# CHAPTER

# II

# CHAPTER-II

## SPIN STUDIES OF NUCLEONS IN A STATISTICAL MODEL

## 2.1 Introduction

The composition of nucleons, in terms of fundamental quarks and gluons degrees of freedom have been modeled variously to account for its observed properties. It is important to calculate as many nucleonic parameters as possible in these models to check their merits and their domains of validity. The naive valence picture of nucleon structure may be regarded as a first order approximation to the real system[1]. Models with one constituent gluon [2] and with one quark-antiquark $q\bar{q}$ pair [3-5], in addition to the three valence quarks, are capable of giving better account of nucleonic properties. In another class of models, it is assumed that nucleons consist of valence quarks surrounded by a "sea" which, in general, contains gluons and virtual quark-antiquark pairs, and is characterized by its total quantum number consistent with the quantum number of nucleons[6,7].

In the chiral quark model of Manohar and Georgi [8], QCD quarks propagate in the nontrivial QCD vaccum having $\bar{q}q$ condensates and this leads to the generation of extra mass to the quarks. As a consequence of this spontaneous chiral symmetry breaking, massless pseudoscalar bound $\bar{q}q$ Goldstone bosons are generated, and this leads to the nontrivial sea structure of the nucleon. In the instanton model [9], the quark-antiquark sea in a nucleon results from a scattering of a valance quark off a nonperturbative vacuum fluctuation of the gluon field, instanton. In the instanton induced interaction described by 't Hooft effective lagrangian, the flavor of the produced quark-antiquark is different from the flavor of the initial valance quarks, and there is a specific correlation between the sea quark helicity and the valance quark helicity. In the chiral-quark soliton model [10], the large $N_c$ model of QCD becomes an effective theory of mesons with the baryons appearing as solitons. Quarks are described by single particle wave functions which are solutions of the Dirac equation in the field of the background pions. In the statistical approach, the nucleon is treated as a collection of massless quarks, antiquarks and gluons in thermal equilibrium within a finite size volume[11]. The momentum distributions for quarks and antiquarks follow a Fermi-Dirac distribution function characterized by a common temperature and a chemical potential which depends on the flavor and helicity of the quarks.

Recently, a new statistical model has been proposed in which a nucleon is taken as an ensemble of quark-gluon Fock states [12,13]. In this model, using the principle of balance that every Fock state should be balanced with all of the nearby Fock states [13], or using the principle of detailed balance that any two nearby Fock states should be balanced with each other [12], the probability of finding every Fock state of the proton accounting upto ≈ 98% of the total Fock state has been obtained. It has been shown that the model gives an excellent description of the light flavor sea asymmetry (i.e, $\bar{u} \neq \bar{d}$) without any parameter.

From the above brief review, it is clear that most of the analytical calculations in the literature on the properties of nucleons related to its spin are done in (i) "minimal" quark model which contains at most a gluon or a quark-antiquark pair, in addition to the three valence quarks, or (ii) models where sea acts only as a background specified only by its quantum numbers with no active role in determining the nucleonic properties. In this article, using statistical ideas, we construct such Fock states of a nucleon which have definite color and spin quantum numbers, and definite symmetry property. The resulting total flavor-spin-color wave function of a spin-up nucleon consists of Fock states with three valence quarks and a sea containing up to five constituents (quark-antiquarks and gluons). We have used this model wave function to calculate the light quark spin content of nucleons, the ratio of their magnetic moments, the semileptonic decay constant of neutron, and the ratio of SU(3) reduced matrix elements for the axial current.

The mode of correlation among the constituents of a Fock state cannot be decided merely by a statistical consideration, and this requires, possibly, some dynamical input. To check the stability of results obtained, under variation in correlation, we introduce two modifications of our primary model and repeat calculations of nucleonic properties in these models as well.

## 2.2. Sea and its Structure

In Ref.[12,13], treating the proton as an ensemble of quark-gluon Fock states, the proton state has been expanded in a complete set of such states as

$$|p\rangle = \sum_{ijk} C_{ijk} |uud, i, j, k\rangle \quad (2.1)$$

where i is the number of $\bar{u}u$ pairs, j is the number of $\bar{d}d$ pairs, and k is the number of gluons. The probability to find a proton in the Fock state $|uud,i,j,k\rangle$ is

$\rho_{ijk} = |C_{ijk}|^2$,

where $\rho_{ijk}$ satisfies the normalization condition,

$$\sum_{ijk} \rho_{ijk} = 1 \quad (2.2)$$

Then, using the detailed balance principle or balance principle, and with sub processes $q \Leftrightarrow q\,g$, $g \Leftrightarrow \bar{q}q$ and $g \Leftrightarrow gg$ considered, all $\rho_{ijk}$ have been calculated explicitly. Interestingly, the model predicts an asymmetry in the sea flavor of $\bar{u}$ and $\bar{d}$ as $\bar{d}$ - $\bar{u}$~0.124 in surprising agreement with the experimental data 0.118±0.012. These quarks and gluons have to be understood as "intrinsic" partons of the proton as opposed to the "extrinsic" partons generated from the QCD hard bremsstrahlung and gluon splitting as a part of the lepton nucleon scattering interaction [14]. The $\bar{q}q$ pairs and gluons, which are multiconnected non-perturbatively to the valence quarks, will collectively be referred to as the sea. Since the proton should be colorless and a $q^3$ state can be in color state $1_c$, $8_c$ and $10_c$, the sea should also be in the corresponding color state to form a color singlet proton. Furthermore, if the sea is in an S-wave state relative to the $q^3$ core, conservation of angular momentum restricts that the spin of the sea can only be 0,1or 2 to give a spin-1/2 proton. The case of the sea with one $\bar{q}q$ pair, where the sea or at least one of the quarks is needed to be in a relative P-wave to meet the positive parity requirement of the proton, will be treated separately. We take the probabilities of finding various quark-gluon Fock states in a proton from Ref.[13], and assume that the quarks and the gluons can be treated nonrelativistically for our problem, and also that, in general, these are in S-wave motion. The effect of the relativistic motion of the constituents will be discussed later. The case of a neutron will be treated in an analogous way using isospin symmetry.

