our Results are also compared with the static approximation results of Joachain et al (1977 b), Optical model calculation of Joachain et al (1980). Results of Rao and Desai (1983) are also compared. Experimental results of Lloyd et al (1974) and Williams (1975) are also compared.

We found in gengeral that there has been a considerable amount of improvement in the results obtain through the refinement in the HHOB theory. We find that for $\theta \leq 50^\circ - 70^\circ$ results are good in agreement when it is compared with theoretical and experimental results. Our results agree very well with the EBS results of Byron and Joachain (1977) in the angular range $\theta \leq 60^o$. As the incident energy is taken greater then 300 eV the 2nd term of the real part of the 2nd Born term i.e. $O(k_i^{-2})$ is very less in the angular region $\theta \leq 30^o$. Contribution of the real term increases as θ increases and is decreases with increase in the incident energy.

We found that contribution of the imaginary term and the first term of the real part of the 2nd Born term has improved as a part of the refinement applied. We find that there has been a considerable amount of improvement in the results.

We found that effect of the refinement in the HHOB theory at incident energy greater than 100 eV and in the angular region $\theta \leq 60^o$ is quite good. The effect of the exchange on the scattering amplitude is treated using the Ochkur approximation. Contribution of the first order exchange is almost negligible over the entire angular range. This is due to the fact that at large momentum transfer (i.e. fixed k_i) there is a poor convergence of the real term and good convergence at higher incident energies (i.e. fixed θ).

Hence as expeceted there has been a considerable amount of improvement in the values of DCS and at various incident energies and in the angular range discussed above.

Hence by means of including certain low lying energy states in the calculation i.e. inclusion of s state ,p state and so on improves the results further.

Table II.I represent the values of various term of the scattering amplitude in the refined HHOB theory for incident energy 100 eV. DCS are compared with and without exchange. We include the first order exchange in the calculation. Figure (II.I) shows the present DCS (solid curve) along with the other theoretical and experimental results. The solid curve is plotted using table II.I. This curve is compared with the experimental data of Ø - Van Wingerden et al (1977) and theoretical results of, + - EBS results of Byron and Joachain (1981), ⊙ - UEBS results of Byron et al (1983), * - EOM (Elicit Optical Model) of Mc Carthy et al (1981). One can observe the importance of the term $O(k_i^{-2})$ when the curve is compared with the other results.

Figure II.II presents the DCS at incident energies 100 (set A), 200 (set B) and 400 (set C) eV in the angular range ○ ≤ 60°. Where the results are compared with the theoretical results of Δ - CCSOPM (Coupled - Channel Second Order Potential Model) of Bransden et al (1982), experimental results of Lloyd et al (1974), Williams (1975). The agreement between the experimental results are better then with the other theoretical results. Here we use the results given in the table II.II, II.III.

Figure (II.III, II.IV) shows the importance of the exchange and the term of the order $O(k_i^{-2})$ to obtain agreement. One can further include the second order or higher order exchange term to improve the results further.

Table II.IV presents the comparison of the results of DCS in the energy range 100 eV after the refinement is applied. We find that there has been a considerable amount of improvement in the results as well as our results are found close to the EBS results.

Table II.V presents the result of the DCS for the energy range from 100 eV to 700 eV in the abgular range 0 to 60 deg. It can be observed that the present exchange corrections are small, at $\theta \geq 50^{\circ}$ (fixed energy) and at $E \geq 600$ eV (fixed angle).

We have also plotted the results of DCS at different energy for further comparision and to testify the effect of the refinement in the HHOB theory. We have also compare our results at different energies with the various experimental and theoretical methods available. We conclude that by means of including certain number of low lying energy state the accuracy can be improved further.

TABLE II.I

Behaviour of the individual terms of the refinement in the HHOB theory and scattering ampltude for hydrogen atom scattering at 100 eV expressed in a.u.

Angle Deg.	$f^{(1)}_{i\to f}$	Re1 $f^{(2)}_{HEA}$	Re2 $f^{(2)}_{HEA}$	$f^{(3)}_{GES}$	g_{Och}	Im $f^{(2)}_{HEA}$	DCS without Exch.	DCS with Exch.
5.0	0.97939	0.41	0.195	0.0236	0.26455	1.34	4.0962	4.355
10.0	0.92203	0.139	0.181	0.0633	0.243	0.9330	2.215	2.441
20.0	0.74420	0.0181	0.1405	0.117	0.182	0.464	0.830	0.966
30.0	0.55938	0.0085	0.1091	0.128	0.122	0.283	0.381	0.449
40.0	0.41324	0.0108	0.0934	0.1175	0.078	0.209	0.203	0.236
50.0	0.30953	0.0121	0.087	0.0938	0.0337	0.1488	0.1225	0.1386
60.0	0.23819	0.0118	0.0873	0.0938	0.0337	0.1488	0.0816	0.0896
80.0	0.15445	0.0099	0.0896	0.0768	0.0166	0.1181	0.0449	0.0475
120.0	0.08852	0.0068	0.09382	0.05928	0.0064	0.0857	0.0225	0.0231

TABLE II.II

Behaviour of the individual terms of the refinement in the HHOB theory and scattering ampltude for hydrogen atom scattering at 200 eV expressed in a.u.

Angle Deg.	$f^{(1)}_{i\to f}$	Re1 $f^{(2)}_{HEA}$	Re2 $f^{(2)}_{HEA}$	$f^{(3)}_{GES}$ (-Ve)	g_{Och} (-ve)	Im $f^{(2)}_{HEA}$	DCS without Exch.	DCS with Exch.
5.0	0.95954	0.1141	0.09559	0.0200	0.128	0.841	2.0051	2.128
10.0	0.85432	0.01639	0.08179	0.04598	0.1100	0.47369	1.0436	1.1376
20.0	0.58637	0.0018	0.0530	0.06429	0.0652	0.2090	0.3790	0.417
30.0	0.37875	0.00409	0.07014	0.0560	0.0345	0.13762	0.1428	0.1509
40.0	0.25137	0.00498	0.04104	0.0480	0.0184	0.10832	0.0639	0.0674
50.0	0.17588	0.00455	0.04319	0.0408	0.0103	0.08974	0.0341	0.0386
60.0	0.12978	0.00383	0.04464	0.03546	0.0062	0.0760	0.0239	0.0246
80.0	0.08064	0.00270	0.04663	0.02835	0.0027	0.05731	0.0126	0.0128
120.0	0.04502	0.00163	0.04838	0.0212	0.0009	0.03913	0.0063	0.0064

TABLE II.III

Behaviour of the individual terms of the refinement in the HHOB theory and scattering ampltude for hydrogen atom scattering at 400 eV expressed in a.u..

Angle Deg.	$f^{(1)}_{i\to f}$	Re1 $f^{(2)}_{HEA}$	Re2 $f^{(2)}_{HEA}$	$f^{(3)}_{GES}$	g_{Och} (-Ve)	Im $f^{(2)}_{HEA}$	DCS without Exch.	DCS with Exch.
5.0	0.92185	0.0161	0.04515	0.01586	0.4552	0.06098	1.141	1.197
10.0	0.74227	0.00112	0.03417	0.02949	0.2204	0.04543	0.6054	0.6392
20.0	0.40536	0.00216	0.02246	0.0291	0.10175	0.01909	0.1710	0.1788
30.0	0.22497	0.00287	0.02167	0.02285	0.0723	0.0077	0.0567	0.0583
40.0	0.13749	0.00232	0.0227	0.0182	0.0557	0.00345	0.0240	0.0245
50.0	0.09273	0.00177	0.02352	0.01515	0.0443	0.00174	0.0124	0.0126
60.0	0.06701	0.00136	0.02406	0.01296	0.0361	0.00097	0.0075	0.0075
80.0	0.04089	0.00089	0.02461	0.01009	0.0259	0.0004	0.0036	0.0036
120.0	0.02262	0.00051	0.02499	0.0073	0.017	0.00013	0.0016	0.0016

TABLE II.IV

Comparison of differential cross section in (a_0^2 /sr) for elastic electron atomic -hydrogen scattering at 100 eV, as obtained from several approximation, namely, eikonal - Born series(EBS), Wallace (W),and Uniterised EBS (UEBS) and third -order optical model (OM) of Byron and Joachain (1981).
[comparison is made with the exchange calculation]

theta deg.	EBS	OM	UEBS	OURS
5.0	4.81	4.73	4.46	4.78
10.0	2.63	2.57	2.40	2.59
20.0	9.41(-1)	9.02(-1)	8.46(-1)	9.32(-1)
30.0	3.96(-1)	3.75(-1)	3.50(-1)	3.78(-1)
40.0	1.88(-1)	1.77(-1)	1.63(-1)	1.80(-1)
50.0	9.92(-2)	9.28(-2)	8.43(-2)	9.56(-2)
60.0	5.79(-2)	5.37(-2)	4.80(-2)	5.57(-2)
70.0	3.68(-2)	3.36(-2)	2.96(-2)	3.43(-2)
80.0	2.52(-2)	2.25(-2)	1.96(-2)	2.39(-2)
90.0	1.83(-2)	1.60(-2)	1.38(-2)	1.71(-2)
100.0	1.40(-2)	1.19(-2)	1.02(-2)	1.23(-2)
120.0	9.37(-3)	7.51(-3)	6.43(-3)	8.43(-3)
140.0	7.20(-3)	5.50(-3)	4.72(-3)	6.68(-3)
160.0	6.20(-3)	4.59(-3)	3.96(-3)	5.81(-3)
180.0	5.91(-3)	4.33(-3)	3.73(-3)	4.89(-3)

TABLE II.V

Value of the diffrential cross section for elastic scattering of electrons by hydrogen atom. For various incident energy range from 100 to 700 eV. [expressed in a.u.] Results given here includes exchange also.

E(ev)	THETA (deg.)						
	5	10	20	30	40	50	60
100	4.355	2.441	0.970	0.339	0.216	0.1186	0.0886
200	2.128	1.1380	0.417	0.1689	0.0801	0.0426	0.0272
300	1.5109	0.8211	0.1710	0.05659	0.0240	0.01184	0.0076
400	1.197	0.6401	0.1698	0.0579	0.245	0.0119	0.0078
500	1.0549	0.5421	0.1298	0.04047	0.01657	0.0085	0.0051
600	0.9527	0.46591	0.10134	0.02936	0.01178	0.00600	0.0036
700	0.8794	0.40702	0.07988	0.02224	0.00879	0.00446	0.0026

Fig II-I

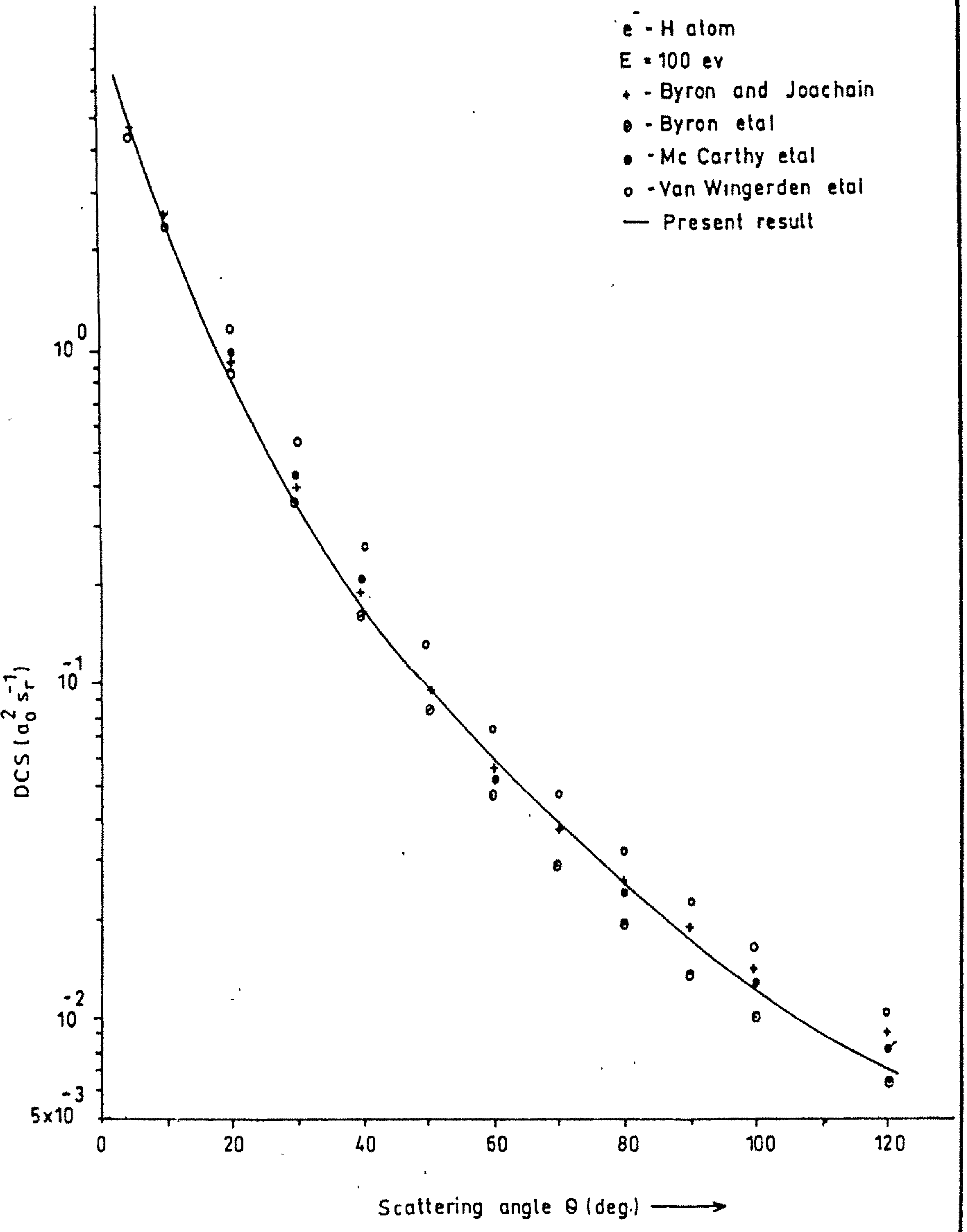
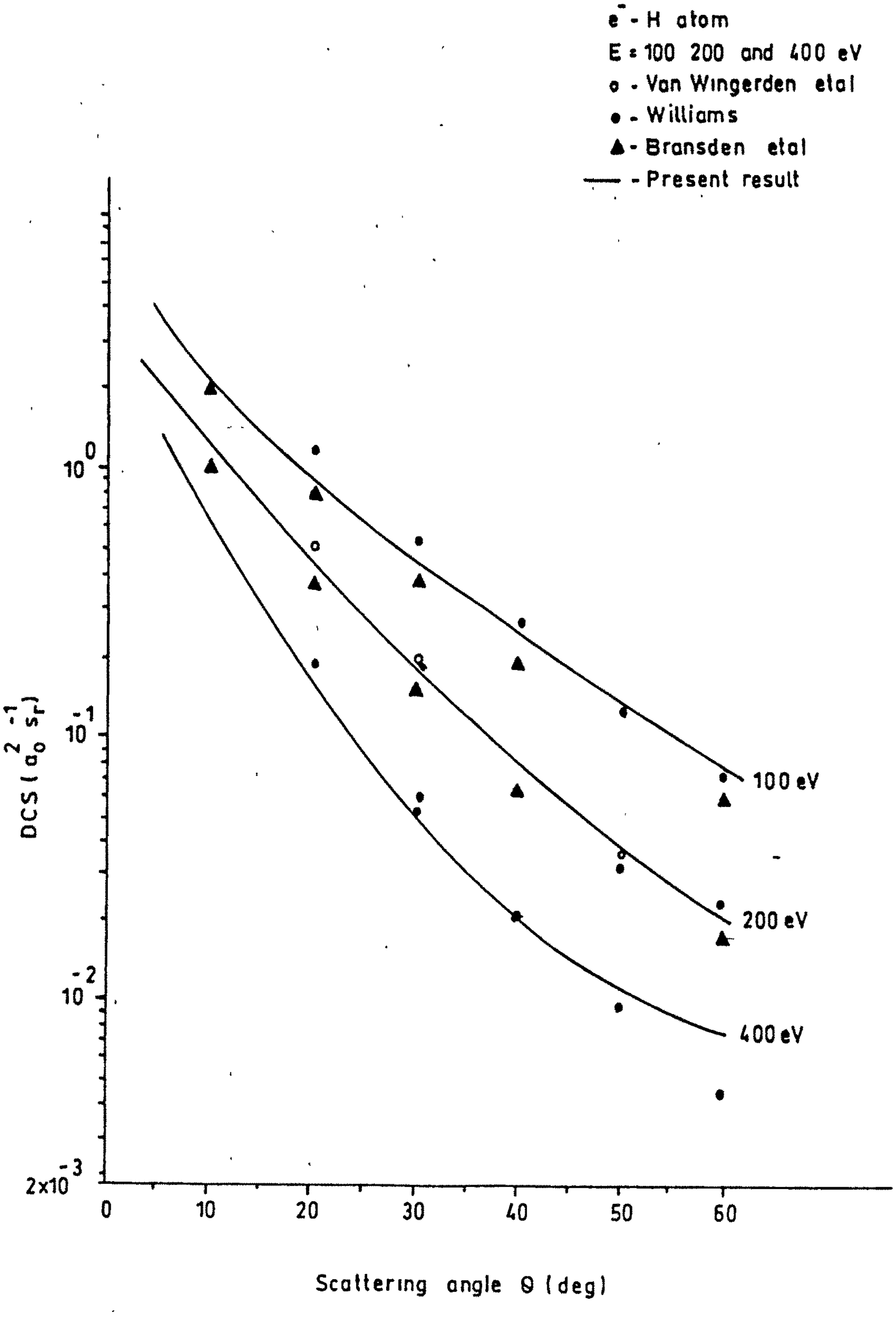

e^- - H atom
E = 100 ev
+ - Byron and Joachain
ϴ - Byron etal
● - Mc Carthy etal
o - Van Wingerden etal
— Present result
DCS ($a_0^2 s_r^{-1}$)
10^0
10^{-1}
10^{-2}
5×10^{-3}
0
20
40
60
80
100
120
Scattering angle θ (deg.) ⟶