Nonrelativistic treatments of quarks in nucleon models are well known [1,4-6]. There are phenomenological evidences that gluons also behave as massive particles with mass ≥0.5GeV[15]. There is a firm evidence from lattice calculation also that gluons behave as massive particles at low momenta (≤4GeV)[16]. It has been shown in Ref [5] that the sum of the relativistic quark spin and orbital angular momentum (derived from QCD Lagrangian ) is equal to the sum of the non-relativistic quark spin and orbital angular momentum,

$$\vec{S}_q + \vec{L}_q = \vec{S}^{NR}_q + \vec{L}^{NR}_q \tag{2.3}$$

Furthermore, it has been shown that on truncating the Fock space to contain only $|q^3\rangle$ and $|q^3\bar{q}q\rangle$ component, the quark orbital angular momentum contribution comes out to be negligible or small [5]. This contribution should decrease on inclusion of Fock states with more "intrinsic" partons, since then each parton will have a lesser linear momentum share, and hence, smaller orbital angular momentum too.

Following Ref.[6], we write the possible combination of $q^3$ and sea wave function, which can give a spin ½ flavor octet, color singlet state as

$\Phi_1^{(1/2)}H_0G_1$ , $\Phi_8^{(1/2)}H_0 G_8$ , $\Phi_{10}^{(1/2)}H_0G_{\overline{10}}$, $\Phi_1^{(1/2)}H_1G_1$, $\Phi_8^{(1/2)}H_1G_8$,

$\Phi_{10}^{(1/2)}H_1G_{\overline{10}}$ and $\Phi_8^{(3/2)}H_1G_8$ , $\Phi_8^{(3/2)} H_2G_8$ . (2.4)

In the above $\Phi^{(1/2,3/2)}{}_{1,8,10}$ is the $q^3$ wave function in obvious notation, while $H_{0,1,2}$ and $G_{1,8,\overline{10}}$ denote spin and color sea wave functions respectively which satisfy

$$\langle H_i \mid H_j\rangle = \delta_{ij}, \quad \langle G_k \mid G_l\rangle = \delta_{kl}$$

The total flavor-spin-color wave function of a spin up proton which consists of three valence quarks and sea components can be written as :

$$|\Phi_{1/2}^{\uparrow}\rangle = (1/N)\ [\Phi_1^{(1/2\uparrow)}H_0G_1 + a_8\ \Phi_8^{(1/2\uparrow)}H_0G_8 + a_{10}\ \Phi_{10}^{(1/2\uparrow)}H_0G_{\overline{10}} + b_1(\Phi_1^{(1/2)} \otimes H_1)^{\uparrow}G_1 + b_8(\Phi_8^{(1/2)} \otimes H_1)^{\uparrow}G_8 + b_{10}\ (\Phi_{10}^{(1/2)} \otimes H_1)^{\uparrow}G_{\overline{10}} + c_8\ (\Phi_8^{(3/2)}\otimes H_1)^{\uparrow}G_8 + d_8(\Phi_8^{(3/2)}\otimes H_2)^{\uparrow}G_8] \quad (2.5)$$

where $N^2 = 1 + a_8^2 + a_{10}^2 + b_1^2 + b_8^2 + b_{10}^2 + c_8^2 + d_8^2$, and $(\Phi_1^{(1/2)}\otimes H_1)^{\uparrow}$, etc. have to be written properly with appropriate CG coefficients and by taking into account the symmetry property of the component wave function .

Here, we suggest a possible way to construct the sea wave function using the statistical model of Zhang et al [13]. However, unlike Ref. [6], we will also take into account the "active" sea contribution of the sea in which the relevant operators act on the sea quarks as well. Furthermore, we will use an approximation in which quarks in the $q^3$ core will not be antisymmetrized with the identical quarks appearing in the sea. Use of different labels for valance and sea quarks has been justified with the assumption that the valance and the sea quarks have very different momentum distributions, with the valance quarks being "hard" and the sea quarks "soft", and that the overlap region between the two momentum distributions is negligible[17]. Consequently, this classification can work where one is concerned with matrix elements having zero momentum transfer and only require that the overlap region between valance and sea quark momentum distribution be negligibly small. Nevertheless, we will use this separation for the problem of quark contribution to the nucleon spin as well.

We assume that the statistical decomposition of the proton state in various quark-gluon Fock states, as obtained by Zhang et al.[13] and which is expected to work at a $Q^2 \sim 1GeV^2$ for the quark system, can be extended down to the proton's rest frame. Since the quarks and gluons in the Fock states are "intrinsic", there should be no problem in this extension as far as color quantum numbers are concerned. However, it has been shown by Ma and Zhang [18] that the Melosh rotation [19] generated by the internal transverse momentum spoils the usual identification of the $\gamma^+\gamma_5$ quark current matrix element with the total rest frame spin projection $s_z$, thus resulting in a reduction of $g_A$. It has also been observed [20,21] that the physical value of the anomalous magnetic moment is reduced from its non-relativistic value due to the Melosh rotation. We will estimate the changes in weak decay constant and the ratio of the magnetic moments of the nucleons due to the Melosh rotation towards the end.

Next, we decompose each one of the Fock states $|uud,i,j,k\rangle$ in terms of the set of states appearing in Eq.(2.5) following a statistical approach.

(i) Consider the decomposition of a state $|uud,0,0,2\rangle$ or $|gg\rangle$ sea (two gluons in the sea ).

Spin : uud : $1/2 \otimes 1/2 \otimes 1/2 = 2(1/2) \oplus 3/2$,

gg : $1 \otimes 1 = 0_s \oplus 1_a \oplus 2_s$,

Color : uud : $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$,

gg : $8 \otimes 8 = 1_s \oplus 8_s \oplus 8_a \oplus 10_a \oplus \overline{10}_a \oplus 27_s$.