Fig II-II



e^- - H atom
E = 100 200 and 400 eV
○ - Van Wingerden etal
● - Williams
▲ - Bransden etal
— - Present result
$DCS\ (a_o^2\ s_r^{-1})$
10^0
10^{-1}
10^{-2}
2×10^{-3}
0
10
20
30
40
50
60
Scattering angle Θ (deg)
100 eV
200 eV
400 eV

Fig II-III

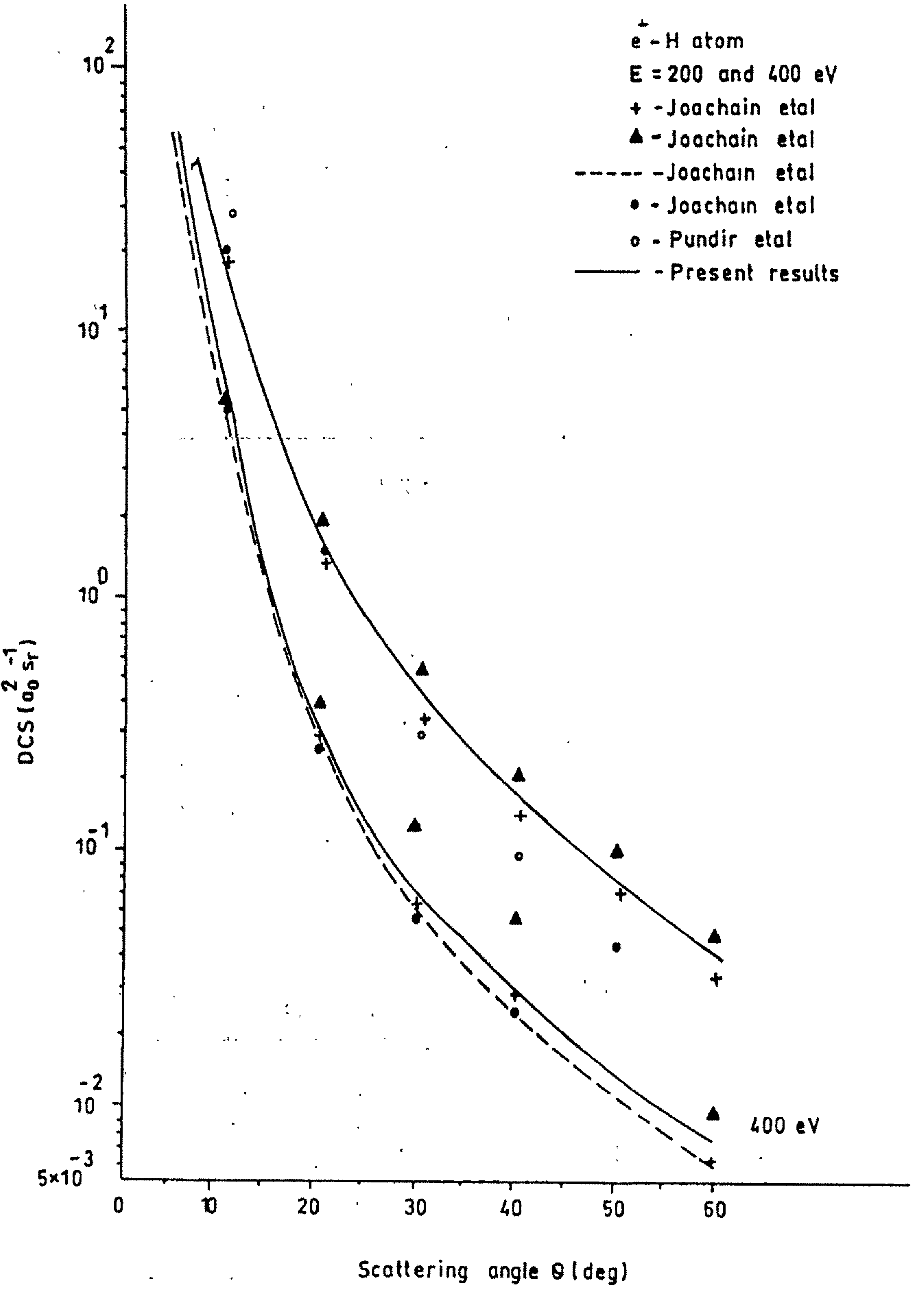
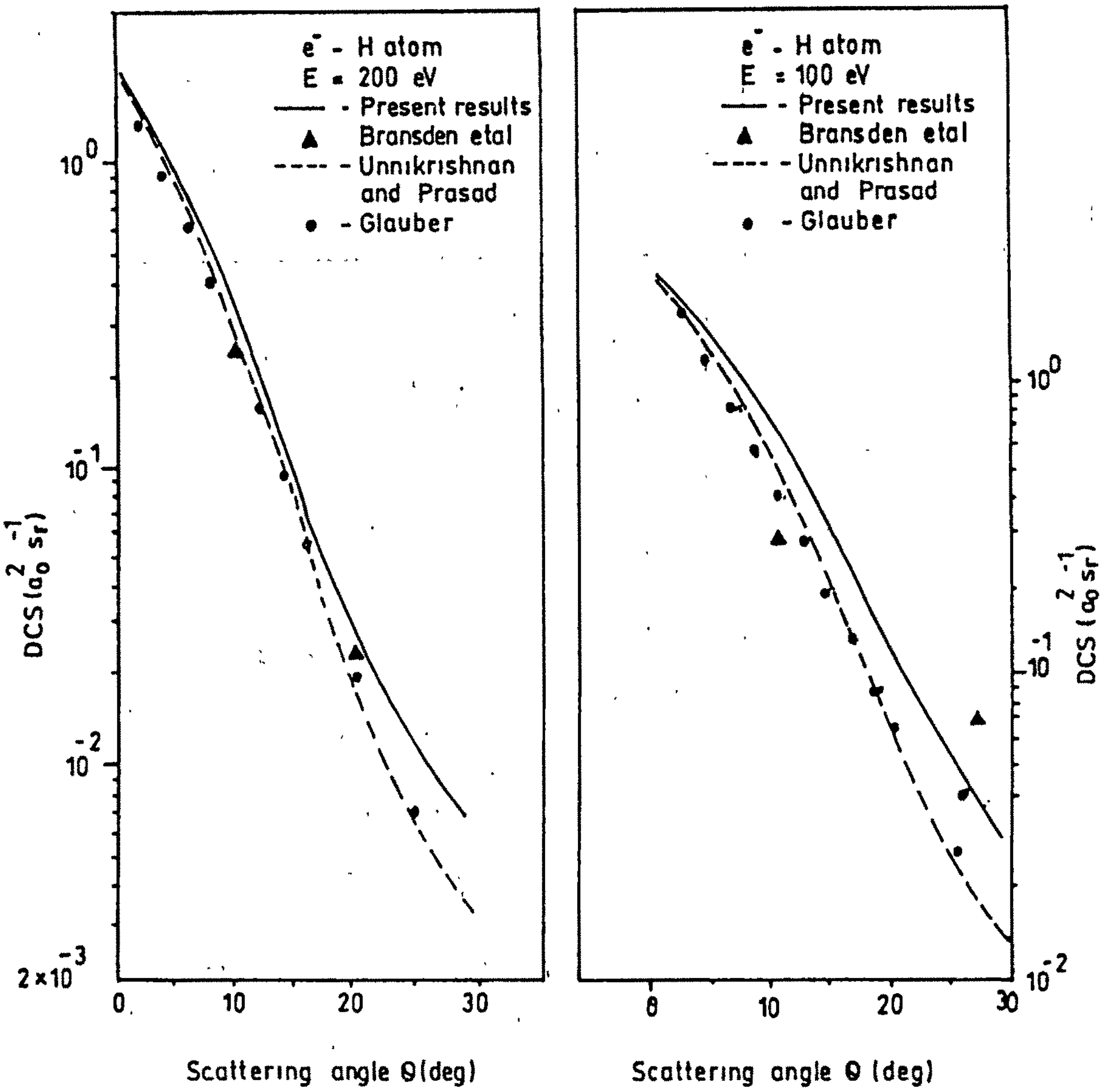

e⁻ - H atom
E = 200 and 400 eV
+ - Joachain etal
▲ - Joachain etal
------ - Joachain etal
● - Joachain etal
○ - Pundir etal
—— - Present results
400 eV
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
5×10^{-3}
DCS ($a_0^2\ sr^{-1}$)
0
10
20
30
40
50
60
Scattering angle θ (deg)

Fig II-IV


e⁻ - H atom
E = 200 eV
- Present results
Bransden etal
- Unnikrishnan and Prasad
- Glauber
DCS (a_0^2 sr^{-1})
10^0
10^{-1}
10^{-2}
2×10^{-3}
0
10
20
30
Scattering angle Θ(deg)
e⁻ - H atom
E = 100 eV
- Present results
Bransden etal
- Unnikrishnan and Prasad
- Glauber
10^0
10^{-1}
10^{-2}
DCS (a_0^2 sr^{-1})
0
10
20
30
Scattering angle Θ (deg)

Elastic Scattering of e^- by helium atom :

There is large amount of work reported for helium ($z=2$) such as (Bromberg, 1969 ; Crooks and Rudd, 1971 ; Bromberg, 1974 ; Jansen et al, 1974 ; Sethuraman et al, 1974 ; Mc Conkey and Preston, 1975 ; Jansen et al, 1976 ; Byron and Joachain, 1977) and so many other, are also available for the comparision of present theory.

Inspired by the success for the hydrogen atom, we now extend our work in this section to study the elastic scattering of e^- by helium atom. We write the straight forward equation after the refinement is applied to the 2nd Born term.

From the theoretical point of view the situation is nearly identical to that of atomic hydrogen, with the only difference being that for helium we must rely on the approximate wave functions derived through different models and theories. The well known wave function for the ground state of helium atom is given by

$$\psi_{1s}(r_1, r_2) = \phi_0(r_1)\, \phi_o(r_2)$$

$$= \left(\frac{P + Q}{4 \Pi}\right)^{1/2} \left(\frac{R + S}{4 \Pi}\right)^{1/2} \qquad (39),$$

where

$p = A \exp(-y' r_1)$, ; $Q = B \exp(-y'' r_2)$,

$R = A \exp(-y' r_2)$, ; $S = B \exp(-y' r_2)$.

The normalization constants, and the exponential parameters are given as

$A = 2.60505$; $B = 2.08144$; $y' = 1.41$ and $y'' = 2.61$

The product of the initial and the final states for elastic scattering can be written as

$$\psi_i(r_1, r_2)\ \psi_f^*(r_1, r_2) = \frac{1}{16\pi^2}\left[\ (p + Q)^2\ (R + S)^2\ \right]$$

$$= \frac{1}{16\pi^2}\left[\ (P^2R^2 + Q^2S^2 + 4\,P\,Q\,R\,S) + (P^2S^2 + Q^2R^2) + 2\,(PQS^2 + Q^2R\,S) + 2\,(P\,R^2\,Q + P^2\,R\,S)\ \right]$$

$$= \frac{1}{16\pi^2}\left[\ (P^2\,R^2 + Q^2\,S^2 + 4\,P\,Q\,R\,S) + (2\,P^2\,S^2) + 2(2\,P\,Q\,S^2) + 2\,(2\,P\,R^2\,Q)\ \right] \qquad (40).$$

All the terms in this expression can be written in the derivative form. Consider the typical term of above equation i.e.

$$2\,P\,R^2\,Q = 2\,A^3\,B\,\exp(-(y' + y'')\,r_1)\,\exp(-2\,y''\,r_2)$$

$$= K\,\exp(-y(I)\,r_1)\,\exp(\,y(J)\,r_2)\ .$$

$$= K\,D^n(y)\ \exp(-y_1 r_1)\left(-\frac{1}{r_1}\right)\ D^m(y_2)\ \exp(-y_2 r_2)\left(-\frac{1}{r_2}\right) \qquad (41),$$

where

$m = n = 1$, $K = 2\,A^3\,B$, $y_1 = y' + y''$, $y_2 = 2\,y''$, like this the other terms of the above terms of the product of the wave function can be written. We consider the above equation of the derivative form to obtain the scattering amplitudes (equation).

The interaction potential between the incident electron and the target helium atom can be written as

$$V_d = - \frac{2}{r_0} + \frac{1}{|r_0 - r_1|} + \frac{1}{|r_0 - r_2|} \qquad (42),$$

where r_0, r_1 and r_2 are the position vectors of the incident electronand the target electrons with respect to the target nuclei. Substitution of the above two equation in the expression for the scattering amplitude (equations.........). We will get the expression for imaginary and real term in the HHOB approximation as follows:

the term corresponds to the first Born term is

$$f^{(1)}_{i-->f} = - \frac{1}{2\Pi} \int dr_0 \; e^{i\, q \cdot r_0} \; V_{fi}(r_0) \qquad (43),$$

where

$$V_{fi}(r_0) = \frac{1}{16}\Pi^2 \int \int dr_1 \, dr_2 \left[- \frac{2}{r_0} + \frac{1}{|r_0 - r_1|} + \frac{1}{|r_0 - r_2|} \right]$$

$$((P^2R^2 + Q^2S^2 + 4\,P\,Q\,R\,S) + (2\,P^2S^2) + 2(2\,P\,Q\,S^2) +$$

$$2\,(2\,P\,R^2Q\,)\,) \qquad (44),$$

$$= \frac{1}{16}\Pi^2 \int \int d\mathbf{r}_1 \, d\mathbf{r}_2 \; \psi_i(\mathbf{r}_1, \mathbf{r}_2) \; \nabla_d \, \psi_f^*(\mathbf{r}_1, \mathbf{r}_2)$$

where,

$$P^2 R^2 = A^4 \exp(-y\, y'(r_1 + r_2))$$

$$Q^2 S^2 = B^4 \exp(-y\, y''(r_1 + r_2))$$

$$P^2 S^2 = A^2 B^2 \exp(-y\,(y' r_1 + y'' r_2))$$

$$4\, P\, Q\, R\, S = 4\, A^2 B^2 \exp(-y\, y'''(r_1 + r_2))$$

$$2\, P\, Q\, S^2 = 2\, B^3 A \exp(-y\,(y'' r_2 + y''' r_1))$$

$$2\, P\, R^2 Q = 2\, A^3 B \exp(-y\,(y' r_2 + y''' r_1))$$

here $y = 2$ and $y''' = \frac{y' + y''}{2}$. For the evaluation of the $d\mathbf{r}'_0$ $d\mathbf{r}_1$ and $d\mathbf{r}_2$ integrals in the equations (41) and (42) consider the typical term of the equation (42),

$$P^2 S^2 = A^2 B^2 \exp(-y\,(y' r_1 + y'' r_2))$$

$$= V \exp(-y\,(M r_1 + N r_2)) \qquad (45),$$

substitution of this term for (.....) terms in the equation (34) We will obtain the closed form of equation (42) for the equations (45).

$$f_i^t = \frac{-V}{2\pi\, 16\pi^2} \int d\mathbf{r}_0 \ \exp\ (i\ \mathbf{q}\cdot\mathbf{r}_0) \iint d\mathbf{r}_1\ d\mathbf{r}_2\ [\ -\ \frac{2}{r_0} +$$

$$\frac{1}{|\mathbf{r}_0 - \mathbf{r}_1|} + \frac{1}{|\mathbf{r}_0 - \mathbf{r}_2|}\ ]\ \exp\ (-y\ (\ Mr_1 + Nr_2)$$

$$= \frac{V}{2\pi}\ (y^{-4})(M\,N\,)^{-3} \int d\mathbf{r}_0\ \exp\ (i\mathbf{q}\cdot\mathbf{r}_0)\ [\ (\ M + \frac{1}{r})\exp\ (-y\ \ M\ \ r_0)$$

$$+ (\ N\ + \frac{1}{r})\ \exp\ (\ -\ N\ r_0\ y)\ ]$$

$$= \quad 2V\ (y)^{-4}(MN)^{-3}\ [\ \frac{(8M^2 + q^2)}{(4M^2 + q^2)^2} + \frac{(8N^2 + q^2)}{(4N^2 + q^2)^2}\ ] \qquad (46),$$

if M = N then

$$= 4\ V\ (y)^{-4}\ (MN)^{-3}\ [\ \frac{(8M^2 + q^2)}{(4M^2 + q^2)^2}\ ]\ \Big|\ y = 2 \qquad (47).$$

using equation (46) and (47) we get the closedform of equation (43). The reduced form of this Born amplitude can be obtainerd as

$$f^{(1)}_{i\to f} = [\ A^4\ (y)^{-2}(y')^{-6} + A^2B^2\ (y)^{-2}\ (y'\,y'')^{-3} + \frac{A^3}{y}\ (y'\ \ y''')^{-3}$$

$$[\ \frac{(8y'^2 + q^2)}{(4y'^2 + q^2)^2}\ ] + [\ \frac{A^2B^2}{y\ y'^2\, y''^3} + \frac{B^4}{y^2\, y'^6} + \frac{B^3A}{y\ y'''\ y''^3}\ 2]$$

$$[\ \frac{(8y''^2 + q^2)}{(4y''^2 + q^2)^2}\ ] + [\ \frac{B^3A}{y\ y'''\ y''^3} + \frac{A^2B^2}{y'''^6} + \frac{A^3B}{y\ y''\ y'''^3}\ 3]$$

$$\left[\frac{(8y'''^{3}+\underline{q}^{2})}{(4y^{2''}+q^{2})^{2}} \right]$$

$$= \sum_{k=1}^{3} C_k \left[\frac{(8Y_k^{2}+\underline{q}^{2})}{(4Y_k^{2}+q^{2})^{2}} \right] \qquad (48),$$

where C_k's and y_k's are constants given as

$C_1 = 2.420884$; $C_2 = 0.23336732$; $C_3 = 1.33543$

$y_1 = 1.41$; $y_2 = 2.61$; $y_3 = 2.01$

Equation (48) represents the first Born approximation for the elastic scattering of electron by helium atom process.

Now the imaginary part of the second Born amplitude equation (18), for the interaction potential V_d (equation, 42) can be written as,

$$\mathrm{Im}\, f^{(2)}_{HEA} = \frac{4\pi^3}{k_i} \int d\underline{p}\, U^{(2)}_{fi} \left(\underline{q} - \underline{p} - \beta_i \hat{y} \ ; \ \underline{p} + \beta_i \hat{y} \right) \qquad (49),$$

where

$U^{(2)}_{fi} (\underline{q} - \underline{p} - \beta_i \hat{y} \ ; \ \underline{p} + \beta_i \hat{y})$ for helium atom is written as,

$$= \langle \psi^{*}_{f} (\underline{r}_1, \underline{r}_2) \mid \bar{V}(\underline{q} - \underline{p} - \beta_i \hat{y} \ ; \ \underline{p} + \beta_i \hat{y}) \ \ \bar{V}(\underline{p} + \beta_i \hat{y})$$
$$\mid \psi_i(\underline{r}_1, \underline{r}_2) \rangle$$

$\bar{V}(\ldots.)$'s substituted from equation (10) in the above expression. Hence,

$$= \frac{1}{4\pi^4 (|\underline{q} - \underline{p}|^2 + \beta_i^2)\ (p^2 + \beta_i^2)} \int\int d\underline{r}_1\, d\underline{r}_2\, \psi^{*}_{f}(r_1, r_2) \psi_i(r_1, r_2)$$

$$\left[\exp\left(i(\underline{q} - \underline{p})\cdot \underline{b}_1 - i\beta_i z_1\right) + \exp\left(i(\underline{q} - \underline{p})\cdot \underline{b}_2 - i\beta_i z_2\right) - 2 \right]$$

$$[\ \exp(i\,\underline{p}\cdot\underline{b}_1 + i\,\beta_i z_1) + \exp(i\,\underline{p}\cdot\underline{b}_2 + i\,\beta_i z_2) - 2\ ]$$

$$= \frac{1}{4\Pi^4(|\underline{q} - \underline{p}|^2 + \beta_i^2)\ (p^2 + \beta_i^2)} \int\int d\underline{r}_1\ d\underline{r}_2\ \psi_f^*(r_1, r_2)\psi_i(r_1, r_2)$$

$$[\exp(i\ \underline{q}\cdot\underline{b}_1 - 2\exp(i(\underline{q} - \underline{p})\cdot\underline{b}_1 - i\beta_i z_1) - 2\exp(i\underline{p}\cdot\underline{b}_1 + i\beta_i z_1)$$

$$+ \exp(i\underline{q}\cdot\underline{b}_2) - 2\exp(i(\underline{q} - \underline{p})\cdot\underline{b}_2 - i\beta_i z_2) - 2\exp(i\underline{p}\cdot\underline{b}_2 + i\beta_i z_2)$$

$$+ \exp(i(\underline{q} - \underline{p})\cdot\underline{b}_1 + i\beta_i z_2 + i\ \underline{p}\cdot\underline{b}_2 - i\beta_i z_1) + 4 + \exp(i(\underline{q} - \underline{p})$$

$$\cdot\underline{b}_2 + i\beta_i z_1 + i\ \underline{p}\cdot\underline{b}_1 - i\beta_i z_2)\ ] \qquad (50),$$

Substitution of exponential term (41) for product of the wave function (equation, 40), we can evaluate the above $d\underline{r}_1$ and $d\underline{r}_2$ integrals. We have

$$U_{fi}^t\ (\ldots.\ ;\ \ldots..) = \frac{\underline{k}\ \underline{D(Y_1)}\ \underline{D(Y_2)}}{4\ \Pi^4(|\underline{q} - \underline{p}|^2 + \beta_i^2)\ (p^2 + \beta_i^2)} \int\int d\underline{r}_1\ d\underline{r}_2$$

$$\exp(-y_1 r_1)\ (r_1)^{-1}\ \exp(-y_2 r_2)\ [\ \text{-------}\ ]$$

The evaluation of the integrals $d\underline{r}_1$ and $d\underline{r}_2$ is performed in a similar way applied to the hydrogen atom. We write the closed form of the above integrals as

$$= \frac{\underline{k}\ \underline{(4\Pi)}^2}{4\ \Pi^4(|\underline{q} - \underline{p}|^2 + \beta_i^2)\ (p^2 + \beta_i^2)}\ [\ D(Y_1)\ \frac{1}{Y_1}\ 2\ D(Y_2)\ (\frac{1}{q^2 + Y_2^2} + \frac{1}{Y_2}\ 2\ -$$

$$\frac{\underline{D(Y_2)}}{(|\underline{q} - \underline{p}|^2 + \beta_i^2 + Y_2^2)} - \frac{\underline{D(Y_2)}}{(p^2 + \beta_i^2 + Y_2^2)}) + D(Y_2)\frac{1}{Y_2}2\ D(Y_1)$$

$$(\frac{1}{q^2+y^2} + \frac{1}{Y_1^2} \frac{D(Y_1)}{(|\mathbf{q}-\mathbf{p}|^2+\beta_i^2+Y_2^2)} - \frac{D(Y_1)}{(p^2+\beta_i^2+Y_1^2)}) + \frac{D(Y_1)}{Y_1^2}$$

$$\frac{D(Y_2)}{Y_2^2} - \frac{D(Y_1)\ D(Y_2)}{(|\mathbf{q}-\mathbf{p}|^2+\beta_i^2+Y_1^2)Y_2^2} - \frac{D(Y_1)\ D(Y_2)}{Y_1^2\ (p^2+\beta_i^2+Y_2^2)} + D(Y_1)\ D(Y_2)$$

$$[\ (|\mathbf{q}-\mathbf{p}|^2+\beta_i^2+Y_1^2)\ (p^2+\beta_i^2+Y_2^2)\ ]^{-1} + (\frac{D(Y_1)}{Y_1^2})\ (\frac{D(Y_2)}{Y_2^2}) -$$

$$D(Y_2)\ D(Y_1)(|\mathbf{q}-\mathbf{p}|^2+\beta_i^2+Y_2)^{-1}\ Y_1^{-1} - D(Y_2)D(Y_1)\ Y_2^{-1}(p^2+\beta_i^2+Y_1^2)^{-1}$$

$$+ D(Y_2)\ D(Y_1)\ (|\mathbf{q}-\mathbf{p}|^2+\beta_i^2+Y_2^2)^{-1}(p^2+\beta_i^2+Y_1^{-1})]\ ] \qquad (51),$$

Substitution of (41) in equation (39) we get the close form of equation (31) of the product of the wave function (30). Hence,

$$\mathrm{Im}\, f^{t}_{HEA} = \frac{(4\Pi)^2 k}{k} \int \frac{d\mathbf{p}}{(p^2+\beta_i^2)\ (|\mathbf{q}-\mathbf{p}|^2+\beta_i^2)}\ [\ D(Y_1)\ \frac{1}{Y_1^2}\ D(Y_2)$$

$$(\frac{q^2+2Y_2^2}{q^2+Y_2^2} - \frac{D(Y_2)}{(|\mathbf{q}-\mathbf{p}|^2+\beta_i^2+Y_2^2)} - \frac{D(Y_2)}{(p^2+\beta_i^2+Y_2^2)}) + D(Y_2)\frac{1}{Y_2^2}$$

$$D(Y_1)\ (\frac{q^2+2Y_1^2}{q^2+Y_1^2} - \frac{D(Y_1)}{(|\mathbf{q}-\mathbf{p}|^2+\beta_i^2+Y_1^2)} - \frac{D(Y_1)}{(p^2+\beta_i^2+Y_1^2)}) +$$

$$(D(Y_1)(\frac{1}{Y_1^2} - (|\mathbf{q}-\mathbf{p}|^2+\beta_i^2+Y_1)^{-1})D(Y_2)(\frac{1}{Y_2^2} - (p^2+\beta_i^2+Y_2^2)^{-1}))$$

$$(D(Y_2)(\frac{1}{Y_2^2} - (|\mathbf{q}-\mathbf{p}|^2+\beta_i^2+Y_2^2)^{-1}\ D(Y_1)(\frac{1}{Y_1^2}\ (p^2+\beta_i^2+Y_1^2)^{-1}\ )\ ] \qquad (52),$$

using the solution obtained for elastic scattering of e^- by hydrogen atom we can write the solution of the imaginary term as

$$= \frac{(4\Pi)^2 k}{k} \left[-\frac{2}{Y_1^3} D(Y_2) \frac{H(Y_2)}{Y_2^2} - \frac{2}{Y_2^3} D(Y_1) \frac{H(Y_1)}{Y_1^2} + 2D(Y_1)\, D(Y_2) \right.$$

$$\left. I_4'\,(Y_1^2, Y_2^2)\ (Y_1 Y_2)^{-2} \right] \tag{53}$$,

if $Y_1 = Y_2$ then,

$$= \frac{(4\Pi)^2 k}{k} \left[-\frac{4}{Y_2^3} D(Y_2) \frac{H(Y_2)}{Y_2^2} + 2D(Y_1)D(Y_2)\ I_4'(Y_1^2, Y_2^2)\ (Y_2)^{-4} \right] \tag{54}$$,

where the quantities $H(Y_1)$ and $I_4'(Y_1^2, Y_2^2)$ are given as

$$H(Y_1) = 2I_1(\beta_1^2, Y_1^2) - \frac{q^2}{q^2 + Y_1^2} I_1'(\beta_1^2, 0) \quad \text{and}$$

$$I_4'(Y_1^2, Y_2^2) = \int \frac{d\mathbf{p}}{(|\mathbf{q} - \mathbf{p}|^2 + \beta_1^2 + Y_1^2)(p^2 + \beta_1^2 + Y_2^2)}$$

using the same treatment applied to the hydrogen atom process and the results derived in the appendix the closed form of the above equation can further be simplified in the following way to have the final form of the imaginary term of the 2nd Born term,

$$\mathrm{Im}\, f^{(2)}_{HEA} = (\Pi\ k_i)^{-1} \left[-\left(4 \frac{A^4}{Y_1^3} + 4 \frac{A^2B^2}{Y_2^3} + 8 \frac{A^3B}{Y_3^3} \right] D(Y_1) \frac{H(Y_1)}{Y_1^2} \right.$$

$$-\left(4 \frac{B^4}{Y_2^3} + 4 \frac{A^2B^2}{Y_1^3} + 8 \frac{B^3A}{Y_3^3} \right) D(Y_2) \frac{H(Y_2)}{Y_2^2} - \left(16 \frac{A^2B^2}{Y_3^2} + 8 \frac{A^3B}{Y_1^3}\right.$$

$$+ 8 \frac{B^2A_3}{Y_2})\ D(Y_3) \frac{H(Y_3)_2}{Y_3} + 2 A^4 D(Y_1) D(Y_1) (Y_1)^{-4} I_3'(Y_1^2 , Y_1^2)$$

$$+ 2 B^4 D(Y_2) D(Y_2) (Y_2)^{-4} I_4'(Y_2^2, Y_2^2) + 8 A^2B^2 D(Y_3)D(Y_3) (Y_3)^{-4}$$

$$I_4'(Y_3^2, Y_3^2) + 4 A^2B^2 D(Y_1) D(Y_2) (Y_1Y_2)^{-2} I_4'(Y_1^2, Y_2^2) + 8 B^3A$$

$$D(Y_2) D(Y_3) (Y_2Y_3)^{-2} I_4'(Y_2^2 , Y_3^2) + 8 A^3B D(Y_3) D(Y_1) (Y_1Y_3)^{-2}$$

$$I_4'(Y_1^2, Y_3^2)]$$

$$= (\Pi k_i)^{-1} \sum_{\substack{k=1,2,3 \\ j=1,2,3}} [- A_k D(Y_k) H(Y_k) Y_k^{-1} + B_{kk} D(Y_k) D(Y_k) (Y_k Y_k)^{-2}$$

$$I_4'(Y_k^2 , Y_k^2) + B_{kj} D(Y_k) D(Y_j) (Y_kY_j)^{-2} I_4'(Y_k^2 , Y_j^2)] \quad (55),$$

where A_k's , B_{kk}'s and Y_k's , B_{kj}'s are constant given as,

$Y_1 = 2.82$, $A_1 = 13.573$, $B_{11} = 92.1074$, $B_{12} = 117.604$

$Y_2 = 5.22$, $A_2 = 8.665$, $B_{22} = 37.5392$, $B_{23} = 187.931$

$Y_3 = 4.02$, $A_3 = 21.689$, $B_{33} = 235.208$, $B_{31} = 294.376$

Now to evaluate the real part of the 2nd Born term of order k_i^{-1} can be written as equation (16). The basic difference between the evaluation of the imaginary and real term is the evaluation of the principal value integral dp_z' evaluation of the dr_1 and dr_2 integrals are same as the imaginary part. Replacing β_i in equation (50) by p_z and using the results obtained for hydrogen atom we can write the closed form of equation (16) for the product of wave function (equation , 38) as,

$$\mathrm{Rel}\, f^{(2)}_{\mathrm{HEA}} = (4\Pi)^2\, k\, (\Pi k_i)^{-1} \left[-\frac{2}{Y_1^3}\, D(Y_2)\, H(Y_2) - \frac{1}{Y_2^2} + -\frac{2}{Y_2^3}\, D(Y_1) \right.$$

$$\left. H(Y_1) - \frac{1}{Y_1^3} - 2\, D(Y_1)\, D(Y_2)\, I_4'(Y_1^2\,,\, Y_2^2\,)\, (Y_1 Y_2)^{-2} \right] \qquad (56).$$

If $Y_1 = Y_2$ a similar expression can be obtained as equation (52) for the equation (54). Where $H'(Y_1)$ and $I_4'(Y_1^2, Y_2^2\,)$ are given as,

$$H'(Y_1) = 2\, I_2(\,\beta_i^2\,,\, Y_1^2) - q^2(q^2 + Y_1^2)^{-1}\, I_2'(\beta_i^2\,,\, 0) \qquad \text{and}$$

$$I_4(Y_1^2,\, Y_2^2) = \mathcal{P} \int \frac{dP_z}{(p_z - \beta_i)} \int \frac{d\underline{p}}{(|\underline{q} - \underline{p}|^2 + p_z^2 + Y_1^2)\,(p^2 + p_z^2 + Y_2^2)}$$

The term $I_4(\ldots.)$, $I_2(\ldots..)$ are defined in the appendix. Using the above expression and making use of equation (38) we get the close form for the real part through $O(k_i^{-1})$ as,

$$\mathrm{Rell}\, f^{(2)}_{\mathrm{HEA}} = \frac{1}{\Pi}-2\, k_i^{-1} \sum_{\substack{k=1,2,3\\ j=2,3,1}} \left[A_k\, D(Y_k)\, H(Y_k) - \frac{1}{Y_k^2} - B_{kk}\, D(Y_k)\, D(Y_k) \right.$$

$$\left. I_4(Y_k^2\,,\, Y_k^2)\, (Y_k^2\, Y_k^2)^{-1} \right] - B_{kj}\, D(Y_k)\, D(Y_j)\, I_4(Y_k^2,\, Y_j^2)\, (Y_k^2\, Y_j^2)^{-1} \,] \qquad (57).$$

All constants appered in the above amplitude are given under equation (55).

The real part through order $O(k_i^{-2})$ of the second Born approximation equation (17) can be written as

$$\mathrm{Re}2\, f^{(2)}_{HEA} = -\frac{2\pi^2}{k^2} D' \mathcal{P} \int d\underline{p} \int_{-\infty}^{+\infty} \frac{dp_z\, (p^2 + p_z^2)}{p_z - \beta_i} U^{(2)}_{fi}(\underline{q} - \underline{p} - p_z\hat{y};\ \underline{p} + p_z\hat{y}) \qquad (58).$$

By comparing the results terms of the amplitude (equations 55,57) with the equations derived for the hydrogen atom case (equations 31,33), we can directly write the closed form of equation (58) of the wave functions product equation (40).

$$\mathrm{Re}2\, f^{(2)}_{HEA} = \frac{(4\pi)^2 k}{2\pi^2 k^2} D' \left[\frac{2}{y_1} 3\, D(y_2)\, H''(y_2) + \frac{2}{y_2} 3\, D(y_1)\, H''(y_1) - D(y_1)\, D(y_2) \frac{I_5(y_1^2,\ y_2^2)}{y_1^2\, y_2^2} \right] \qquad (59)$$

where $H''(Y_2)$ and $I_5(y_1^2,\ y_2^2)$ are given as,

$$H''(y_2) = \frac{I_3(\beta_1,\ 0)}{(q^2 + y_1^2)} + \frac{I_3(\beta_1, y_1^2)}{y_1^2} - I_2(\beta_1^2,\ y_1^2)$$

and

$$I_5(y_1^2,\ y_2^2) = \mathcal{P} \int_{-\infty}^{+\infty} \frac{dp_z}{(p_z - \beta_i)} \int \frac{d\underline{p}\, (p^2 + p_z^2)}{(|\underline{q}-\underline{p}|^2 + p_z^2 + y_1^2)\, (p^2 + p_z^2 + y_2^2)}$$

$$+ \mathcal{P} \int_{-\infty}^{+\infty} \frac{dp_z}{(p_z - \beta_i)} \int \frac{d\underline{p}\, (p^2 + p_z^2)}{(|\underline{q}-\underline{p}|^2 + p_z^2 + y_2^2)\, (p^2 + p_z^2 + y_1^2)}$$

$$= I_3(\beta_1^2,\ y_1^2) - I_4(y_1^2,\ y_2^2)\, (y_1^2 + y_2^2) + I_3(\beta_1^2,\ y_2^2)$$

The above three terms are obtained by adding abd deducing y_1^2 and y_2^2 in the preceding two terms of $I_5(y_1^2,\ y_2^2)$, and making use of the prevous results. Using equation (59), the real part (58) can be obtained through the equations (40,49).

$$\mathrm{Re}\,2\, f^{(2)}_{\mathrm{HEA}} = (2\Pi^2 k_i^2)^{-1} \sum_{\substack{k=1;2;3\\ j=2;3;1}} D'[\, A_k D(y_k)\, H''(y_k) - \frac{B_{kk}}{2} D(y_k)\, D(y_k) \frac{I_5(y_k^2, y_k^2)}{y_k^2\, y_k^2} - \frac{B_{kj}}{2} D(y_k) D(y_j) \frac{I_5(y_k^2, y_j^2)}{y_k^2\, y_j^2} \,] \qquad (60).$$