The subscripts s and $a$ denote symmetry and asymmetry respectively under the exchange of two identical bosons (gluons above). Call $\rho_{j1\,j2}$ as the probability that the $q^3$ core and gg sea are in angular momentum states $j_1$ and $j_2$ respectively, and they finally add to give total angular momentum 1/2. Let us compare such probabilities.

$$\rho_{1/2\,0s} / \rho_{1/2\,1a} = \frac{(4/8).(1/9).1}{(4/8).(3/9).(2/6)} = 1,$$

$$\rho_{1/2\,0s} / \rho_{3/2\,2s} = \frac{(4/8).(1/9).1}{(4/8).(5/9).(2/20)} = 2,$$

$$\rho_{3/2\,1a} / \rho_{3/2\,2s} = \frac{(4/8).(3/9).(2/12)}{(4/8).(5/9).(2/20)} = 1,$$

$$\rho_{1/2\,1a} / \rho_{3/2\,1a} = \frac{(4/8).(3/9).(2/6)}{(4/8).(3/9).(2/12)} = 2.$$

The first factor in the numerator or denominator in the r.h.s is the relative probability for the core quarks to have spin $j_1$, the second factor is the same for the two gluons to have spin $j_2$, and finally the third one is the same for $j_1$ and $j_2$ to have resultant 1/2. In future, we will omit the factor which is common in the numerator and the denominator.

Similarly we can compare the probabilities for the $q^3$ core and gg to be in different color substates which finally give a color singlet proton. In obvious notations:

$$\rho_{1\,1s}/\rho_{8\,8s} = \frac{(1/27).(1/64).1}{(16/27).(8/64).(1/64)} = 1/2 = \rho_{1\,1s}/\rho_{8\,8a},$$

$$\rho_{1\,1s}/\rho_{10\,\overline{10}} = \frac{(1/27).(1/64).1}{(10/27).(10/64).(1/100)} = 1.$$

The product of probabilities in spin and color spaces can be written in terms of one common parameter c as

$\rho_{1/2\ 0s}\,[\rho_{1\,1s}, \rho_{8\,8s}] = 2c\,(1,2)$,

$\rho_{1/2\,1a}\,[\,\rho_{8\,8a}, \rho_{10\,\overline{10}}\,] = 2c\,(2,1)$,

$\rho_{3/2\ 1a}\,[\rho_{8\,8a}] = 2c\,, \quad \rho_{3/2\,2s}\,[\rho_{8\,8s}] = 2c.$

There is no contribution to $H_0G_{\overline{10}}$ and $H_1G_1$ sea from two gluon states because $H_0$ and $G_1$ are symmetric whereas $H_1$ and $G_{\overline{10}}$ are antisymmetric under exchange of the two gluons making these product wave functions antisymmetric and hence unacceptable for a bosonic system. The sum of all these probabilities is taken from Ref.[13] and this determines the unknown parameter c :

$\rho_{uud\,gg} = 0.081887,\ c = 0.005118,$

giving us above products of probabilities.

It is clear that the numbers on the r.h.s in above equations containing products of probabilities give the probabilities for finding the Fock state with two gluons in the sea in various substates with specified spin and color quantum numbers. Thus, for instance $\rho_{1/2\,0s}\,\rho_{1\,1s} = 0.01024$ means that the probability for finding the three quark core in spin ½ and color singlet state along with the two-gluon sea to be in a scalar and color singlet state is 0.01024.

Similar decomposition will hold good for $|\, \bar{q}q\,\bar{q}q \rangle$ sea also. By proceeding on a similar line, we get

$\rho_{1/2\ 0s}\,[\rho_{1\ 1s}, \rho_{8\ 8s}];\ \rho_{1/2\ 1a}\,[\rho_{8\ 8a}, \rho_{10\ \overline{10}}];\ \rho_{3/2\ 1a}\,[\rho_{8\ 8a}]\,;\ \rho_{3/2\ 2s}\,[\rho_{8\ 8s}]$

$= 0.000904(1,2;2,1;2;2)$ for $|\, \bar{u}u\bar{u}u \rangle$,

$= 0.014571(1,2;2,1;2;2)$ for $|\, \bar{d}d\bar{d}d \rangle$.

(ii) For decomposition of $|\, g\, \bar{q}q \rangle$ and $|\, \bar{u}u\, \bar{d}d \rangle$ sea, symmetry consideration is not needed. Here we have assumed that $\bar{q}q$ carries the quantum numbers of a gluon due to the sub processes $g \Leftrightarrow \bar{q}q$. This gives the relative probability density in color space as $\rho_{1\ 1}/\rho_{8\ 8} = 1/4$. The ratio $\rho_{1\ 1}/\rho_{10\ \overline{10}}$ and the relative densities in spin space remain the same as in (i). Proceeding as in the previous case, the products of densities in spin and color spaces come out as

$\rho_{1/2\ 0}\,[\rho_{1\ 1}, \rho_{8\ 8}, \rho_{10\ \overline{10}}];\ \rho_{1/2\ 1}\,[\rho_{1 1}, \rho_{8\ 8}, \rho_{10\ \overline{10}}];\ \rho_{3/2\ 1}\,[\rho_{8\ 8}];\ \rho_{3/2\ 2}\,[\rho_{8\ 8}]$

$= 0.00344\ (1,4,1;1,4,1;2;2)$ for $|\, g, \bar{u}u \rangle$,

$= 0.00517\ (1,4,1;1,4,1;2;2)$ for $|\, g, \bar{d}d \rangle$,

$= 0.00366(1,4,1;1,4,1;2;2)$ for $|\, \bar{u}u, \bar{d}d \rangle$.

(iii) $|gg\,\bar{q}q \rangle$, $|\, \bar{q}q\,\bar{q}q\, g \rangle$ sea : First we take the product of two spin 1 states and two color octet states as in (i). These are further multiplied with spin 1 and color octet state respectively. The new results needed are

Spin : $1 \otimes 2 = 1 \oplus 2 \oplus 3$,

Color: $10 \otimes 8 = 8 \oplus 10 \oplus 27 \oplus 35$,

$27 \otimes 8 = 8 \oplus 10 \oplus \overline{10} \oplus 2(27) \oplus 35 \oplus 35 \oplus 64$.