Now to get the closed form of the terms corresponds to imaginary and real through $O(k_i^{-2})$ we use the relation

$$U^{(2)}_{fi}(\, \underline{q} - \underline{p} - \beta_i \hat{y};\ \underline{p} + \beta_i \hat{y},\ \underline{r}_1, \underline{r}_2) =$$

$$\langle \psi_f(\underline{r}_1, \underline{r}_2) \mid \bar{V}(\underline{q} - \underline{p} - \beta_i \hat{y};\ \underline{r}_1, \underline{r}_2)\ \bar{V}(\underline{p} + \beta_i \hat{y},\ \underline{r}_1, \underline{r}_2) \mid \psi_i(\underline{r}_1, \underline{r}_2) \rangle - \langle \psi_f(r_1 r_2) \mid \bar{V}(\underline{q} - \underline{p} - \beta_i \hat{y},\ \underline{r}_1) \mid \psi_i(r_1 r_2) \rangle \langle \psi_f(r_1 r_2) \mid \bar{V}(\underline{p} + \beta_i \hat{y},\ r_2) \mid \psi_f(r_1, r_2) \rangle \qquad (61).$$

Now the imaginary (equation,49) and real parts (equation, 58) can be derived. The closed form of the imaginary part in the elastic scattering of electrons by helium atom can be obtained as,

$$\mathrm{Im}\, f^{(2)}_{\mathrm{HEA}} = \frac{4\Pi^3}{k_i} \int d\underline{p}\ U^{(2)}_{fi}(\underline{q} - \underline{p} - \beta_i \hat{y};\ \underline{p} + \beta_i \hat{y},\ r_1, r_2) \qquad (62),$$

$$= \frac{1}{\Pi k_i} \int \frac{d\underline{p}}{(|\underline{q}-\underline{p}|^2 + \beta_i^2)\ (p^2 + \beta_i^2)} \int d\underline{r}_1 \int d\underline{r}_2\, [\ e^{i\underline{q}\cdot\underline{b}_1} {}^{-i(\underline{p}\cdot\underline{b}_1} {}^{+ \beta_i z_1]}$$

$$\psi_i\, \psi_f^{*} \int d\underline{r}_1 \int d\underline{r}_2\, \exp(i(|\underline{q} - \underline{p}\,| \cdot \underline{b}_2 + i\beta_i z_2)\ ]\ \psi_i\, \psi_f^{*} \qquad (63).$$

Substituting the product of initial and final states of Hartree -Fock wave functions (equations,40 ,41) and following the procedure of $d\underline{r}_1$, $d\underline{r}_2$ and $d\underline{p}$, the closed form of the above equation (63) can be written as

$$\mathrm{Im}\ f^{(2)}_{\mathrm{HEA}} = - \frac{1}{\Pi\, k_i} \sum_{i=1}^{3} D(y_1) \frac{A_i I_1'(\beta_i^2,\ 0)}{(q^2 + y_i^2)} - \frac{1}{\Pi\, k_i} \sum_{\substack{i=1,3\\ j=1,3}} [\ A_{ij}\ D(y_i)$$

$$D(y_j)\ (y_i^2\ y_j^2)^{-1}\ \{\ I_1'(\beta_i^2,\ 0) - I_1(\beta_i^2,\ y_i^2) - I_1(\beta_i^2,\ y_j^2) +$$

$$I_4'(y_i^2,\ y_j^2)\ \}\ ] \qquad (64).$$

Similarlly real parts through $O(k_i^{-2})$ can be derived through equations (40,41 and 63). The closed form of these term can be written as it is written in case of the hydrogen atom. Hence,

$$\mathrm{Re1}\ f^{(2)}_{\mathrm{HEA}} = \frac{1}{\Pi^2 k_i} \sum_{i=1}^{3} D(y_i) \frac{A_i\ I_2'(\beta_i^2, 0)}{(q^2 + y_i^2)} - \frac{1}{\Pi^2 k_i} \sum_{\substack{i=1,3\\ j=1,3}} [\ A_{ij}\ D(y_i)$$

$$D(y_j)\ (y_i^2 y_j^2)^{-1}\ \{\ I_2'\ (\beta_i^2,\ 0) - I_2(\ \beta_i^2,\ y_i^2) - I_2(\beta_i^2,\ y_j^2) +$$

$$I_4\ (y_i^2,\ y_j^2)\ \}\ ] \qquad (65),$$

and

$$\mathrm{Re2}\ f^{(2)}_{\mathrm{HEA}} = (\Pi^2 k_i^2)^{-1} \sum_{i=1}^{3} A_i\ D'\ D(y_i) \frac{I_3'(\beta_i^2, 0)}{(q^2 + y_i^2)} - (2\Pi^2 k_i^2)^{-1} \sum_{\substack{i=1,3\\ j=1,3}}$$

$$A_{ij}\ D' D(y_i)\ D(y_j)\ (y_j)^{-2}\ [\ I_2(\beta_i^2,\ y_i^2) - I_4(y_j^2,\ y_i^2)\ ] \qquad (66).$$

All the constants A_i, y_i and A_{ij}, y_j can be obtained from the equation (39),

$A_1 = 6.7863$, $A_2 = 4.3324$, $A_3 = 10.845$

$y_1 = 2.82$, $y_2 = 5.22$, $y_3 = 4.02$

$A_{11} = 46.0537$, $A_{22} = 18.7696$, $A_{33} = 117.60431$

$A_{12} = 29.4009$, $A_{21} = 29.4009$, $A_{31} = 73.5939$

$A_{13} = 73.5939$, $A_{23} = 46.9327$, $A_{32} = 46.9827$

We use equations (64,65,66 and 38) in equation (22) for the calculation of the differential cross section for the process to study the elastic scattering of electrons by helium atom. We also perform the calculation of the term corresponds to $\beta = 0$ and for finite value of β as it is done for the hydrogen atom after the refinement is applied for the above said process. We also calculate the third term in GES i.e. $f^{(3)}_{GES}$ and the exchange term g_{och} in the Ochkur approximation which is described as follows:

Calculation of third term in GES ($f^{(3)}_{GES}$) for helium atom :

We use the expression of scattering amplitude obtained in Glauber Eikonal Series method given as,

$$f^{(n)}_{GES}(q,k_i) \doteq 2\Pi k_i \frac{(-Q)^n}{(\Pi k)^n} \frac{1}{n!} \int \frac{d\underline{p}_1}{p_1{}^2} \text{------} \int d\underline{p}_{n-1} (p^2_{n-1} + |\underline{q} - \underline{p}|^2$$

$$< \psi_f | B(\underline{p}_1)\, B(\underline{p}_{n-1}) \ldots . B(|\underline{q} - \underline{p}|) \,|\, \psi_i > .$$

Considering the product of the wave function of the helium atom and the interaction potential of helium atom in the calculation we can simplify the above term.

Singh and Tripathi (1980) evaluated the 3rd term of GES for elastic and inelastic scattering of e^- by helium atom. We use the simplified form of the same as,

$$f^{(3)}_{GES} = 3\,(\Pi k_i)^{-2}\,[\,X_{31} + X_{32}\,]$$

where X_{31} and X_{32} are well defined quantities given by Singh and Tripathi (1980).

Calculation of the exchange amplitude for helium atom :

We use the equation (39) and Ochkur approximation (Joachain, 1975) for the evaluation of the exchange amplitude for He atom. We write the straight form as follows :

$$g_{och} = - 32 (k_i)^{-2} (q^2 + 4)^{-2}.$$

RESULTES AND DISCUSSION :

Results obtained through the refinement in the HHOB theory for the differential cross sections are compared with the two sets of measurements viz., those of Bromberg (1969,1974) and Jansen etal (1976) and theoretical results of Dewangan and Walters (1977) of DWBSA method, EBS and UEBS resultes of Byron and Joachain (1974, 1976). We also compare experimental results of Crooks and Rudd, Vuskovic et al and Jost etal. Theoretical results of the optical eikonal method of Byron and Joachian are also compared. We also compare theoretical results of Rao and Desai and Suja and Desai. The choice for choosing these groups of data are that, firstly they tend to support one another and together cover a large energy range (100 eV to 3 KeV) and, secondly the systematics of the improvements in agreements between these data and our results as the energy is increased, and therefore the comparison is made in this manner.

We find in general that for the incident energy E $\geq$ 200 eV gives better results and it improves further as the energy of incidence is increased further. We find that our results agrees well with the EBS results as compare to the results of Rao and Desai (1983) and Suja and Desai (1987). Further due to the refinement results at large scattering angle also improves further as compare to the earlier methods. When the results obtained through the refinement is compared with the optical model they found to differ. This may be due to the fact that optical model treats the static potential (which is very important in large angle scattering) exactly to all orders of perturbation theory, the optical model results should be preferred at large angles, while the results obtained through the refinement in the HHOB theory agrees well over the entire energy range. As the energy of incidence increases further the accuracy also increses further.

As the energy of incidence is increased the contribution of the 2nd term of the real part of the 2nd term is getting less.

The effect of the exchange is treated separately using the Ochkur approximation. The difference between our results, EBS results and optical model results are mainly due to large angle behaviour of differential cross sections. Further, due to refinement the contribution of the imaginary term is significant for the energy E $\geq$ 200 eV.

Tables II.VI, II.VII shows the behaviour of the individual terms of the HHOB approximation at 200 eV and 400 eV respectively.

In table II.VII we compare our results with the results of Singh and Tripathi (1980) and results of Rao and Desai (1983). We find that there is remarkable difference in the results. But when it compared with theoretical result of the optical model of Byron and Joachain (1977) and the experimental results of Crooks and Rudd (1971), Bromberg etal (1974) , Sethuraman etal (1974), we find that there is good amount of improvements in the results for E ≥ 200 eV. We also compare our results with the results of EBS method, results of Jansen etal, HHOB method and the results of Suja and Desai in the table II.VIII. We find that our results agrees well with these methods. We also compare our results for E ≥ 400 eV in the table II.IX and II.XI with the experimental results and the other theoretical results available.

We have also compared our results in the graphical manner also. Figure II.V shows the values of the differential cross section for the energy of incidence 200 eV. We find that there is a good amount of improvement in the results. We have compared our resuls with the experimental and theoretical results for the angular range 10 to 130 deg. Similarly firgure II.VI shows the results for 200 eV as well as for 400 eV. Here also we compare our results with the theoretical and other experimental results also.

In figure II.VII the results are plotted for the energy of incidence 200 eV and 400 eV respectively. Hence looking to the comparison we can say that through refinement i.e. by including certain low lying energy states in the calculation results can be improved upon. We also calculate the differential cross section for the energy of incidence 100 eV to 700 eV which is given in the table II.XIII. Results are expressed in a_0^2 /sr. We also conclude at the same time that the effects of the higher order terms of the exchange can improve the results further.

Hence looking to the easiness in computing the results through the refinement in the HHOB theory and the improvement obtained in the results we extend our method to study the elastic scattering of electrons by lithium atom followed in the following section of this chapter.

TABLE II.VI

Behaviour of the individual terms of the refinement in the HHOB theory and scattering amplitude for helium atom scattering at 200 eV.

Angle Deg.	$f^{(1)}_{i\to f}$	$\mathrm{Re1f}^{(2)}_{\mathrm{HEA}}$	$\mathrm{Re2f}^{(2)}_{\mathrm{HEA}}$	$f^{(3)}_{\mathrm{GES}}$	g_{och}	$\mathrm{Imf}^{(2)}_{\mathrm{HEA}}$	DCS with exch.
10.0	0.73730	0.1359	0.0835	0.129	0.1248	0.2186	1.338
20.0	0.61030	0.037	0.051	0.126	0.0992	0.9261	5.78(-1)
30.0	0.47430	0.0218	0.0488	0.137	0.0724	0.0496	2.80(-1)
40.0	0.36220	0.0195	0.0503	0.117	0.0513	0.0336	1.52(-1)
50.0	0.27930	0.0185	0.052	0.123	0.03626	0.0332	8.78(-2)
70.0	0.17751	0.0157	0.0563	0.102	0.01921	0.0199	3.43(-2)
90.0	0.12470	0.0128	0.060	0.094	0.0113	0.1376	1.75(-2)
110.0	0.09582	0.01072	0.0625	0.086	0.0075	0.1165	1.11(-2)
130.0	0.07940	0.00936	0.064	0.0802	0.00557	0.1009	7.96(-3)

TABLE II.VII

Behaviour of the individual terms of the refinement in the HHOB theory and scattering amplitude for helium atom scattering at 400 eV.

Angle Deg.	$f^{(1)}_{i\to f}$	$\mathrm{Re1f}^{(2)}_{\mathrm{HEA}}$	$\mathrm{Re2f}^{(2)}_{\mathrm{HEA}}$	$f^{(3)}_{\mathrm{GES}}$ (-ve)	g_{och} (-ve)	$\mathrm{Im}\ f^{(2)}_{\mathrm{HEA}}$	DCS ref.1	with DCS ref.2	exchange Present results
10.0	0.6885	0.0274	0.029	0.068	0.05753	0.368	5.05(-1)	0.67	7.71(-1)
20.0	0.4936	0.0103	0.0241	0.069	0.0382	0.206	2.1 (-1)	0.29	3.72(-1)
30.0	0.3345	0.0093	0.028	0.0657	0.02313	0.1522	9.0 (-2)	0.132	1.83(-1)
40.0	0.231	0.0086	0.0271	0.058	0.01396	0.1184	4.2 (-2)	0.066	8.89(-2)
50.0	0.1656	0.0076	0.0287	0.052	0.0087	0.1103	2.1 (-2)	0.0376	4.98(-2)
70.0	0.0972	0.00535	0.0313	0.0432	0.00387	0.0835	7.4 (-3)	0.016	2.06(-2)
90.0	0.0657	0.00317	0.033	0.0374	0.00205	0.0602	3.5 (-3)	0.0082	1.06(-2)
110.0	0.0497	0.00322	0.034	0.0334	0.00127	0.047	2.00(-3)	0.0052	6.72(-3)
130.0	0.0407	0.00269	0.0343	0.0307	0.00090	0.0390	1.4 (-3)	0.0038	4.91(-3)

Ref.1 Results of SIngh and Tripathi (1980)
Ref.2 Results of Rao and Desai (1983)

TABLE II.VIII

Differential Cross Section (in a_0^2/Sr) for the elastic scattering for e^- - He atom at incident energy E = 200 eV.

Angle (Deg.)	Present results	Jansen et al	Crooks & Rudd	HHOB	EBS	Suja & Desai
10.0	1.338	1.08	1.93	1.3113	1.34	1.3249
20.0	5.78(-1)	5.28(-1)	7.18(-1)	6.0973(-1)	5.83(-1)	6.1141(-1)
30.0	2.80(-1)	2.81(-1)	3.25(-1)	3.1266(-1)	2.88(-1)	3.1662(-1)
40.0	1.52(-1)	1.51(-1)	----	1.8634(-1)	1.54(-1)	1.7582(-1)
50.0	8.78(-2)	8.85(-2)	1.08(-1)	1.0794(-1)	8.81(-2)	1.0400(-1)

TABLE II. IX

Differential Cross Section (in a_0^2 /Sr) for the elastic scattering for e^- - He atom at incident energy E = 400 eV.

Angle (Deg.)	Present results	Jansen et al	Bromberg	HHOB	EBS	Suja & Desai
10.0	7.71(-1)	7.87(-1)	8.10(-1)	6.7601(-1)	7.61(-1)	6.8589(-1)
20.0	3.72(-1)	3.65(-1)	3.67(-1)	1.3190(-1)	3.79(-1)	2.8982(-1)
30.0	1.83(-1)	2.47(-1)	1.76(-1)	1.3190(-1)	1.78(-1)	1.2847(-1)
40.0	8.89(-2)	1.69(-1)	8.85(-2)	6.6050(-2)	8.79(-2)	6.1727(-2)
50.0	4.98(-2)	9.85(-2)	4.81(-2)	3.7670(-2)	4.76(-2)	3.5025(-2)

TABLE II.X

Comparison of the differential cross sections for elastic electron -helium atom scattering at an incident -electron energy of 200 eV. [expressed in a_0^2/ sr.]

theta deg.	Present results	Theoretical		Experimental		
		EBS method	Optical Eikonal	Croock & Rudd	Vuskovic et al	Jost et al
5.0	1.81	2.12	2.97	---	---	2.36
10.0	1.338	1.34	1.28	1.93	1.72	1.50
20.0	5.78(-1)	5.83(-1)	5.81(-1)	7.13(-1)	3.63(-1)	6.07(-1)
30.0	2.80(-1)	2.88(-1)	2.90(-1)	3.25(-1)	1.41(-1)	3.20(-1)
50.0	8.78(-2)	8.76(-2)	8.42(-2)	1.03(-1)	6.08(-2)	1.00(-1)
70.0	3.43(-2)	3.61(-2)	3.08(-2)	4.23(-2)	2.39(-2)	4.11(-2)
90.0	1.75(-2)	1.97(-2)	1.41(-2)	2.33(-2)	1.45(-2)	2.00(-2)
110.0	1.11(-2)	1.32(-2)	7.85(-3)	1.41(-2)	9.47(-3)	1.21(-2)
130.0	7.96(-3)	1.01(-2)	5.17(-3)	1.05(-2)	7.37(-3)	8.57(-3)
150.0	6.89(-3)	8.60(-3)	3.95(-3)	8.43(-3)	6.43(-3)	---

TABLE II.XI

Comparison of the differential cross sections for elastic electron -helium atom scattering at an incident -electron energy of 400 eV. [expressed in a_0^2/ sr.]

theta	Present	Theoretical		Experimental		
deg.	results	EBS method	Optical Eikonal	Croock & Rudd	Chamberlain et al	Jost et al
5.0	1.10	1.18	1.14	---	1.04	1.39
10.0	7.71(-1)	6.89(-1)	6.75(-1)	7.61(-1)	6.22(-1)	8.93(-1)
20.0	2.72(-1)	2.85(-1)	2.82(-1)	3.17(-1)	2.37(-1)	3.64(-1)
30.0	1.83(-1)	1.23(-1)	1.20(-1)	1.41(-1)	9.48(-2)	1.55(-1)
50.0	3.41(-2)	2.96(-2)	2.78(-2)	3.34(-2)	---	3.57(-2)
70.0	1.06(-2)	1.09(-2)	9.29(-3)	1.17(-2)	---	1.21(-2)
90.0	4.78(-3)	5.34(-3)	4.16(-3)	6.60(-3)	---	5.18(-3)
110.0	3.72(-3)	3.37(-3)	3.10(-3)	3.33(-3)	---	3.00(-3)
130.0	2.26(-3)	2.44(-3)	1.55(-3)	2.32(-3)	---	1.96(-3)
150.0	1.89(-3)	2.00(-3)	1.20(-3)	1.83(-3)	---	---

TABLE II. XII

Comparison of various theoretical and experimental differential cross sections for elastic electron - helium scattering at inicdent -electron energy 200 eV. Results expressed in a_0^2 /sr.