Using the subscript s and $a$ for symmetry and asymmetry under the exchange of first two bosons, the relative probability densities in spin space are:

$$\rho_{1/2\,0a}/\rho_{1/2\,1a} = \frac{(1/27).1}{(3/27).(2/6)} = 1, \quad \rho_{1/2\,0a}/\rho_{1/2\,1s} = \frac{(1/27).1}{(6/27).(4/12)} = \frac{1}{2},$$

$$\rho_{1/2\,1a}/\rho_{3/2\,1a} = \frac{(3/27).(1/3)}{(3/27).(1/6)} = 2 = \rho_{1/2\,1s}/\rho_{3/2\,1s},$$

$$\rho_{3/2 1a}/\rho_{3/2\,2a} = \frac{(3/27).(2/12)}{(5/27).(2/20)} = 1, \quad \rho_{3/2\,1s}/\rho_{3/2\,2s} = \frac{(6/27).(4/24)}{(5/27).(2/20)} = 2.$$

The ratio of the probability densities in color space are:

$$\rho_{1\,1s}/\rho_{8\,8s} = \frac{(1/27).(1/512).1}{(16/27).(32/512).(1/64)} = 1/8,$$

$$\rho_{1\,1s}/\rho_{10\,\overline{10}s} = \frac{(1/27).(1/512).1}{(10/27).(20/512).(1/100)} = \frac{1}{2} = \rho_{1\,1a}/\rho_{10\,\overline{10}a},$$

$$\rho_{1\,1a}/\rho_{8\,8a} = \frac{(1/27).(1/512).1}{(16/27).(32/512).(1/64)} = 1/8.$$

The combined probabilities in spin and color space can be written as

$\rho_{1/2\ 0a}[\rho_{1\ 1a}, \rho_{8\ 8a}, \rho_{10\ \overline{10}a}]$; $\rho_{1/2\ 1a}[\rho_{1\ 1a}, \rho_{8\ 8a}, \rho_{10\ \overline{10}a}]$; $\rho_{1/2\ 1s}[\rho_{1\ 1s}, \rho_{8\ 8s}, \rho_{10\ \overline{10}s}]$; $\rho_{3/2\ 1a}[\rho_{8\ 8a}]$;

$\rho_{3/2\ 1s}[\rho_{8\ 8s}]$; $\rho_{3/2\ 2a}[\rho_{8\ 8a}]$ $= 0.00051(1,8,2;1,8,2;2,16,4;4;8;4)$ for $|\,gg, \bar{u}u\,\rangle$,

$= 0.00076(1,8,2;1,8,2;2,16,4;4;8;4)$ for $|\,gg, \bar{d}d\,\rangle$,

$= 0.00007\,(1,8,2;1,8,2;2,16,4;4;8;4)$ for $|\,\bar{u}u\,\bar{u}u, g\,\rangle$,

$= 0.00025(1,8,2;1,8,2;2,16,4;4;8;4)$ for $|\,\bar{d}d\,\bar{d}d, g\rangle$.

(iv) | $\bar{u}u\,\bar{d}d$ g ⟩ sea : Here, there is no symmetry requirement. Ratios of probability densities are

$\rho_{1/2\,0}/\rho_{1/2\,1} = 1/3, \quad \rho_{1/2\,0}/\rho_{3/2\,2} = 1, \quad \rho_{1/2\,1}/\rho_{3/2\,1} = 2, \quad \rho_{3/2\,1}/\rho_{3/2\,2} = 3/2.$

in spin space, and $\rho_{1\,1}/\rho_{8\,8} = 1/8, \quad \rho_{1\,1}/\rho_{10\,\overline{10}} = 1/2$

in color space. Their products can be written as

$$\rho_{1/2\,0}[\rho_{11}, \rho_{8\,8}, \rho_{10\,\overline{10}}];\ \rho_{1/2\,1}[\rho_{11}, \rho_{8\,8}, \rho_{10\,\overline{10}}];\ \rho_{3/2\,1}[\rho_{8\,8}];\ \rho_{3/2\,2}[\rho_{8\,8}]$$
$$= 0.00048(1, 8, 2;3, 24, 6;12; 8).$$

(v) | ggg⟩ sea :

The wave function for this sea should be completely symmetric under the exchange of any two gluons . Among the product spin function, the total spin S= 0 is completely antisymmetric and one S=1 is completely symmetric. Among the product color functions, there is one color singlet state and one color octet state which are completely antisymmetric; and there is one color singlet state and one color octet state which are completely symmetric. This gives

$\rho_{1/2\,0}/\rho_{1/2\,1} = 1, \quad \rho_{1/2\,1}/\rho_{3/2\,1} = 2, \quad \rho_{1\,1a,s}/\rho_{8\,8} = 1/2.$

This gives us the product of probabilities in spin and color spaces as

$$\rho_{1/2\,0a}[\rho_{1\,1a}, \rho_{8\,8a}];\ \rho_{1/2\,1s}[\rho_{11s}, \rho_{8\,8s}];\ \rho_{3/2\,1s}[\rho_{8\,8s}]$$
$$= 0.00534(1, 2; 1, 2;1).$$

A confined gluon in the sea may be divided into TE (transverse electric ) modes with $J^{pc} = 1^{+-}$ and the TM (transverse magnetic) modes with $J^{pc} = 1^{--}$ . The Fock states with a single gluon in the sea may be considered to be consisting of a TE gluon [22]. Clearly, a gluon in the sea will contribute only to the $H_1G_8$

component of the sea. From this decomposition we get the following numbers for the coefficients in the expansion in Eq.(2.5) of the proton state:

$a_8^2$=0.5043, $a_{10}^2$= 0.0892, $b_1^2$=0.1037, $b_8^2$=1.8133, $b_{10}^2$ = 0.2220, $c_8^2$ = 0.9067, $d_8^2$ = 0.2630 and $N^2$ = 4.9024.

However, the treatment of a $\bar{q}q$ pair in the sea requires special attention, since as stated earlier, to keep the parity of the system positive, one or a group of the five particles is required to be in a P-wave state. This requires detailed knowledge of spatial wave function. To get the contribution of this particular Fock space, we have borrowed the result from Ref.[5] and scaled it to give the same probability which we are using, as given in Ref.[13]. Thus, we have also introduced non-zero orbital angular momentum states, albeit for only one type of Fock states among the several Fock states considered, in our nucleon wave function. Unlike our treatment, the total wave function in[5] has been properly antisymmetrized. All the above states taken together constitute ≈ 86% of the total Fock space. The cases with three $q\bar{q}$ pairs, four gluons and two $q\bar{q}$ pairs with two gluons, and other higher Fock states have not been considered due to smaller probabilities associated with these Fock states in the statistical model, and due to more involved analysis in their decomposition. We believe, we can get sufficient insight in the problem under consideration even at the cost of directly excluding higher Fock states. We assume that the rest of the quark-gluon sea spanning ~14% of the Fock space of the nucleon also decomposes in color and spin subspaces in approximately the same proportion as the one which we have worked out explicitly above. The number of strange quark-antiquark pairs in the statistical model is 0.05 in the nucleon as compared to the average number of particles which is 5.57[13]; hence we neglect the contributions of the s-quark and other higher mass quarks in calculations of nucleonic properties.

For calculating physical quantities related to the spin of a nucleon, it is useful to introduce two parameters, $\alpha$ and $\beta$ as [6]

$$\alpha = (1/N^2).(4/9).(2a+2b+3d+\sqrt{2}\,e),$$

$$\beta = (1/N^2).(1/9).(2a-4b-6c-6d+4\sqrt{2}\,e),$$

where,

$$a = (1/2).(1-b_1^2/3),\ b=(1/4).(a_8^2-b_8^2/3),\ c=(1/2).(a_{10}^2-b_{10}^2/3)$$

$$d = (1/18).(5c_8^2-3d_8^2),\ e=(\sqrt{2}/3).b_8c_8\,.$$

The importance of these parameters lies in the fact that they are connected with the numbers of spin-up ($n(q\uparrow)$) and spin-down ($n(q\downarrow)$) quarks in the spin-up proton. If $\Delta q = n(q\uparrow)-n(q\downarrow)+n(\bar{q}\uparrow)-n(\bar{q}\downarrow)$, $q = u, d$ ,then $\Delta u = 3\alpha$ and $\Delta d = -3\beta$. Contributions to the parameters $\alpha$ and $\beta$ from the sea excluding the single $\bar{q}q$ components have been denoted by $\alpha_1$ and $\beta_1$ respectively, whereas those from the single $\bar{q}q$ components have been denoted by $\alpha_2$ and $\beta_2$: $\alpha = \alpha_1+\alpha_2$, $\beta = \beta_1+\beta_2$. Numerical values of all these parameters

have been listed in Table 2.1 (Model C). These can be used to calculate various physical quantities as done in Ref.[6], where the sea plays a role of "passive" background and the relevant operators act only on the three-quark core. When the operator $\sum_i e_i^{\,2}\sigma_z^i$ acts on the sea minus the single $\bar{q}q$ component, i.e. when the sea plays the "active" role, the result has been denoted by $\Delta I_1^p$ and $\Delta I_1^n$ for the proton and neutron respectively .There is no such contribution to the magnetic moments due to the "active" sea, since the $q\bar{q}$ pairs carry the quantum numbers of the parent gluons. The total contribution to the nucleon spin from the spins of the quarks, denoted by $I_1^p$ and $I_1^n$, have been displayed in Table 2.2 (Model C) and compared with the revised EMC result [23]. We should note that EMC value is for $Q^2 \approx 10$ $GeV^2$, whereas our result for $I_1^p$ and $I_1^n$ should be considered to work at $Q^2 \sim 1 GeV^2$ where the Fock state decomposition of the nucleon state [13] used in this work applies. To estimate $(g_A/g_V)$, we use Bjorken sum rule

$$(\bar{g}_1^p - \bar{g}_1^n) = \int_0^1 dx[g_1^p(x) - g_1^n(x)] = \frac{1}{6}\left|\frac{g_A}{g_V}\right|[1 - \frac{\alpha_s(Q^2)}{\pi}]$$
$$= 0\ 191 \pm 0\ 002$$

written upto $O(\alpha_s^3/\pi^3)$ [24]. We have considered three values of $\alpha_s$ from the recent literature. Authors of Ref.[25] have used $\alpha_s(1GeV^2) \approx 0.5$ for the same purpose as ours. Particle Data Group[26] average value is $\alpha_s(m_c) = 0.357$, which we modify as $\alpha_s(1GeV^2) = 0.375$ for our use. Authors of Ref.[27] use $\alpha_s(0) = 0.35$ (to fit the bound states in QCD). The values of $(g_A/g_V)$ obtained for each one of these values have been displayed in Table 2.2 (Model C). The F/D value has been obtained from $\alpha$ and $\beta$ as per the prescription given in Ref.[6].

**TABLE 2.1** : $\alpha$ and $\beta$ as defined in Ref [6]: $\alpha_1$ and $\beta_1$ are the contributions from the sea excluding the single $q\bar{q}$ components; $\alpha_2$ and $\beta_2$ are the contribution from the single $q\bar{q}$ components of the sea. $\Delta I_1^p$ and $\Delta I_1^n$ are the contribution to $I_1^p$ and $I_1^n$ respectively when the operator $\sum_i e_i^2 \sigma_z^i$ acts on the sea excluding the single $q\bar{q}$ component. Model C is our first statistical model described in the text. In model P, $q\bar{q}$ pairs have been taken as colorless pseudoscalars, whereas model D is the one in which suppressed higher multiplicity states appear.

| Model Type | $\alpha_1$ | $\alpha_2$ | $\alpha = \alpha_1 + \alpha_2$ | $\beta_1$ | $\beta_2$ | $\beta = \beta_1 + \beta_2$ | $\Delta I_1^p$ | $\Delta I_1^n$ |
|---|---|---|---|---|---|---|---|---|
| Model C | 0.1821 | 0.0417 | 0.2237 | 0.0549 | 0.0186 | 0.0736 | 0.0308 | 0.0406 |
| Model P | 0.2136 | 0.0417 | 0.2552 | 0.0660 | 0.0186 | 0.0846 | 0.0000 | 0.0000 |
| Model D | 0.2223 | 0.0417 | 0.2639 | 0.0521 | 0.0186 | 0.0707 | 0.0151 | 0.0179 |

**TABLE 2.2 :** Comparison of our calculated results of various physical parameters with the experimental numbers. Quantities in [a] are without Melosh rotation and those in [b] are with Melosh rotation for parameters from Ref.[18]. $g_A/g_V$ in [a] are obtained using Bjorken sum rule.