			Experimental values		
theta deg.	OM BJ 1977	Crooks and Rudd 1971	Bromberg 1974	Sethuraman et al 1974	Ours
5.0	1.98	---	1.73	---	1.81
10.0	1.25	1.93	1.12	---	1.338
20.0	5.75(-1)	7.13(-1)	5.27(-1)	---	5.78(-1)
30.0	2.91(-1)	3.25(-1)	2.76(-1)	2.63(-1)	2.80(-1)
40.0	1.55(-1)	---	1.52(-1)	1.57(-1)	1.52(-1)
50.0	8.86(-2)	1.03(-1)	8.91(-1)	9.30(-2)	8.78(-2)
60.0	5.43(-2)	---	5.57(-2)	5.38(-2)	4.89(-2)
70.0	3.57(-2)	4.23(-2)	3.72(-2)	3.57(-2)	3.43(-2)
80.0	2.49(-2)	---	2.63(-2)	2.47(-2)	2.51(-2)
90.0	1.84(-2)	2.33(-2)	1.90(-2)	1.77(-2)	1.75(-2)
100.0	1.42(-2)	---	1.45(-2)	1.40(-2)	1.36(-2)
110.0	1.14(-2)	1.41(-2)	1.18(-2)	1.15(-2)	1.11(-2)
120.0	9.51(-3)	---	---	9.80(-3)	8.89(-3)
130.0	8.20(-3)	1.05(-2)	---	8.00(-3)	7.96(-3)
140.0	7.28(-3)	---	---	6.80(-3)	7.12(-3)
150.0	6.65(-3)	8.43(-3)	---	6.00(-3)	6.89(-3)

TABLE II.XIIA

Values of differential cross section for e^- – He elastic scattering at various incident energies and scattering angles. [expressed in a_0^2 /sr].

theta			ENERGY (eV)			
deg.	100	200	300	400	500	700
5.0	2.93	1.68	1.55	1.38	1.09	8.90(-1)
10.0	1.98	1.338	1.01	7.71(-1)	5.89(-1)	5.01(-1)
20.0	1.38	5.78(-1)	5.43(-1)	3.72(-1)	3.32(-1)	3.02(-1)
30.0	6.78(-1)	2.80(-1)	3.89(-1)	1.83(-1)	1.30(-1)	1.02(-1)
40.0	4.34(-1)	1.52(-1)	1.11(-1)	8.89(-2)	7.89(-2)	8.89(-2)
50.0	2.56(-1)	8.78(-2)	8.47(-2)	4.98(-2)	3.78(-2)	4.92(-2)
60.0	1.68(-1)	5.69(-2)	4.39(-2)	3.21(-2)	2.01(-2)	1.01(-2)
70.0	1.43(-1)	4.01(-2)	2.21(-2)	2.06(-2)	1.01(-2)	8.32(-3)
80.0	1.01(-1)	3.32(-2)	1.54(-2)	1.56(-2)	8.65(-3)	7.11(-3)
90.0	7.85(-2)	2.32(-2)	1.01(-2)	1.06(-2)	5.68(-3)	4.32(-3)
100.0	6.45(-2)	1.68(-2)	7.59(-3)	6.01(-3)	4.93(-3)	2.89(-3)
120.0	4.56(-2)	1.01(-2)	5.81(-3)	5.89(-3)	3.33(-3)	1.02(-3)
140.0	4.01(-2)	9.43(-3)	5.11(-3)	4.03(-3)	1.23(-3)	8.89(-4)
160.0	3.67(-2)	7.54(-3)	3.89(-3)	2.65(-3)	9.99(-4)	6.43(-4)
180.0	3.01(-2)	6.23(-3)	2.56(-3)	1.89(-3)	8.53(-4)	5.59(-4)

Fig II-V

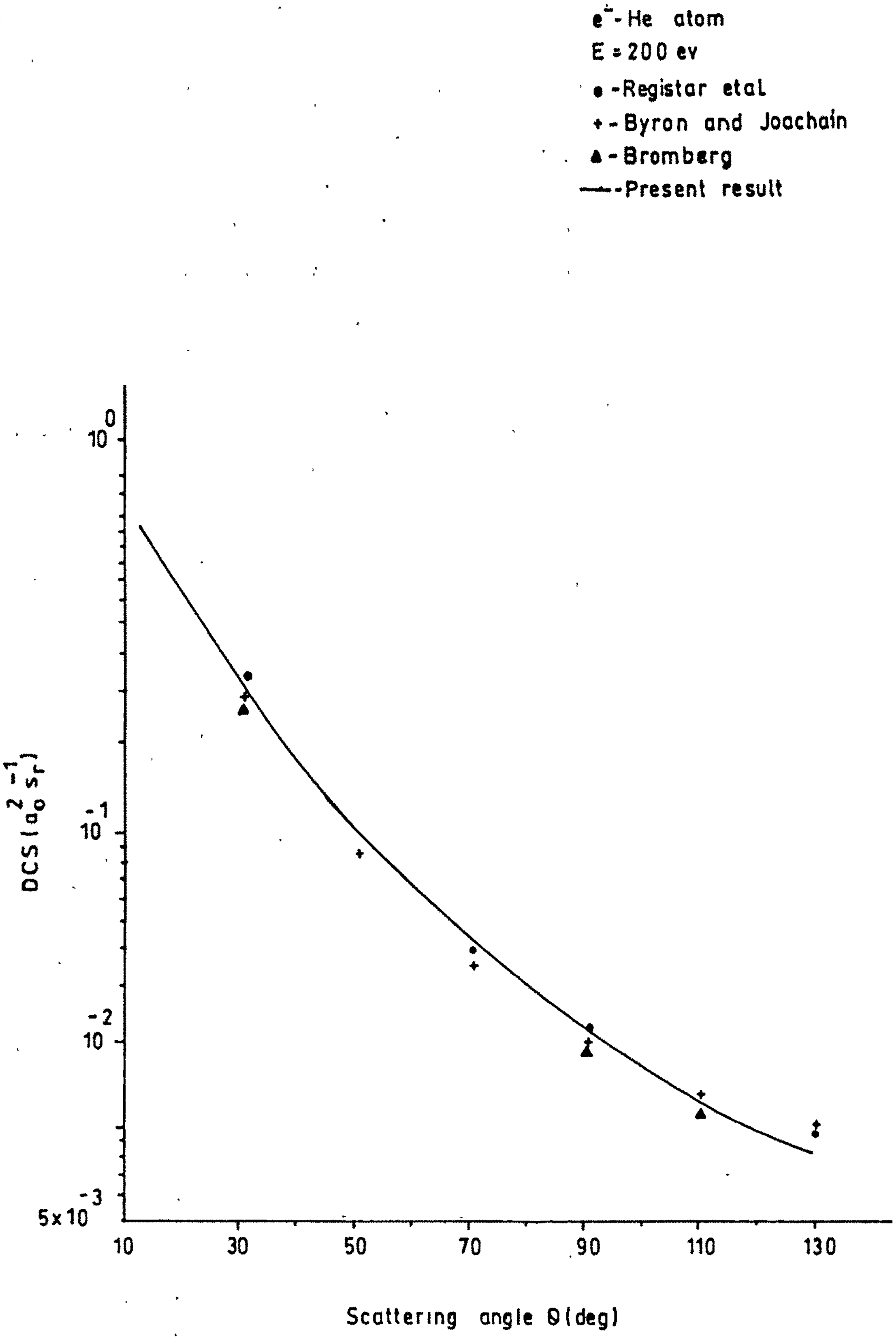

e$^-$- He atom
E = 200 ev
● - Registar etal
+ - Byron and Joachain
▲ - Bromberg
—— - Present result
10^0
10^{-1}
10^{-2}
5×10^{-3}
DCS ($a_0^2\,sr^{-1}$)
10
30
50
70
90
110
130
Scattering angle θ (deg)

Fig II-VI

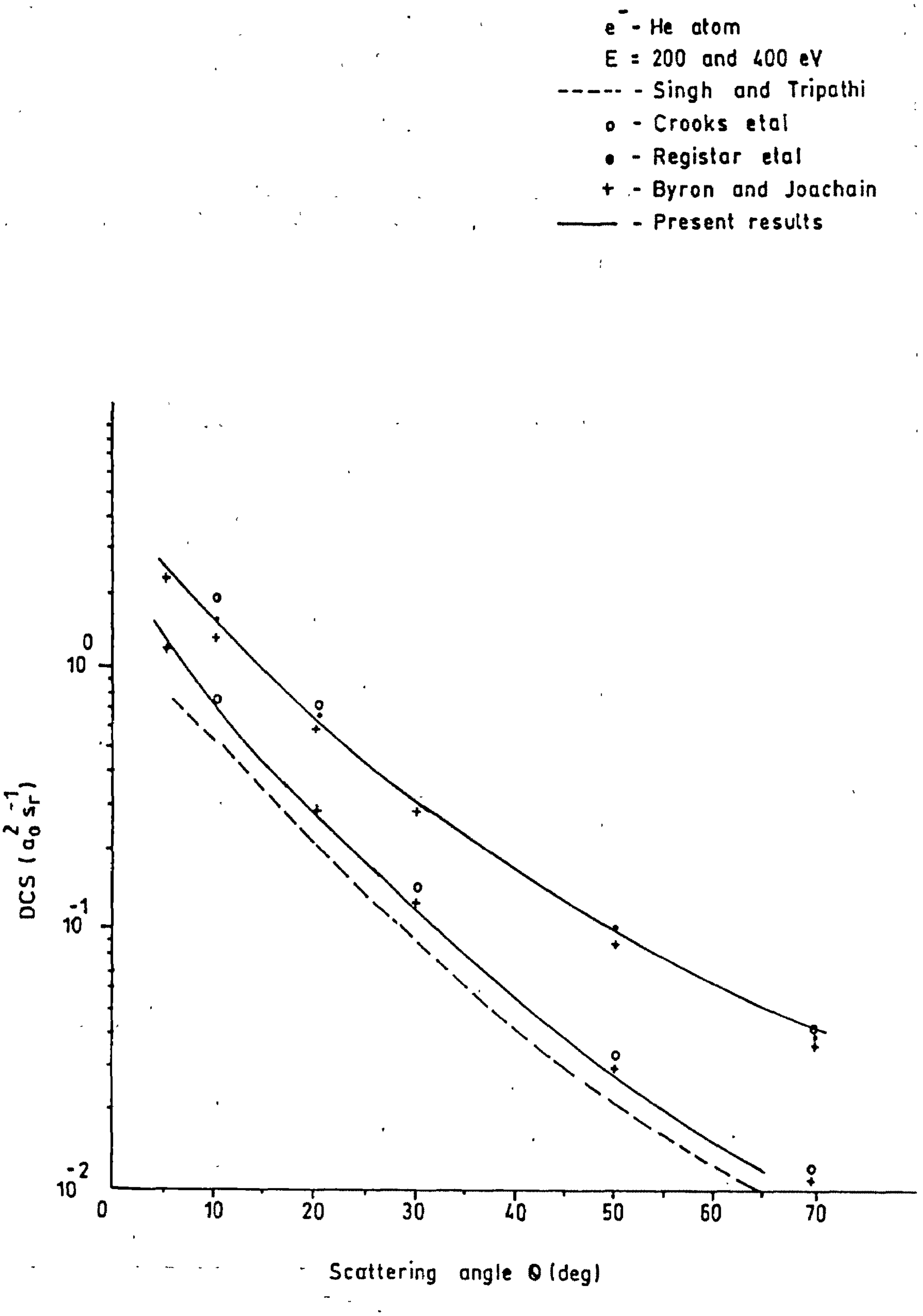
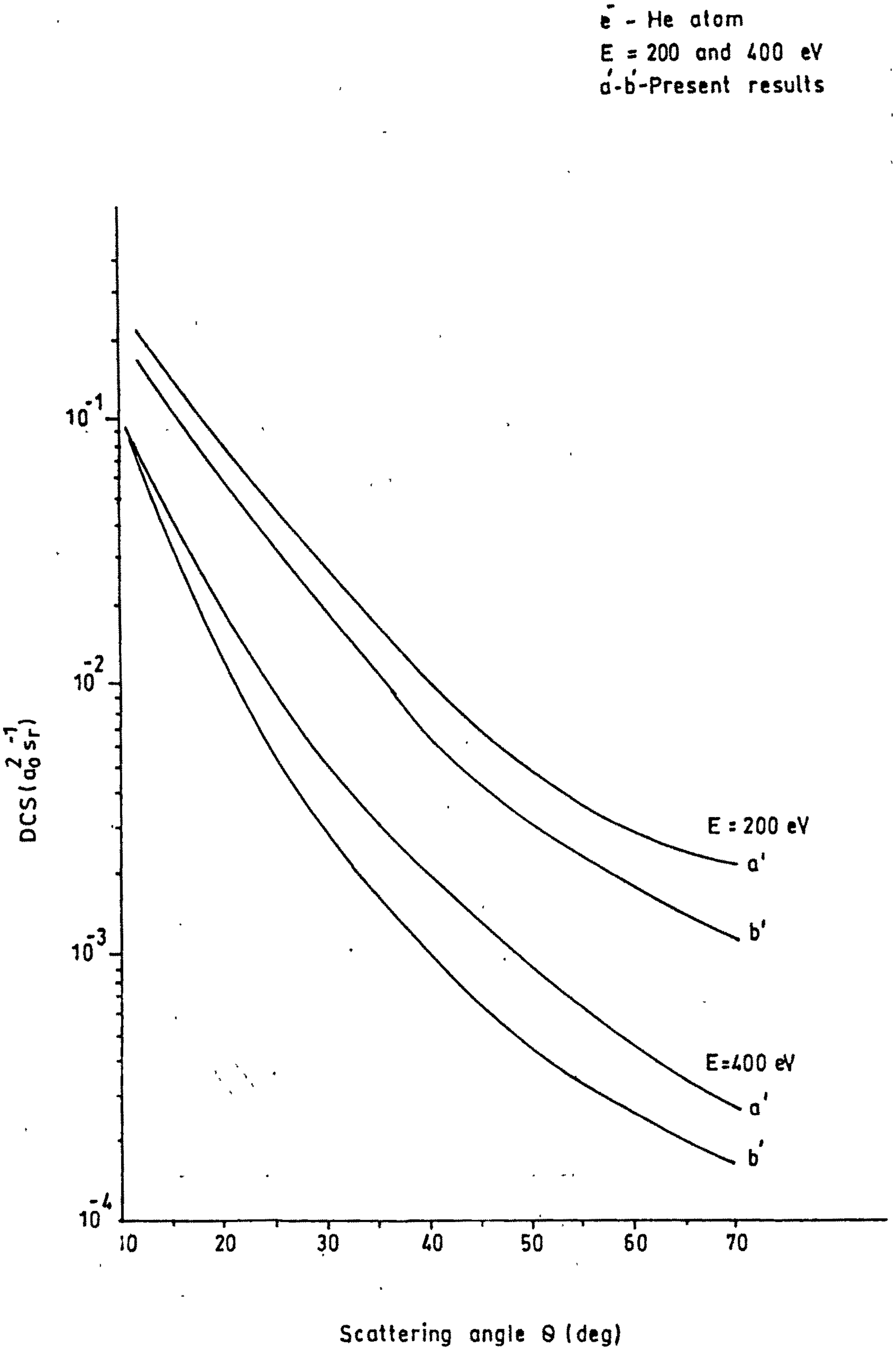

e⁻ - He atom
E = 200 and 400 eV
----- - Singh and Tripathi
o - Crooks etal
● - Registar etal
+ - Byron and Joachain
—— - Present results
DCS ($a_0^2 sr^{-1}$)
10^0
10^{-1}
10^{-2}
0
10
20
30
40
50
60
70
Scattering angle Θ (deg)

Fig II-VII


e⁻ - He atom
E = 200 and 400 eV
a'-b'-Present results
DCS ($a_0^2 sr^{-1}$)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10
20
30
40
50
60
70
Scattering angle θ (deg)
E = 200 eV
a'
b'
E=400 eV
a'
b'

e^{-}-Li ,elastic scattering using the refinement in the HHOB theory :

The study of electron - alkali atom - collision became an interesting area in the recent years both theoreticllay and experimentally. Among them is the discovery of alkali atoms in the atmosphere. The theoretical work was stimulated in the intermediate and high energy range due to availability of the experimental results of Willams and Crowe (1976). The part played by the alkali atoms in the Magneto -hydrodynamics is very important in the present day of energy crisis.