| Model Type | $I_1^p$ | $I_1^n$ | $\mu_p/\mu_n$ [a] | $\mu_p/\mu_n$ [b] | $g_A/g_V$ [a] $\alpha_s = 0.35$ | $g_A/g_V$ [a] $\alpha_s = 0.37$ | $g_A/g_V$ [a] $\alpha_s = 0.5$ | $g_A/g_V$ [b] | F/D [a] | F/D [b] |
|---|---|---|---|---|---|---|---|---|---|---|
| Model C | 0.168 | 0.029 | -1.405 | -1.535 | 1.019 | 1.045 | 1.243 | 1.318 | 0.603 | 0.690 |
| Model P | 0.156 | -0.014 | -1.402 | -1.532 | 1.249 | 1.282 | 1.525 | 1.505 | 0.601 | 0.688 |
| Model D | 0.179 | 0.015 | -1.477 | -1.598 | 1.210 | 1.241 | 1.476 | 1.502 | 0.651 | 0.732 |
| Expt. Valu | 0.136 | -0.030 | -1.460 | | 1.267 | | | | 0.575 | |
| [Ref.] | [23] | [23] | [26] | | [26] | | | | [29] | |

The effect of Melosh rotation on physical quantities related to the spin structure of the nucleon has been discussed in recent literature [18-21].Basically, Melosh rotation effect comes from the relativistic effect of the quark intrinsic transversal motion inside the nucleon. As a result of this, $\Delta q$, measured in polarized deep inelastic scattering and defined as the quark spin in the light-cone formalism, can not be identified with $\Delta q_{QM}$, the spin carried by each quark flavor in the proton rest frame or the quark spin in the quark model . The quark helicity

$$\Delta q = \langle M_q \rangle \Delta q_{QM},$$

where $<M_q>$ is the averaged value of the (dimensionless) Melosh rotation factor for the quark q, and is less than 1.

In Ref.[20], authors have considered a relativistic three-quark model formulated on the light-cone and concluded that Melosh rotation results in a ≈ 25 % reduction of the non-relativistic predictions for the anomalous magnetic moment, the axial vector coupling, and the quark helicity content of the proton leading to a better agreement with the observation . The model of the nucleon by Ma et al. [21], which include the three quark component and a baryon–meson state with a pseudoscalar meson, is nearer to our case because of its sea. We use their result to estimate the effect due to Melosh rotation on quantities related to the nucleon spin. In effect, it results in a replacement of $\alpha \rightarrow \alpha/<M_u>$ and $\beta \rightarrow \beta/<M_d>$, where numerically $<M_u>=$ 0.624 and $<M_d> = 0.912$. This makes the parameters $\alpha$ and $\beta$ closer to the ones used in Ref.[6] on phenomenological ground. For the nucleon spin problem, $\alpha$ and $\beta$, and not their above scaled values, will be used, since the quark helicity $\Delta q$ observed in polarized DIS is actually the quark spin defined in the light-cone formalism for which $\alpha$ and $\beta$ are appropriate quantities.

In order to check the stability of our results against some plausible changes in some physical parameters, we consider two modifications of the above model. It appears reasonable to assume that in determining low energy hadronic observables, the long range and confining forces leading to specific correlations among the constituents, in addition to the statistical consideration, will have a role to play. Based upon this point of view, we introduce the following two models:

## 2.3 Sea with Pseudoscalars

In the statistical formulation of Ref.[12,13], a quark-antiquark pair is created from a gluon splitting: $g \Leftrightarrow \bar{q}q$. This pair, naturally, carries the quantum numbers of the parent gluon. However, this is not an energetically favorable situation even within the hadronic boundary [28] ; the pair on exchange of a soft gluon with the rest of the system, and also possibly on a spin flip, will evolve to a colorless pseudoscalar form, called internal Goldstone boson [28-30]. We will assume that all the $\bar{q}q$ pairs are in one or the other pseudoscalar form practically for whole of their lifetimes giving no contribution to the spin or the color charge of the proton. In case of $|gg\,\bar{q}q\rangle$ state, in order to compensate the odd parity of the $\bar{q}q$ pair, one of the gluons will be assumed to be in TE mode while the other in TM mode. With these assumptions, we can decompose the Fock states, considered earlier, in spin and color spaces as:

$|\,q\bar{q}\,\rangle \sim H_1G_1,$

$|\,u\bar{u}d\bar{d}\,\rangle,\ |\,q\bar{q}\ q\bar{q}\,\rangle \sim H_0G_1,$

$|\,q\bar{q}, g\rangle,\ |\,u\bar{u}d\bar{d}, g\rangle,\ |\,q\bar{q}\ q\bar{q}, g\,\rangle \sim |g\rangle,$

$|\,q\bar{q}, gg\,\rangle \sim |gg\rangle'.$

As a consequence of the last result, we will have

$$\rho_{1/2\ 0}\,[\rho_{1\,1}, \rho_{8\,8}, \rho_{10\,\overline{10}}];\ \rho_{1/2\,1}\,[\rho_{1\,1}, \rho_{8\,8}, \rho_{10\,\overline{10}}];\ \rho_{3/2\,1}\,[\rho_{8\,8}];\ \rho_{3/2\,2}\,[\rho_{8\,8}]$$
$$= 0.00474\,(1,4,1;1,4,1;2;2) \quad \text{for } |\,q\bar{q}, gg\,\rangle\ .$$

The Fock states $|g\rangle$, $|gg\rangle$ and $|ggg\rangle$ can be decomposed as in the previous case. Thus, we get the following contribution to the expansion coefficients in Eq.(2.5) of the proton state:

$a_8^2 = 0.22143$, $a_{10}^2 = 0.02161$, $b_1^2 = 0.04247$, $b_8^2 = 1.25408$, $b_{10}^2 = 0.06825$, $c_8^2 = 0.62704$, $d_8^2 = 0.0898$.

This sea will not "actively" contribute to the spins or the magnetic moments of the nucleons. With this sea, the results of the spin distribution of nucleons come closer to the data as is evident from results of Model P in Table 2.2. There is hardly any change in the values of the ratios $\mu_p/\mu_n$ and F/D from the previous case. Matching the values of $g_A/g_V$ with the experimental numbers favors the smaller values of $\alpha_s$.

## 2.4 Sea with Suppressed Higher Multiplicity States

We propose a second modification of the model in which the contribution to the states with higher multiplicities is suppressed. Within the hadronic boundary, pseudoscalar exchange has been found to dominate over vector exchange and even gluon exchanges [5,28-30]. Although we are not using any dynamical model, we tend to believe that the states with larger number of gluons (having corresponding smaller probabilities) approximate the ones with saturated gluons for which color neutrality is achieved over a certain scale, which is called 'saturation scale'[31,32]. In Landshoff-Nachtmann model, quark–quark and hadron–hadron scatterings are assumed to arise due to exchanges of two non-perturbative gluons having vacuum quantum numbers[33] It is believed that pomeron and odderon exchanges are associated with the exchanges of a family of glueballs which are colorless but of different spins [33]. It is reasonable to assume that when a set of 'intrinsic' gluons exist in a nucleon, they would prefer to be in a similar state.