It is a known fact that the methods which applies well to the elastic scattering by the light atoms will not be too effective in the alkali atoms. It is due to the pecular nature of the alkali atoms. It is because of the quasi degeneracy of the ground and first excited states, there exists a strong coupling between these states. The large polarisability of the alkali atoms can be accounted mainly due to this coupling. The outermost electron in this atom is loosely bound S-electron hence the increased activity of these atoms. The absorption effect (removal of electrons from the elastic to the inelastic channel) also plays a important role in the alkali atom scattering. But, the Li atom being the first member of the alkali atoms, the above discussed deviations from the closed shell atoms will be a least case. Because of this it gives an oppurtinity to test the theoretical model which applies well to the lighter atoms like hydrogen and helium atom. There has been many attempts made to study the elastic e^-- Li atom scattering in the intermediate and high energy region. Sarkar etal (1973) used eikonal approximation to study the e^--Li elastic scattering for a wide energy range 0.8 eV to 500 eV. Gregory and Fink (1974) solved the relativistic Dirac equation to calculate DCS and TCS in the energy range 100 eV to 1.5 KeV. Chan and Chang (1976) applied Glauber approximation to obtain DCS at 100 eV, 200 eV and 400 eV. Vanderpoorten (1976) used a local optical potential consisting of static, polarisation, absorption and exchange effects to evaluate the DCS at 54.4 eV and 60 eV. Mukherjee and Sural (1979) used integral approach to the second order potential (SOP) to calculate DCS and TECS at 10 eV to 200 eV. Gien (1981) investigated the exchange effects in the frozen core Glauber approximation at 20 eV to 1000 eV. Tayal etal (1981) have calculated the DCS and TECS using corrected static approximation and in an approximation which combines the contribution of the non - static parts of the higher order terms in the Glauber approximation with the static part treated exactly for the enrgy range varying from 10 eV to 200 eV. Wadhera (1982) used first Born approximation along with the polarised Born amplitude to obtain integrated elastic cross sections from 500 eV to 1000 eV. Dhal (1982) used two potential formation in which the close encounter collision are treated exactly and the polarisation, exchange and absorption effects are treated through the optical eikonal approximation at 60 eV, 200 eV and 400 eV. Rao and Desai (1983b) used the high energy higher order Born (HHOB) approximation along with the Glauber eikonal series (GES) to calculate the DCS from 50 eV to 1000 eV and TCS from 100 eV to 700 eV. Tayal (1984) applied corrected static approximation to obtain TCS from 10 eV to 200 eV. Vijayshri (1985) evaluated the DCS and TCS using modified Glauber approximation (MGA) 20 eV to 1000 eV. They used the two models mainly, the single particle scattering model (SPSM) and the inert core (IC) model. They ignored the multiple scattering effects. Chandraprabha (1985) used modified Glauber eikonal series (MGES) and (GES) to calculate DCS in the energy region lying in between 100 eV to 800 eV. Yadav and Roy (1987) have calculated the DCS using the Coulomb -projected -Born approximation with Junker's modification to obtain the DCS for energy region 10 eV to 20 eV. Amongs all discussed above mentioned work, the results of Vanderpoorten (1976) and MGA (SPSM) of Vijayshri (1985) are reasonaly close to the experimental data. Suja and Desai (1988) used the extension of Rao and Desai (1983b) theory of HHOB approximation. Where the Li was taken as a three electron system. Where the long range polarisation and absorption effects are accounted.

Inspired by the success of applying the refinement in the HHOB theory we now extend the same for the lithium atom to study the elastic scattering process. The differential cross section are calculated in the energy range 100 eV to 700 eV . Here we also consider the Lithium atom as three electron system. The wave function for the ground state of Li atom has been considered as that of Veselov et al (1961) as quoted by Chan and Chang (1976),

$$\psi = \frac{1}{(3!)}^{1/2} \alpha \ et \ [\ \phi_{1SI}, \phi_{1SI}, \phi_{2SI}\ ] \tag{67}$$

with
$$\phi_{1S} = \left(\frac{\alpha^3}{\Pi}\right)^{1/2} e^{-\alpha r} \tag{68}$$

$$\phi_{2S} = \left[\frac{3\beta^5}{\Pi(\alpha^2 - \alpha\beta + \beta^2)}\right]^{1/2} \left(1 - \frac{\alpha + \beta}{3}\right) r \ e^{-\beta r} \tag{69}$$

where $\alpha = 2.694$ and $\beta = 0.767$. This wave function gives an energy of -7.414 a.u. against the experimental value of -7.478 a.u..

$$\psi \psi^* = \frac{1}{3!} \mid \det \ (\phi_{1SI} , \phi_{1SI} , \phi_{2SI}) \mid^2$$

$$= \phi^2_{1S}(r_1)\phi^2_{1S}(r_2)\phi^2_{2S}(r_3) - \phi^2_{1S}(r_1)\ \phi_{1S}(r_2)\ \phi_{2S}(r_2)\ \phi_{1S}(r_3)\ \phi_{2S}(r_3) \tag{70}$$

further,

$$\phi^*_{1S}\,\phi_{1S} = \frac{\lambda^3}{8\Pi}\ e^{-\lambda_1 r}\ , \quad \lambda_1 = 2\alpha \tag{71}$$

$$\phi^*_{2S}\phi_{2S} = 8\ N^2\ \mathrm{DOP}\ (\lambda_3, \lambda_2)\ \frac{1}{\lambda_2{}^3}\ \frac{\lambda_2{}^3}{8\Pi}\ e^{-\lambda_2 r} \tag{72}$$

where,

$$N^2 = \frac{-3\beta^5}{(\alpha^2 - \alpha\beta + \beta^2)}\ , \quad \lambda_2 = 2\beta\ , \quad \lambda_3 = \alpha + \beta \tag{73}$$

and
$$\mathrm{DOP}\ (\lambda_3, \lambda_2) = \left(1 + \frac{2\lambda_3}{3}\ \frac{\partial}{\partial\lambda_2} + \frac{\lambda_3{}^2}{9}\ \frac{\partial^2}{\partial\lambda_2{}^2}\right) \tag{74}$$

The interaction between the incident electron and the target lithium atom can be written as,

$$V_d = -\frac{3}{r_0} + \frac{1}{|r_0 - r_1|} + \frac{1}{|r_0 - r_2|} + \frac{1}{|r_0 - r_3|} \qquad (75),$$

where, r_0, r_1, r_2, r_3 are the position vectors of the incident and target electrons with respect to the target nuclie.

Now the closed form of the first Born approximation can be written as,

$$f^{(1)}_{HEA} = -\frac{1}{2\pi} \int dr_0\, e^{iq \cdot r_0} \int dr_1\, \psi_i^* \psi_f \left\{ -\frac{3}{r_0} + \frac{1}{|r_0 - r_1|} + \right.$$

$$\left. + \frac{1}{|r_0 - r_2|} + \frac{1}{|r_0 - r_3|} \right\} \qquad (76),$$

substitution of equn. (71,72,73) in the above expression yields the final form of the first Born term as,

$$f^{(1)}_{HEA} = 4\, \frac{(q^2 + 2\lambda_1^2)}{(q^2 + \lambda_1^2)^2} + 8\, N^2\, DOP\, (\lambda_3, \lambda_2)\, \frac{2}{2}3 \left(\frac{(q^2 + 2\lambda_2^2)}{(q^2 + \lambda_2^2)} \right)$$

(77).

similarly the imaginary term Im $F^{(2)}_{HEA}$ can be written as,

$$\text{Im}\, f^{(2)}_{HEA} = \frac{4\pi^3}{k_i} \int dp\, U^{(2)}_{fi}\, (q - p\, \beta_i \hat{y}\ ;\ p + \beta_i \hat{y}) \qquad (78),$$

where, $U^{(2)}_{fi}(\, q - p + \beta_i \hat{y}\ p + \beta_i \hat{y}\,) = \langle \psi_f^*(\, r_1, r_2, r_3\,) |$

$$\bar{V}\, (q - \beta_i \hat{y}\ ;\ r_1, r_2, r_3)\ \bar{V}\, (q - p - \beta_i \hat{y}\ ;\ r_1, r_2, r_3)$$

$$| \psi_i(r_1, r_2, r_3) > \qquad (79),$$

substitution of $\overline{V}$ from (9,10) and $\psi^* \psi$ from (70) we can write,

$$U^{(2)}_{fi}(\underline{q} - \underline{p} + \beta_i \hat{y} \,;\, \underline{p} + \beta_i \hat{y}) = \frac{1}{4\Pi^4(|\underline{q} - \underline{p}|^2 + \beta_i^2)(p^2 + \beta_i^2)}$$

$$\iiint [\, dv_1 dv_2 dv_3\, \phi^2_{1s}(r_1)\, \phi^2_{1s}(r_2) \phi^2_{2s}(r_3) - \phi^2_{1s}(r_1)\; \phi_{1s}(r_2)\; \phi_{2s}(r_2)$$

$$\phi_{2s}(r_3)\, \phi_{1s}(r_3)\,] \quad \sum_{j=1}^{3} \sum_{i=1}^{3} [\, e^{i\underline{p}\cdot \underline{b}_j + ip_z z_j} - 1][\, e^{i\underline{p}\cdot \underline{b}_i + ip_z z_i} - 1\,]$$

(80).

carrying out the integration of (78) and using (80) we can write the imaginary term as follows:

$$\mathrm{Im}\, f^{(2)}_{HEA} = \frac{1}{\Pi\, k_i}\Big[-3 + \frac{16\, \alpha^3 \lambda_1}{(\lambda_1^2 + q^2)^2} + 4N^2\, \mathrm{DOP}\left(-\frac{\partial}{\partial \lambda_2}\right)\left(\frac{1}{\lambda_2^2 + q^2}\right)$$

$$I_1(q^2, \beta_i^2, \beta_i^2) + 32\, \alpha^6 \left(-\frac{\partial}{\partial \lambda_1}\right) \frac{1}{\lambda_1^2} \left(-\frac{\partial}{\partial \lambda_1}\right) \frac{1}{\lambda_1^2}\; I(q^2, \gamma_1^2, \gamma_2^2) \; +$$

$$16\alpha^3 \left(-\frac{\partial}{\partial \lambda_1}\right) \frac{1}{\lambda_1^2}\; I(q^2, \gamma_1^2, \beta_i^2) + 8N^2\, \mathrm{DOP}\left(-\frac{\partial}{\partial \lambda_2}\right) \frac{1}{\lambda_2^2}\; I(q^2, \gamma_2^2, \beta^2)$$

$$+\, 64\, N^2 \alpha^2 \left(-\frac{\partial}{\partial \lambda_1}\right) \frac{1}{\lambda_1^2}\; \mathrm{DOP}\left(-\frac{\partial}{\partial \lambda_2}\right) \frac{1}{2}\, 2 \;\; I_1(q^2, \gamma_1^2, \gamma_2^2) + \frac{32}{\Pi k_i}\Big[\frac{4}{(\lambda_3)}\, 6\, B^2$$

$$-\, \frac{6BB^1}{\lambda_3} + \frac{9\, B'^2}{\lambda_3^2}\; I_1(q^2, \beta_i^2, \beta_i^2) - \frac{4}{\lambda_3^2} \Big\{ \left(\frac{8\, B\, B'}{\lambda_3^3} + \frac{12\, B'^2}{\lambda_3^4} \right) \frac{\partial}{\partial \beta}\, \frac{1}{\lambda_3^2} \Big\}$$

$$I_1(q^2,u^2,\beta_i^2) + B\left(-\frac{\partial}{\partial\beta}\right)\frac{1}{\lambda_3^2}\left(-\frac{\partial}{\partial\beta}\right)\frac{1}{\lambda_3^2} + 2BB'\left(-\frac{\partial}{\partial\beta}\right)\frac{1}{\lambda_3^2}\frac{\partial^2}{\partial\beta^2}$$

$$+ B'^2 \frac{\partial^2}{\partial\beta^2}\frac{1}{\lambda_3^2}\frac{\partial^2}{\partial\beta^2}\frac{1}{\lambda_3^2} I_1(q^2,u_1^2,u_2^2)] , \quad (81)$$

where, $\gamma_1^2 = \beta_i^2 + \lambda_1^2$; $\gamma_2^2 = \beta_i^2 + \lambda_2^2$; $u^2 = \beta_i^2 + \lambda_3^2$ and

$$B = \left(\frac{\alpha^3}{\Pi}\right)^{1/2} \left[\frac{3\beta^5}{\Pi\ (\alpha^2 - \alpha\beta + \beta^2)} \right]^{1/2} \text{ and } B' = \left(\frac{\lambda_3}{3}\right) B.$$

Integrals of the form $I_1(\ldots\ldots)$ is defined in the appendix. Here $\beta_i = \Delta E/k_i$, ΔE the excitation energy. Here for lithium atom $\Delta E = 0.08825$ is taken, which was calculated by Vijayshri (1985). Further, the terms corresponding to real part i.e. Rel $f^{(2)}_{HEA}$ can be written as follows :

$$\text{Rel } f^{(2)}_{HEA} = \text{Rel 1 } f^{(2)}_{HEA} + \text{Rel 2 } f^{(2)}_{HEA} \quad (82),$$

where,

$$\text{Rel 1 } f^{(2)}_{HEA} = - \frac{4\Pi^2}{k_i} \mathcal{P} \int d\underline{p} \int_{-\infty}^{+\infty} \frac{dp_z}{p_z - \beta_i}\ U^{(2)}_{fi} \quad (83),$$

$$\text{Rel 2 } f^{(2)}_{HEA} = - \frac{2\Pi^2}{k_i^2} \mathcal{P} \int d\underline{p} \int_{-\infty}^{+\infty} dp_z (\ p^2 + p_z^2\ )\ U^{(2)}_{fi} \quad (84),$$

Where symbols have their usual meanings as defined earlier. Again using the same mathematical treatment for the lithium also we can write the solution of the above equation as follows :

$$\text{Rel } 1\ f^{(2)}_{HEA} = \frac{1}{\Pi^2 k_i} \left[-3 + \frac{16\,\alpha^3 \lambda_1}{(\lambda_1^2 + q^2)^2} + 4N^2\ \mathrm{DOP} \left(-\frac{\partial}{\partial \lambda_2}\right) \frac{1}{(\alpha_2^2 + q^2)^2} \right.$$

$$I_2(q^2, \beta_i^2, 0) + 24\,\alpha^3 \left(-\frac{\partial}{\partial \lambda_1}\right) \frac{1}{\lambda_1^2}\, I_2(q^2, \beta_i^2, \lambda_1^2) + 8N^2\ \mathrm{DOP}$$

$$\left(-\frac{\partial}{\partial \lambda_2}\right) \frac{1}{\lambda_2^2}\, I_2(q^2, \beta_1^2, \lambda_2^2) - 32\,\alpha^6 \left(-\frac{\partial}{\partial \lambda_1}\right) \frac{1}{\lambda_1^2} \left(-\frac{\partial}{\partial \lambda_1}\right) \frac{1}{\lambda_1^2}$$

$$I_4(q^2, \beta_1^2, \lambda_1^2, \lambda_2^2) + 64\,\alpha^6 \left(-\frac{\partial}{\partial \lambda_1}\right) \frac{1}{\lambda_1^2} \left(-\frac{\partial}{\partial \lambda_1}\right) \frac{1}{\lambda_1^2}\, I_2(q^2, \beta_i^2, \lambda_1^2, \lambda_2^2)$$

$$-\,64\,\alpha^3 N^6 \left(-\frac{\partial}{\partial \lambda_1}\right) \frac{1}{\lambda_1^2}\ \mathrm{DOP} \left(-\frac{\partial}{\partial \lambda_2}\right) \frac{1}{\lambda_2^2}\, I_4(q^2\, \beta_i^2, \lambda_1^2, \lambda_2^2) - 2\,(4\Pi)^2$$

$$\left\{ -\frac{4}{\lambda_3^6} \left(B^2 - \frac{6BB'}{\lambda_3} + \frac{9B'}{\lambda_3}\right)^2 I_2(q^2, \beta_i^2, 0) \right\} - \frac{2}{\lambda_3^2} \left(B^2 - \frac{6BB'}{\lambda_3}\right) \left\{ \left(-\frac{\partial}{\partial \beta}\right) \right.$$

$$\left. \frac{1}{\lambda_3^2}\, I_2(q^2\, \beta_i^2, \lambda_3^2) + \frac{6\beta'^2}{\lambda_3^2} \left(-\frac{\partial}{\partial \beta}\right)^2 \frac{1}{\lambda_3^2}\, I_2(q, \beta_1^2 0, \lambda_3^2) \right\} - \frac{2\beta^2}{(\alpha+\beta)^3}$$

$$\left(-\frac{\partial}{\partial \beta}\right) \frac{1}{(\alpha+\beta)^2}\, I_4(q^2, \beta_1^2, 0, \lambda_3^2) + B^2 \left(-\frac{\partial}{\partial \beta}\right) \frac{1}{\lambda_3^2} \left(-\frac{\partial}{\partial \beta}\right) \frac{1}{\lambda_3^2}$$

$$I_4(q^2\, \beta^2, \lambda_3^2, \lambda_3^2) + \frac{4BB'}{\lambda_3^2} \left(-\frac{\partial}{\partial \beta}\right)^2 I_4(q^2\, \beta_i^2, 0, \lambda_3^2) - 2\, B\, B'$$

$$\left(-\frac{\partial}{\partial \beta}\right) \frac{1}{\lambda_3^2} \left(-\frac{\partial}{\partial \beta}\right)^2 \frac{1}{\lambda_3^2}\, I_4(q^2\, \beta_i^2, \lambda_3^2, \lambda_3^2) - \frac{6B'}{\lambda_3^2} \left(-\frac{\partial}{\partial \beta}\right) \frac{1}{\lambda_3^2}$$

$$\left(-\frac{\partial}{\partial \beta}\right) \frac{1}{\lambda_3^2} I_4(q^2, \beta_1^2, \lambda_3^2, \lambda_3^2) \qquad (85).$$

For the calculation of Rel 2 $f_{HEA}^{(2)}$ we have omitted the cross terms in $U_{fi}^{(2)}$. Because contributions are negligibly small. Hence

$$\mathrm{Re}\ 2 f_{HEA}^{(2)} = -\frac{1}{2} (\pi k_i)^{-2} [-6 \alpha^3 \left(\frac{\partial}{\partial \beta_i}\right) \left(-\frac{\partial}{\partial \lambda_1}\right) I_2(q^2, \beta_i^2, \lambda_1^2) +$$

$$32\alpha^6 \left(\frac{\partial}{\partial \beta_i}\right) \left(\frac{\partial^2}{\partial \lambda_1^2}\right) \frac{1}{\lambda_1^2} I_2 (q^2, \beta_i^2, \lambda_1^2, \lambda_1^2) - 16 N^2 \left(\frac{\partial}{\partial \beta}\right) \mathrm{DOP}$$

$$\left(-\frac{\partial}{\partial \lambda}\right)_2 I_2(q^2, \beta_i^2, \lambda_2^2) - 64 \alpha^3 N^2 \left(\frac{\partial}{\partial \beta}\right) \mathrm{DOP} \left(-\frac{\partial}{\partial \lambda}\right)_2 \left(-\frac{\partial}{\partial \lambda}\right)_1$$

$$\frac{1}{\lambda_1^2} I_4(q^2, \beta_i^2, \lambda_1^2, \lambda_1^2) \qquad (86).$$

The integrals I_2 (...) and I_4 (.....) are evaluated in the appendix. The terms corresponding to $\beta = 0$ and for finite value of β is calculated using the equation (25), where the use of the Cox -Bonham parameters are used to evaluate the terms like I's for γ's and λ's. The values of Cox and Bonham (1967) parameters for the lithium atom i.e. values of γ's and λ's are given as follows:

$\gamma_1 = 1.3215$, $\gamma_2 = -0.2273$, $\gamma_3 = 1.3369$, $\gamma_4 = 1.4070$,
$\gamma_5 = -1.6110$, $\gamma_6 = -0.9567$ and,
$\lambda_1 = 0.8737$, $\lambda_2 = 7.9222$, $\lambda_3 = 2.2645$, $\lambda_4 = 3.8024$, $\lambda_5 = 1.3839$,
$\lambda_6 = 2.7065$.