Even within the hadronic boundary, Goldstone boson exchange (GBE) model successfully describes diverse phenomenon [28-30]. In color space, singlets are unique due to confinement, but even there the color octet exchange models, and not any higher color states exchange model, have been successfully used [34]. Larger is color multiciplity of a group of particles (here the sea), larger will be the probability of its interaction with the rest of the particles (the core) and smaller will be its probability of survival. Authors of Ref.[6] have, on phenomenological ground, proposed a set of parameters in which states with higher multiplicities occur with lower probabilities.

In view of these phenomenological evidences, it appears reasonable to propose that higher multiplicity states are suppressed. We parameterize this suppression in a simple way by assuming that probability of a system to be in a spin and color state is inversely proportional to the multiplicity (both in spin and color spaces) of the state. This probability factor is additional to the previously incorporated factors in the probabilities. With this new input, we decompose Fock states as follows.

(i) $|gg\rangle$, $|\bar{q}q\,\bar{q}q\rangle$ sea: Equating the sum of the products of probabilities to the probabilities for finding the above Fock states as done in the previous cases, as done in the previous case, we get

$\rho_{1/2\,0s}[\rho_{1\,1s}, \rho_{8\,8s}, ]; \rho_{1/2\,1a}[\rho_{8\,8a}, \rho_{10\,\overline{10}\,a}]; \rho_{3/2\,1a}[\rho_{8\,8a}]; \rho_{3/2\,2s}[\rho_{8\,8s}]$

$= 0.03903(2,1/16;1/48,1/150;1/192;1/320)$ for $|gg\rangle$,

$= 0.00345(2,1/16;1/48,1/150;1/192;1/320)$ for $|u\bar{u}\,u\bar{u}\rangle$,

$= 0.00694(2,1/16;1/48,1/150;1/192;1/320)$ for $|d\bar{d}d\bar{d}\rangle$.

(ii) $|g q\bar{q}\rangle, |u\bar{u}\,d\bar{d}\rangle$ sea :

$\rho_{1/2\,0}[\rho_{1\,1}, \rho_{8\,8}, \rho_{10\,\overline{10}}]; \rho_{1/2\,1}[\rho_{1\,1}, \rho_{8\,8}, \rho_{10\,\overline{10}}]; \rho_{3/2\,1}[\rho_{8\,8}]; \rho_{3/2\,2}[\rho_{8\,8}]$

$= 0.01912(2, 1/8, 1/50; 2/3, 1/24, 1/150; 1/96; 1/160)$ for $|g u\bar{u}\rangle$,

$= 0.02876\,(2, 1/8, 1/50; 2/3, 1/24, 1/150; 1/96; 1/160)$ for $|g d\bar{d}\rangle$,

$= 0.01090\,(2, 1/8, 1/50; 2/3, 1/24, 1/150; 1/96; 1/160)$ for $|u\bar{u}\,d\bar{d}\rangle$.

(iii) $|gg q\bar{q}\rangle$, $|q\bar{q}\,q\bar{q}\,g\rangle$ sea :

$\rho_{1/2\,0a}[\rho_{1\,1a}, \rho_{8\,8a}, \rho_{10\,\overline{10}\,a}]; \rho_{1/2\,1s}[\rho_{1\,1s}, \rho_{8\,8s}, \rho_{10\,\overline{10}\,s}]; \rho_{3/2\,1a}[\rho_{8\,8a}]; \rho_{3/2\,1s}[\rho_{8\,8s}]; \rho_{3/2\,2a}$

$[\rho_{8\,8a}]$ $= 0.00328\,(1, 1/8, 1/50; 1, 1/8,1/50;1/32;1/32;1/160)$ for $|u\bar{u}\,u\bar{u}\,g\rangle$,

$= 0.00655\,(1, 1/8, 1/50; 1, 1/8,1/50;1/32;1/32;1/160)$ for $|d\bar{d}d\bar{d}\,g\rangle$,

$= 0.01952(1, 1/8, 1/50; 1, 1/8,1/50;1/32;1/32;1/160)$ for $|gg d\bar{d}\rangle$,

$= 0.01307(1, 1/8, 1/50; 1, 1/8,1/50;1/32;1/32;1/160)$ for $|gg u\bar{u}\rangle$.

(iv) $|g u\bar{u}\,d\bar{d}\rangle$ sea :

$\rho_{1/2\,0}[\rho_{1\,1}, \rho_{8\,8}, \rho_{10\,\overline{10}}]; \rho_{1/2\,1}[\rho_{1\,1}, \rho_{8\,8}, \rho_{10\,\overline{10}}]; \rho_{3/2\,2}[\rho_{8\,8}]$

$= 0.02620\,(1/2,1/16,1/100; 1/2,1/16,1/100;1/164;1/160)$.

(v) $|ggg\rangle$ sea:

$\rho_{1/2\,0a}[\rho_{1\,1a}, \rho_{8\,8a}]; \rho_{1/2\,1s}[\rho_{1\,1s}, \rho_{8\,8s}]; \rho_{3/2\,1s}[\rho_{8\,8s}]$

$= 0.02327(1,1/32; 1/3,1/96;1/384)$.

We would like to point out that there is nothing special about the use of the inverse of the multiplicity for suppression of higher multiplicity states. One could have fine tuned the power of the multiplicity to fit the data in a better way. It is only a possible way to suppress the contributions of states with higher multiplicities within the nucleon sea, which might be originally due to some dynamics. In the above calculation we have also included the (active) contribution of sea quarks. Numerical results for this case have been displayed in Model D in Tables 2.1 and 2.2 .

## 2.5 Summary and Conclusion

The statistical approach advocated in Ref.[12,13] was successful in describing the large asymmetry between $\bar{u}$ and $\bar{d}$ quark distributions of the proton. We have extended that approach by decomposing various quark-gluon Fock states into states in which the three quark core and the rest of the stuff (called sea ) have definite spin and color quantum numbers, using the assumption of equal probability for each substate of such a state of the nucleon. We have further used the approximation in which a quark in the core is not antisymmetrized with an identical quark in the sea, and have treated quarks and gluons as nonrelativistic particles moving in S-wave (except for a single $\bar{q}q$ sea) motion. Also, we have not taken into account any contribution of the s-quark and other heavy quarks, and we have covered only $\approx 86\%$ of the total Fock state. With these approximations we have calculated the quarks contribution to the spin of the nucleons, the ratio of the magnetic moments of the nucleons , their weak decay constant , and the ratio of SU(3) reduced matrix elements for the axial current. All of these quantities give integrated result of Bjorken variable. We have also considered two modifications of the above statistical approach with a view to reduce the contributions of the sea components with higher multiplicities, and have done the above calculations for those two cases as well.

The effect of Melosh rotation is to increase the values of the physical quantities related to the nucleon spin, which are measured in the rest frame of the nucleon while keeping the quark contribution to the nucleon spin, measured in the light–cone frame, unchanged. If we treat the Melosh rotation as free parameter, we can reproduce the experimental value of $g_A/g_V$ along with $\mu_p/\mu_n$ =1.415, and F/D =0.610 with the Melosh rotation parameters, $(\langle M_u \rangle, \langle M_d \rangle)$ = (0.699, 0.719), (0.797, 0.827) and (0.825, 0.692) for cases (C), (P) and (D) respectively, while keeping the values of $I_1^p$ and $I_1^n$ as listed in Table 2.2 .

Our results of calculation holds good for a typical hadronic energy scale~1 $GeV^2$ [13]. Experimental results for $I_1^p$ and $I_1^n$ apply for $Q^2 \approx 10$ $GeV^2$, and their values will increase when evolved to a lower energy scale. Hence, our calculated results for $I_1^p$ and $I_1^n$ may well be consistent with the data. Our result for the ratio of magnetic moments of nucleons is within few percent of the data. Weak decay constant has been calculated using Bjorken sum rule, written up to $O(\alpha_s^3/\pi^3)$. There is some controversy in the value of $\alpha_s$ at the low energy~1GeV we are working at, and we have chosen three typical values taken from recent literature. The significance of the Melosh rotation connecting the spin states in the light-front dynamics and the conventional instant form dynamics has been widely recognized. We have tried to construct a spin wave function of a nucleon with a non trivial sea in the nucleon rest frame from a statistical model of a nucleon. Such a wave function, along with a Melosh rotation, is capable of giving a reasonable result for several physical quantities related to the nucleon spin.

## 2.6 References


1. Fayyazuddin and Riazuddin, A Modern Introduction to Particle Physics, World Scientific, Singapore,1992.
2. E. Golowich, E. Haqq, and G. Karl, Phys. Rev.D28, 160(1983); F.E. Close and Z. Li, Phys. Rev. D42, 2194(1990); Z.Li , Phys. Rev.D44, 2841(1991); Z. Li and G Karl, Phys. Rev.D49, 2620 (1994).
3. J.F. Donoghue and E. Golowich, Phys. Rev. D15, 3421 (1977).
4. J. Keppler and H.M. Hofmann, Phys .Rev. D51,3936 (1995).
5. Di Qing, X-S Chen and F. Wang, Phys. Rev. D58, 114032 (1998).
6. X. Song and V.Gupta, Phys. Rev. D49, 2211(1994).
7. V. Gupta, R. Huerta and G. Sanchez- Colon, Int. J. Mod. Phys.A13, 4195 (1998).
8. A. Manohar and H. Georgi, Nucl. Phys. B234,189(1984).
9. A.E. Dorokhov and N.I. Kochelev, Phys. Lett. B259 335(1991); B304 167(1993).
10. D.I. Diakonov et al., Nucl. Phys. B480,341(1996); M. Wakamatsu and T. Kubota, Phys. Rev. D 57, 5755(1998).
11. R.S. Bhalerao, Phys. Rev. C63, 025208(2001); C. Bourrely, J. Soffer and F. Buccella, Eur. Phys. J. C23,487(2002).
12. Y. J Zhang, B. Zhang and B.-Q. Ma, Phys. Lett. B523, 260, (2002).
13. Y.J. Zhang, W.Z. Deng and B.-Q. Ma, Phys. Rev. D 65, 114005 (2002).
14. S.J. Brodsky and B.-Q. Ma, Phys. Lett.B381, 317(1996).

15. J. H. Field, Phys. Rev. D66, 013013 (2002); L. Frankfurt and Strikman, ibid, 031502.

16. A.G Williams et al., hep-ph/0107029.

17. M . Karliner and H.J. Lipkin, hep-ph/0202099; H.J. Lipkin, Phys. Lett B256, 284(1991).

18. B.-Q. Ma and Q.-R. Zhang, Z.Phys.C58, 479(1993)

19. H.J. Melosh, Phys. Rev. D9,1095(1974)

20. S.J. Brodsky and F. Schlumph, Phys. Lett.B329,111(1994)

21. B.-Q. Ma, I. Schmidt and J.-J. Yang, hep-ph0209115.

22. Z. Li in Ref.[2].

23. E143 Collab, P.L. Anthony et al., Phys. Rev. Lett. 74,346(1995): K Abe et al., ibid 75, 25 (1995); Phys. Rev. D58,112003 (1998).

24. J. Ellis and M. Karliner, Phys. Lett. B341, 397(1994).

25. L. Babukhadia and M.D. Scadron, hep-ph/0208242.

26. Particle Data Group, D.E Groom et al., Eur. Phys. J. C15, (2000).

27. M. Baldicchi and G.M. Prosperi in Phys.Rev. D66,O74008(2002).

28. D.O. Riska, Few Body Systems Suppl.11,18(1999); R. F. Wagenbrunn et al., ibid, 25; W. Plessas et al., ibid, 29.

29. T.P. Cheng and Ling-Fong Li, hep-ph/9811279.

30. J. Franklin, Phys. Rev. D66, 033010 (2002).

31. E. Iancu and L. McLerran, Phys. Lett., B 510,145(2001).

32. E. Iancu, A. Leonidov and L. McLerran, hep-ph/0202270.

33. S. Donnachie, G. Dosch, P. Landshoff and O. Nachtman, Pomeron Physics

and QCD, Cambridge University Press, 2002.

34. M. Klasen et al., Phys. Rev. Lett. 89, 032001(2002).