The differential cross sections is obtained using the following relation:

$$\frac{d\sigma}{d\Omega} = | f_{HHOB}^{d} |^2 \qquad (87).$$

where the term f^{d}_{HHOB} has ususal meanings given in equation (21). We obtain the third GES term i.e. fg_3 as follows:

Calculation of third GES term ($f^{(3)}_{GES}$) for Lithium atom :

We obtain the expression for $f^{(3)}_{GES}$ using,

$$f^{(n)}_{i\text{-->}f} = \frac{k!}{2\pi n!} \int d\mathbf{b}_{\emptyset} \; e^{i\,\mathbf{q}\cdot\mathbf{b}_{\emptyset}} \; \langle \psi_f \mid x^n \mid \psi_i \rangle \qquad (88),$$

where substitution of n = 3 yield the expression for Li atom. Hence, for Li atom we can write,

$$f_{G3} = f^{1}_{G3} + f^{2}_{G3} \qquad (89),$$

where,

$$f^{1}_{G3} = 2\, f^{H}_{G3}\,(\lambda_i \,,\, q\,) + 8\, N^2\, \mathrm{DOP}\,(\lambda_2 \,,\, \lambda_3) - \frac{1}{3}\, f^{H}_{G3}\,(\lambda_2 \,,\, q) \qquad (90),$$

where,

$$f^{H}_{G3}\,(\lambda_1, q) = \lambda_1\big(\, 16\big(\frac{\lambda}{2}\big)\, k_1^2\, X^3\big)^{-1} \big(\frac{\partial}{\partial X}\big) \big(\frac{X^4}{1+X^2}\big) \{\, 4\, [\, \ln\big(\frac{1+X^2}{X}\big)]^2 + \frac{\pi^2}{3} - 2A(X)\, \} \qquad (91),$$

$$A(X) = 2\, \ln(X)^2 + \frac{\pi^2}{6} \sum_{n=1}^{\infty} \frac{(-x^2)^n}{n^2} \quad \text{for} \quad X \le 1$$

$$= - \sum_{n=1}^{\infty} \frac{(-X^{-2})^n}{n^2} \quad \text{for} \quad X > 1 \qquad (92),$$

X being equals to q/λ_1. Now, f^{2}_{G3} can be written as

$$= -\,(\underline{k}_i^{-2}) \int_0^\infty \underline{b}d\underline{b}\; J_0(\underline{q}\,\underline{b})\,[I_1I_2 + I_2I_3 + I_4I_1 + I_1^2I_3 - I_5I_6 - I_1I_5^2\,] \tag{93}$$

where,

$$I_1 = \langle\phi_{1s}|AI|\phi_{1s}\rangle \quad ; \quad I_2 = \langle\phi_{1s}|AII|\phi_{2s}\rangle \quad ; \quad I_3 = \langle\phi_{2s}|AI|\phi_{2s}\rangle$$

$$I_4 = \langle\phi_{2s}|AII|\phi_{2s}\rangle \quad ; \quad I_5 = \langle\phi_{1s}|AI|\phi_{2s}\rangle \quad ; \quad I_6 = \langle\phi_{1s}|AI|\phi_{2s}\rangle \qquad |(94),$$

and

$$AI = \ln\left(1 - \frac{2bb_1}{b}\cos\phi_1 + \left(\frac{b_1}{b}\right)^2\right)$$

$$AII = \ln^2\left(1 - \frac{2bb_1}{b}\cos\phi_1 + \left(\frac{b_1}{b}\right)^2\right) \qquad (95),$$

substituting the value of ϕ_{1s} and ϕ_{2s} from (68) and (69) and carrying out the integration over z , we obtain the terms

$$I_1 = (\lambda_1)^3(4\pi)^{-1} \int_0^\infty b_1^2\, db_1\; K_1(\lambda_1, b_1)\; ALN_1(b_1, b) \qquad (96a),$$

$$I_2 = (\lambda_1)^2(4\pi)^{-1} \int_0^\infty b_1^2\, db_1\; K_1(\lambda_1, b_1)\; ALN_2(b_1, b) \qquad (96b),$$

$$I_3 = (2N^2)\,\pi^{-1} \int_0^\infty b_1^2\, db_1\; DOP(\lambda_3, \lambda_2)\; K_1(\lambda_2, b_1)\; ALN_1(b_1, b) \qquad (96c),$$

$$I_4 = (2N^2)\,\pi^{-1} \int_0^\infty b_1^2\, db_1\; DOP(\lambda_3, \lambda_2)\; K_1(\lambda_2, b_1)\; ALN_2(b_1, b) \qquad (96d),$$

$$I_5 = \left(\frac{\lambda_1^3 N^2}{2}\right)^{1/2} \pi^{-1} \int_0^\infty b_1^2\, db_1\; \left(1 + \frac{\lambda_3}{3}\frac{\partial}{\partial\lambda_3}\right) K_1(\lambda_3, b_1)\; ALN_1(b_1, b) \qquad (96e),$$

$$I_6 = \left(\frac{N^2}{2}\right)^{1/2} \Pi^{-1} \int_0^{\infty} b_1^2 \, db_1 \left(1 + \frac{\lambda_3}{3} \frac{\partial}{\partial \lambda_3}\right) K_1(\lambda_3, b_1) \, ALN_2(b_1, b)$$

where, (96f),

$$ALN_1(b_1, b) = \int_0^{2\Pi} Ln \left(1 - \frac{2b_1}{b} \cos\phi_1 + \left(\frac{b_1}{b}\right)^2\right) d\phi_1 \qquad (96g),$$

$$ALN_2(b_1 . b) = \int_0^{2\Pi} Ln^2 \left(1 - \frac{2b_1}{b} \cos\phi_1 + \left(\frac{b_1}{b}\right)^2\right) d\phi_1 \qquad (96h),$$

and $K_1(\lambda, b)$ are the modified Bessel functions of the second kind. The integrals (92) and (93) are then evaluated using the standard numerical techniques (Vijayshri, 1985). Thus using the above set of equations we have obtained the DCS for elastic scattering by Li atom from 100 to 400 eV.

RESULTS AND DISCUSSION :

The present results for the elastic scattering of electrons by Li atom are tabulated in the tables (II.XIV) to (II.XX). Results are tabulated for the energy of incidence varying from 100 to 400 eV. As stated earliar there is no divergent integrals and is computationally very simple. Also if one substitute $\beta = 0$ in the present approximation terms, the corresponding terms in the Glauber Eikonal Born series (GES) is obtained. The imaginary term will not diverge due to the presence of β in the expression. Results are compared with the other theoretical results of Vijayashri (1985), Rao and Desai (1983) , Chandraprabha (1986) and Suja and Desai (1988).

The present results are quite good in agreement with MGA (SPSM) than MGA (IC) of Vijayshri (1985) for all angles. In MGA (IC) core was ignored. Hence the results are lower in small angles. In any model the inclusion of core is necessary due to the deeper penetration of the incident particle into the atomic core (Chan and Chang, 1976). The results of Yadav and Roy (1986) are also compared. They used the single electron system, causing decrease in the results for 100 eV and 200 eV for all angles. Figures II. and II. results of 100 eV and 200 eV are plotted along with the other results. From figures we can say that the results of Rao and Desai are lower estimated for all angles. Where they treat lithium as a single electron system. This emphasis the fact that one must considered the lithium as a three electron system. From the terms of the HHOB theory we can see that contribution of the imaginary term is more important than the terms corresponds to the polarisation in the alkali atom scattering.

The EBS method which gives good results for e^--H and e^-- He gives higher values in case of e^-- Li scattering.This may be due to the fact of the cancellation of the higher order Born terms in e^-- Li scattering. Due to very limited experimental data for e^-- Li scattering in these energy range it is impossible to have a complete analysis for the present calculation. Having obtain success in case of e^-- H and e^-- He scattering we extend it to the e^-- Li scattering. Hence not only the refinement is applied but we considered the lithium as three electron system. The results obtained through are well compared with the other theoretical results and the experimental data available. It is observed that the results have improved when it is compared with the other methods.

Table II.XIV to II.XVII describes the behaviour of the terms of the HHOB scattering amplitudes (equation, 21) for the energy of incidence 100, 300, 200 and 400 eV respectively in the angular range 5 to 120 deg. The terms are expressed in a_0^2 / sr.

Tables II.XVIII to II.XX compares the results produced after the refinement to the HHOB approximation of Yates (1979) for the energy varying from 100, 200 to 400 eV in the angular range 5 to 150 deg. As stated above as the energy of incidence increases beyond 400 eV or so the results are not upto mark.

Figures II. to II. also compares our results with the methods described above.

Hence, here we have considered the lithium atom as a three electron system as well as the refinement is applied to the HHOB approximation, i.e. inclusion of certain low lying energy states in the calculation of the differential cross sections.

Hence we find that in general without going in for much computational complexity of the higher order Born terms, if certain low lying energy states are included then the accuracy in the measurements of the cross sections can also increases further.

TABLE II.XIV

Behaviour of the individual terms of the refinment in the HHOB theory for Lithium atom scattering at 100 eV. [Results are expressed in a_0^2/Sr.]

Angle (deg.)	(1) $f_{i\text{-->}f}$	(2) Im f_{HEA}	(2) Re 1 f_{HEA}	(2) Re 2 f_{HEA}	(3) f_{GES}
5.0	5.3104	4.619	1.315	4.04 (-1)	1.47
10.0	4.3589	2.053	1.305	2.246 (-1)	1.79
15.0	3.286	1.24	1.17	7.10 (-1)	1.86
20.0	2.3875	1.021	1.05	1.017 (-1)	1.77
25.0	1.741	9.53 (-1)	9.56 (-1)	3.68 (-1)	1.64
30.0	1.3077	8.976 (-1)	8.898 (-1)	9.77 (-1)	1.53
40.0	8.2825(-1)	7.637 (-1)	7.90 (-1)	2.28 (-1)	1.40
50.0	5.9833(-1)	6.877 (-1)	7.057 (-1)	1.117 (-1)	1.33
60.0	4.6904(-1)	5.406 (-1)	6.29 (-1)	6.14 (-1)	1.16
70.0	3.8601(-1)	4.69 (-1)	5.608 (-1)	3.37 (-1)	1.11
80.0	3.2863(-1)	4.158 (-1)	5.015 (-1)	1.95 (-1)	1.00
90.0	2.8809(-1)	3.758 (-1)	4.511 (-1)	1.18 (-1)	9.55(-1)
100.0	2.5690(-1)	3.45 (-1)	4.09 (-1)	7.4 (-1)	9.80(-1)
110.0	2.3240(-1)	3.213 (-1)	3.75 (-1)	4.74 (-1)	8.60(-1)
120.0	2.1210(-1)	3.03 (-1)	3.47 (-1)	3.10 (-1)	8.60(-1)

TABLE II.XV

Behaviour of the individual terms of the refinment in the HHOB theory for Lithium atom scattering at 300 eV.

Angle (deg.)	$f^{(1)}_{i\text{-->}f}$	$\text{Im } f^{(2)}_{HEA}$	$\text{Re1 } f^{(2)}_{HEA}$	$\text{Re2 } f^{(2)}_{HEA}$	fg_3
5.0	4.64	1.33	7.82 (-1)	1.27 (-1)	5.77(-1)
10.0	2.83	5.9(-1)	6.442 (-1)	1.23 (-1)	6.0 (-1)
20.0	1.02	4.694 (-1)	4.896 (-1)	1.296 (-2)	4.8 (-1)
30.0	5.48(-1)	3.43 (-1)	3.96 (-1)	3.78 (-3)	4.05(-1)
40.0	3.57(-1)	2.59 (-1)	3.16 (-1)	1.13 (-3)	3.6 (-1)
50.0	2.65(-1)	2.07 (-1)	2.51 (-1)	3.8 (-4)	3.01(-1)
60.0	2.13(-1)	1.736 (-1)	2.01 (-1)	1.3 (-4)	2.87(-1)
70.0	1.68(-1)	1.45 (-1)	1.55 (-1)	3.522 (-5)	2.57(-1)
80.0	1.42(-1)	1.325 (-1)	1.36 (-1)	1.563 (-5)	2.31(-1)
90.0	1.22(-1)	1.193 (-1)	1.153 (-1)	1.12 (-5)	2.26(-1)

TABLE II.XVI

Behaviour of the individual terms of the refinment in the HHOB theory for Lithium atom scattering at 200 eV.

Angle (deg.)	(1) $f_{i \to f}$	(2) Im f_{HEA}	(2) Re 1f_{HEA}	(2) Re 2 f_{HEA}	fg_3
5.0	4.9598	2.16	9.773 (-1)	2.109 (-1)	8.170(-1)
10.0	3.4641	8.826 (-1)	8.48 (-1)	5.94 (-1)	9.300(-1)
15.0	2.198	6.81 (-1)	7.273 (-1)	3.193 (-1)	8.67 (-1)
20.0	1.4213	6.34 (-1)	6.4881(-1)	2.34 (-1)	7.810(-1)
25.0	9.9090(-1)	5.74 (-1)	5.93 (-1)	9.77 (-2)	7.150(-1)
30.0	7.4431(-1)	5.063 (-1)	5.447 (-1)	1.1 (-2)	6.640(-1)
40.0	4.9194(-1)	3.92 (-1)	4.58 (-1)	3.9 (-3)	6.440(-1)
50.0	3.6469(-1)	3.15 (-1)	3.83 (-1)	1.55 (-3)	5.340(-1)
60.0	2.8809(-1)	2.642 (-1)	3.2 (-1)	6.6 (-4)	4.850(-1)
70.0	2.3452(-1)	2.285 (-1)	2.69 (-1)	2.80 (-4)	4.450(-1)
80.0	1.9748(-1)	2.02 (-1)	2.29 (-1)	3.6 (-4)	4.250(-1)
90.0	1.7029(-1)	1.83 (-1)	1.982 (-1)	6.17 (-4)	3.920(-1)
100.0	1.5170(-1)	1.67 (-1)	1.74 (-1)	3.26 (-4)	3.720(-1)
110.0	1.3420(-1)	1.56 (-1)	1.55 (-1)	2.98 (-4)	3.420(-1)
120.0	1.2250(-1)	1.464 (-1)	1.41 (-1)	2.594 (-4)	3.390(-1)

TABLE II.XVII

Behaviour of the individual terms of the refinment in the HHOB theory for Lithium atom scattering at 400 eV.

Angle (deg.)	$f^{(1)}_{HEA}$	Im $f^{(2)}_{HEA}$	Re1 $f^{(2)}_{HEA}$	Re2 $f^{(2)}_{HEA}$	fg_3
5.0	4.36	9.26 (-1)	6.585 (-1)	8.2 (-1)	4.480(-1)
10.0	2.37	4.7 (-1)	5.285 (-1)	7.85 (-1)	4.410(-1)
15.0	1.29	4.38 (-1)	4.5 (-1)	2.25 (-1)	3.810(-1)
20.0	8.08(-1)	3.68 (-1)	3.98 (-1)	1.26 (-1)	3.390(-1)
30.0	4.46(-1)	2.57 (-1)	3.09 (-1)	5.61 (-3)	2.840(-1)
40.0	4.26(-1)	1.92 (-1)	2.357 (-1)	4.2 (-3)	1.900(-1)
50.0	2.49(-1)	1.54 (-1)	1.804 (-1)	1.1 (-3)	1.850(-1)
60.0	1.70(-1)	1.29 (-1)	1.40 (-1)	2.00 (-4)	1.760(-1)
70.0	1.48(-1)	1.1 (-1)	1.12 (-1)	1.90 (-4)	1.700(-1)
80.0	1.19(-1)	9.77 (-2)	9.17 (-2)	1.51 (-4)	1.600(-1)
90.0	9.46(-2)	8.76 (-2)	7.70 (-2)	1.05 (-4)	1.520(-1)
100.0	8.56(-2)	7.97 (-2)	6.635 (-2)	1.3 (-4)	1.420(-1)
110.0	7.51(-2)	7.35 (-2)	5.84 (-2)	1.21 (-4)	1.320(-1)
120.0	6.52(-2)	6.86 (-2)	5.25 (-2)	1.1 (-4)	1.270(-1)

TABLE II.XVIII

Differential cross section ($a_0^2\ S_r^{-1}$) for elastic scattering of electron by Lithium using refinment in the HHOB theory atom at 100 eV.

Angle (Deg.)	MGA (IC)	MGA(SPSM)	EBS	Rao & Desai(1983)	Suja & Desai(1987)	present results
5.0	4.49(+1)	5.53(+1)	5.29(+1)	----	5.3778(+1)	5.18(+1)
10.0	1.75(+1)	2.22(+1)	1.82(+1)	2.797(+1)	2.2760(+1)	1.76(+1)
15.0	7.7	1.05(+1)	7.47	----	9.6669	7.21
20.0	3.48	5.3	3.73	7.212	4.4368	3.98
25.0	1.67	2.88	2.40	----	2.5995	2.49
30.0	8.74(-1)	1.7	1.85	2.613	1.7316	1.782
40.0	3.02(-1)	7.20(-1)	----	1.274	7.0060(-1)	7.803(-1)
50.0	1.31(-1)	3.66(-1)	----	7.446(-1)	5.2992(-1)	5.312(-1)
60.0	6.66(-2)	2.12(-1)	8.89(-1)	4.915(-1)	4.1575(-1)	6.781(-1)
70.0	3.80(-2)	1.35(-1)	----	3.538(-1)	2.6970(-1)	2.891(-1)
80.0	2.38(-2)	9.30(-2)	----	2.721(-1)	1.4017(-1)	1.563(-1)
90.0	1.61(-2)	6.78(-2)	6.16(-1)	2.200(-1)	1.2943(-1)	5.894(-2)
100.0	1.15(-2)	5.18(-2)	----	----	6.4789(-2)	6.672(-2)
110.0	8.74(-3)	4.12(-2)	----	1.605(-1)	3.0652(-2)	3.814(-2)
120.0	6.94(-3)	3.41(-2)	4.99(-1)	1.430(-1)	2.9647(-2)	2.983(-2)
130.0	5.75(-3)	2.91(-2)	----	1.303(-1)	1.8923(-2)	1.982(-2)
140.0	4.95(-3)	2.56(-2)	----	----	1.1558(-2)	1.173(-2)
150.0	-----	2.32(-2)	4.43(-1)	----	1.1246(-2)	1.153(-2)

TABLE II.XIX

Differential cross section ($a_0^2 S_r^{-1}$) for elastic scattering of electron by Lithium atom at 200 eV.

Angle (Deg.)	MGA (IC)	MGA(SPSM)	EBS	Rao & Desai	Suja & Desai	present results
5.0	2.60(+1)	3.12(+1)	2.86(+1)	----	3.3954(+1)	2.891(+1)
10.0	9.47	1.20(+1)	9.93	1.3691(+1)	1.3476(+1)	9.983
15.0	3.26	4.70	3.69	----	5.6950	3.711
20.0	1.20	2.04	1.36	2.3859	2.5331	1.368
25.0	5.11(-1)	1.04	9.89(-1)	----	1.4557	1.017
30.0	2.51(-1)	5.95(-1)	6.70(-1)	7.725(-1)	9.9444(-1)	8.891 (-1)
40.0	8.18(-2)	2.54(-1)	3.90(-1)	3.6646(-1)	4.3140(-1)	4.124 (-1)
50.0	3.49(-2)	1.33(-1)	2.70(-1)	2.1445(-1)	2.7583(-1)	2.731 (-1)
60.0	1.77(-2)	7.95(-2)	2.06(-1)	1.4311(-1)	1.0000(-1)	2.113 (-1)
70.0	6.02(-2)	5.15(-2)	1.68(-1)	1.0411(-1)	9.7470(-2)	1.72 (-1)
80.0	1.42(-3)	3.56(-2)	1.42(-1)	8.0552(-2)	5.4285(-2)	1.371 (-1)
90.0	4.37(-3)	2.58(-2)	1.24(-1)	6.5112(-1)	3.5685(-2)	1.321 (-1)
100.0	3.16(-3)	1.96(-2)	1.11(-1)	4.7356(-2)	2.1343(-2)	1.176 (-1)
110.0	2.41(-3)	1.55(-2)	1.01(-1)	----	1.6391(-2)	1.122 (-1)
120.0	1.93(-3)	1.27(-2)	9.31(-2)	4.2029(-2)	7.9070(-3)	1.0201(-1)
130.0	1.60(-3)	1.08(-2)	8.71(-2)	1.8305(-2)	3.9626(-3)	9.721 (-2)
140.0	1.38(-3)	9.47(-3)	8.26(-2)	----	----	8.762 (-2)
150.0	1.24(-3)	8.55(-3)	7.93(-2)	----	1.0856(-3)	8.543 (-2)

TABLE II.XX
Differential cross section ($a_0^2 S_r^{-1}$) for elastic scattering of electron by Lithium atom at 400 eV.

Angle (Deg.)	GES	MGES	Rao & Desai(1983)	Suja & Desai(1987)	present results
5.0	---	---	---	2.2853(+1)	2.293(+1)
10.0	4.570	4.905	5.6746	6.6259	6.6421
15.0	---	---	---	2.5377	2.5281
20.0	5.422(+1)	6.269(-1)	7.2730(-1)	1.0619	9.874(-1)
25.0	---	---	---	6.0898(-1)	6.113(-1)
30.0	1.620(-1)	2.003(-1)	2.3558(-1)	4.3328(-1)	4.291(-1)
40.0	7.110(-2)	9.292(-2)	1.1378(-1)	3.1653(-1)	2.981(-1)
50.0	3.712(-2)	5.059(-2)	6.7001(-2)	1.1625(-1)	1.142(-1)
60.0	---	---	4.4315(-2)	5.3131(-2)	5.2931(-1)
70.0	1.339(-2)	1.934(-2)	3.1880(-2)	2.9755(-2)	2.9987(-2)
80.0	---	---	2.4191(-2)	----	2.3812(-2)
90.0	6.231(-3)	9.353(-3)	1.9271(-2)	8.3802(-3)	1.9882(-2)
100.0	---	---	---	---	
110.0	3.534(-3)	5.439(-3)	1.3612(-2)	---	1.3542(-2)
120.0	---	---	1.1948(-2)	5.0259(-3)	1.2312(-2)

Fig II-VII

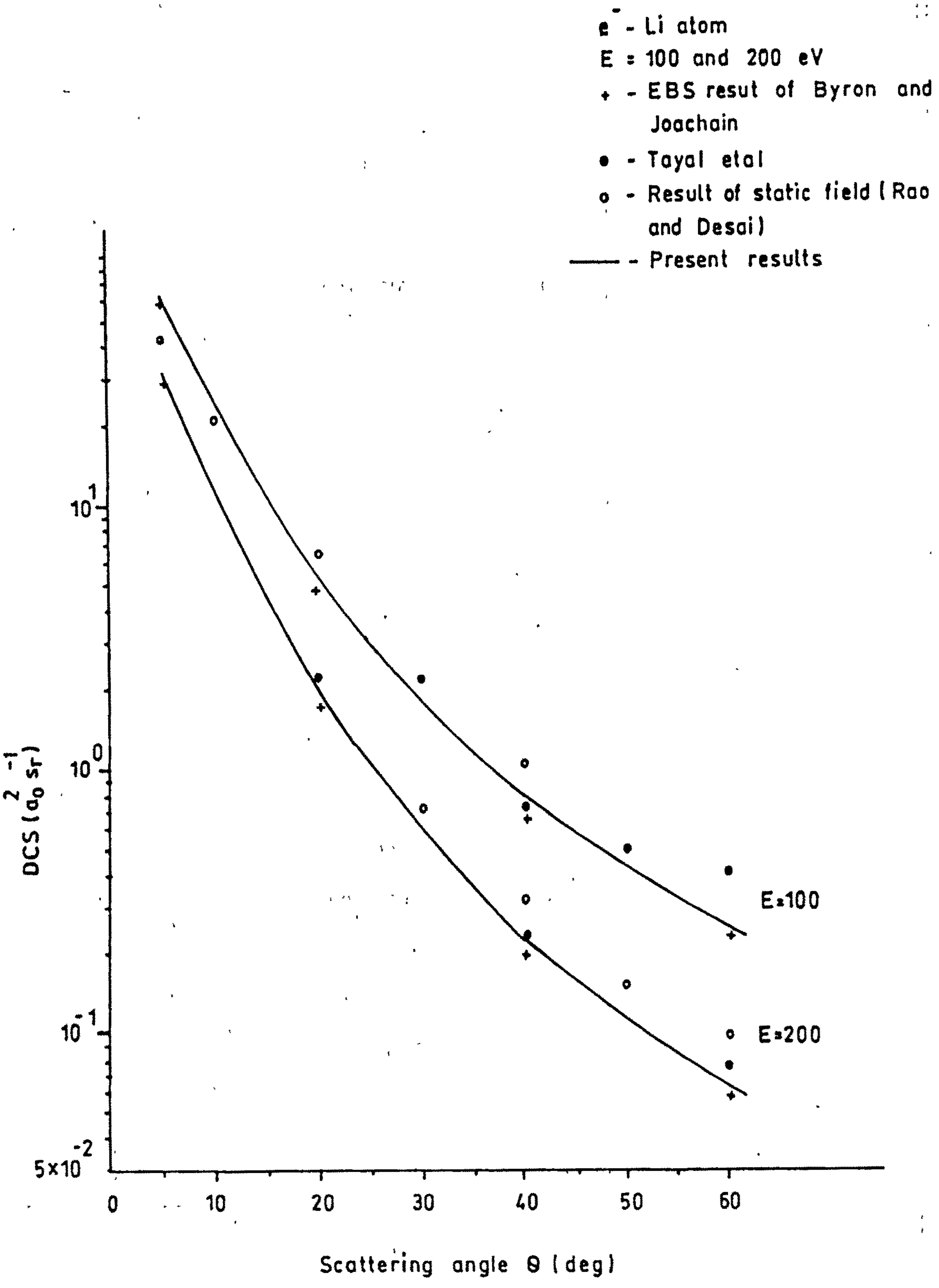

e⁻ - Li atom
E = 100 and 200 eV
+ - EBS resut of Byron and Joachain
● - Tayal etal
○ - Result of static field (Rao and Desai)
—— - Present results
E=100
E=200
DCS (a_0^2 sr^{-1})
10^1
10^0
10^{-1}
5×10^{-2}
0
10
20
30
40
50
60
Scattering angle θ (deg)

Fig II-VIII

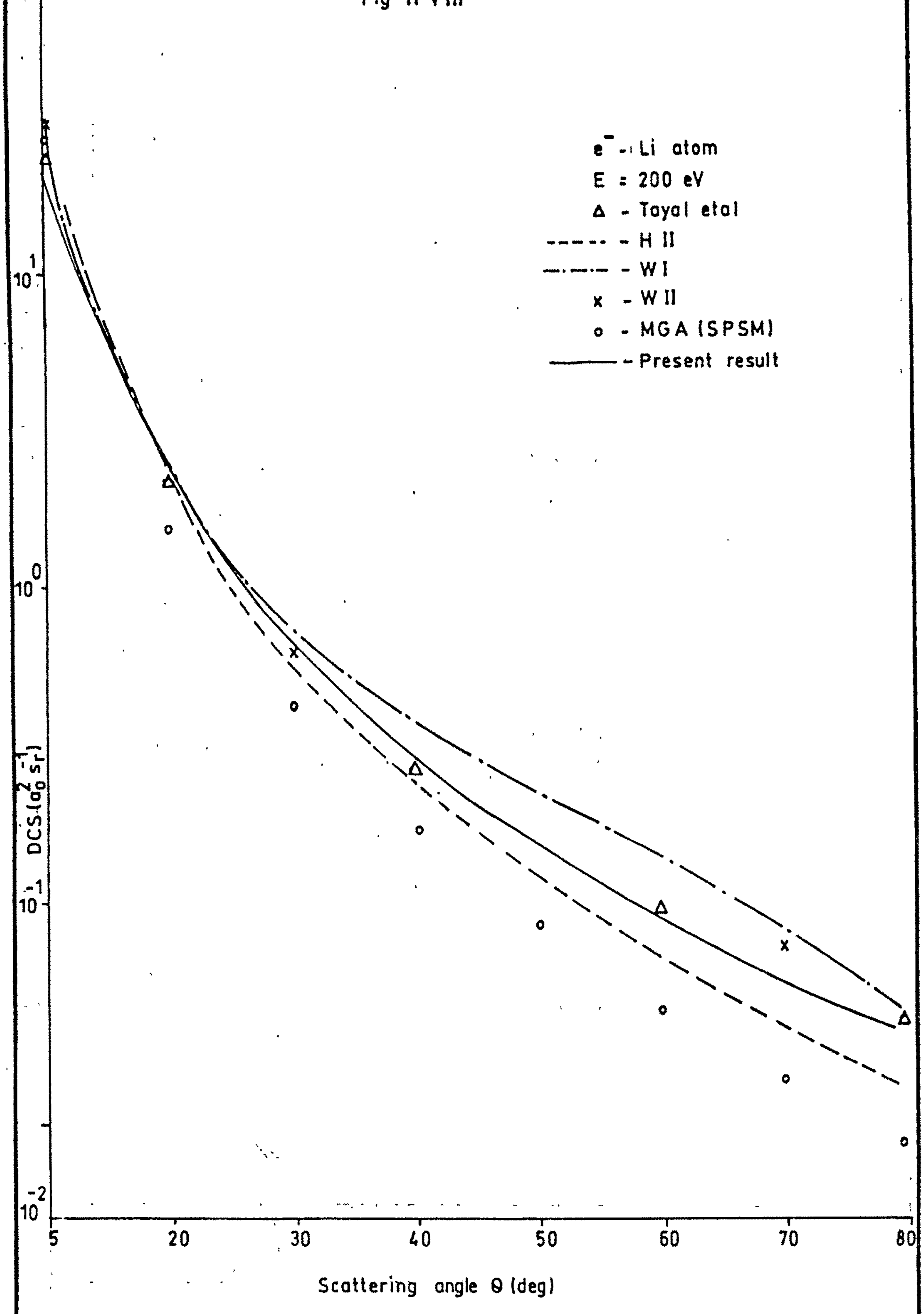
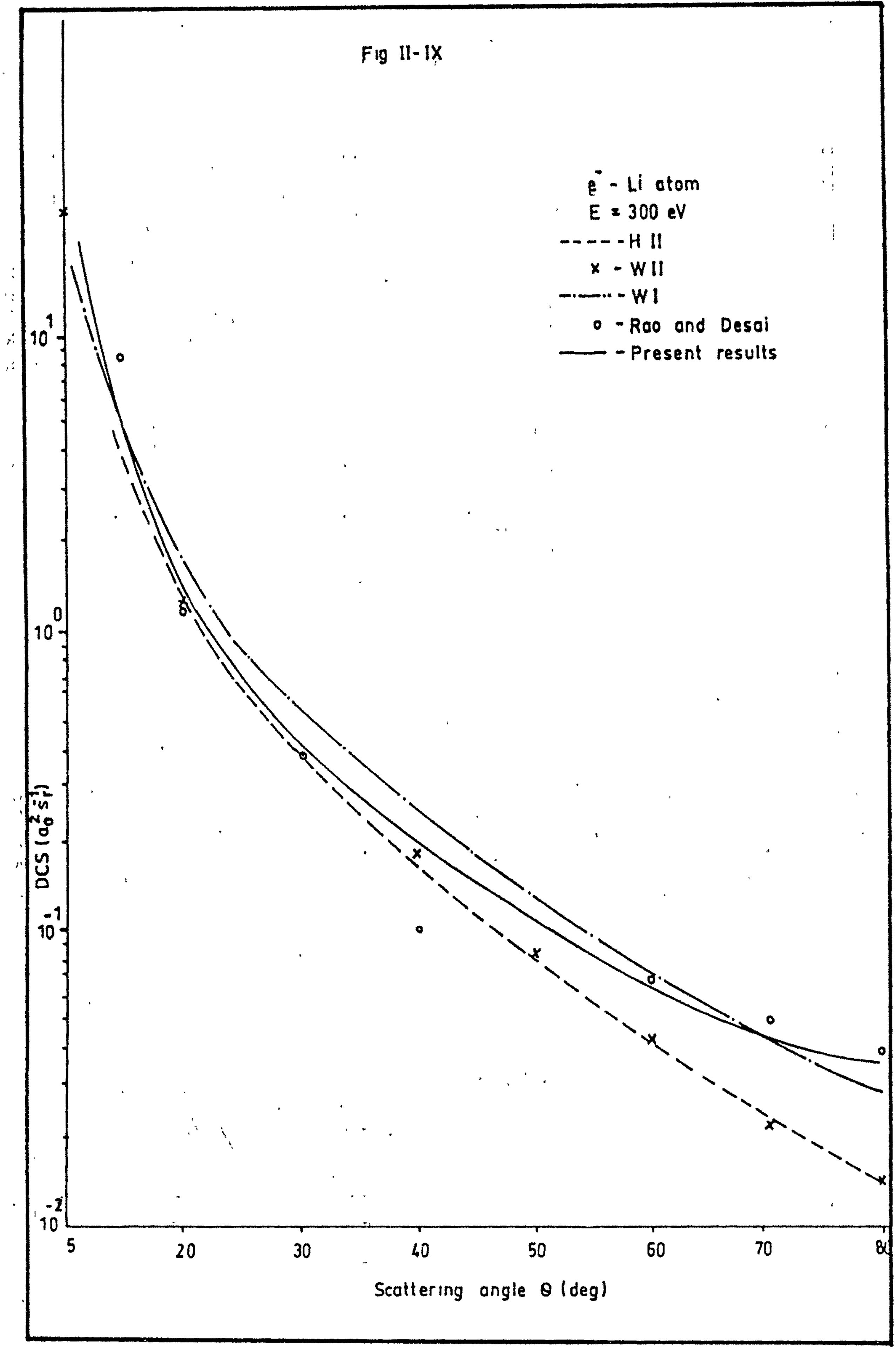

e^- - Li atom
E = 200 eV
Δ - Tayal etal
- H II
- W I
x - W II
o - MGA (SPSM)
- Present result
10^1
10^0
10^{-1}
10^{-2}
DCS $(a_0^2 sr^{-1})$
5
20
30
40
50
60
70
80
Scattering angle θ (deg)

Fig II-IX


e⁻ - Li atom
E = 300 eV
----- H II
× - W II
—·—··- W I
o - Rao and Desai
—— - Present results
DCS (a₀² sr⁻¹)
10¹
10⁰
10⁻¹
10⁻²
5
20
30
40
50
60
70
80
Scattering angle θ (deg)