CHAPTER

III

CHAPTER-III

ISOSPIN BREAKING IN DIAGONAL PION-NUCLEON COUPLING CONSTANT: QCD SUM RULE APPROACH

3.1 Introduction

Determination of meson-nucleon couplings is of particular interest in particle physics as well as in nuclear physics. In particle physics, estimate of these parameters is useful to test the low energy behaviour of the QCD. In nuclear physics, nucleon-nucleon interactions are traditionally viewed as arising from meson exchanges. Pion exchange is linked to spontaneous and explicit chiral symmetry breaking (CSB) of low-energy QCD. According to Goldstone theorem, pions are Goldstone bosons having point like derivative couplings to the nucleons. For intermediate and short distances, heavy mesons have to be included in the modeling.

On the other hand, at energies much below the scales set by the pion mass, it is sufficient to consider four-nucleon interaction only. Starting from nucleon and pion degrees of freedom, effective field theory has been used for a separation of these scales [1]. Accounting higher order terms in the chiral expansion, a form of two-nucleon potential for the neutron-proton system has been developed in so-called modified Weinberg scheme and shown to be close in accuracy to the so-called modern potentials (in some partial waves) [2]. Isospin symmetry is a good symmetry of low-energy hadronic physics and charge symmetry is even better. In low-energy observables isospin violation is typically much smaller. The study of charge symmetry breaking, which is a special case of isospin breaking, in pion-nucleon coupling is an important step for investigation of charge symmetry breaking effects in nucleon-nucleon interactions.

The effect of isospin violating meson-nucleon couplings has recently seen a strong revival of interest in the investigation of charge symmetry breaking phenomenon. On a microscopical level, isospin symmetry is broken by electromagnetic interaction as well as the mass difference of up and down quarks: $m_u - m_d$. We shall examine the difference between the diagonal pion-nucleon coupling constants, $g_{pp\pi^0} - g_{nn\pi^0}$ using the QCD sum rule method.

3.2 A Model Calculation in QCDSR: Proton Mass

QCD sum rule method was originally suggested by Shifman, Vainstein and Zakharov, and has been applied to determine masses and leptonic widths of light mesons (ρ, π, κ*). For these determination, virtuality region is taken of the order of $Q^2 \sim 1 GeV^2$ and $\alpha_s \sim 0.3\text{-}0.4$, so that perturbative terms are small i.e $\alpha_s/\pi \sim 0.1$ and hence only leading logarithmic corrections~$[\alpha_s(Q^2)\ln Q^2/\Lambda^2]$ are taken into account. To illustrate the characteristic features of the method and to use it for our main calculation, we shall show a calculation of the mass of the proton using QCD sum rules.

For this purpose we consider the polarization operator as

$$\Pi(p,k) = i \int d^4x\, e^{ipx} \langle 0 | T\{\eta(x), \bar{\eta}(0)\} | 0 \rangle \qquad (3.1)$$

where η (x) is the quark current with proton quantum numbers and p^2 is chosen to be space-like: $p^2 < 0$, $|P^2| \sim 1 GeV^2$. The current η is colorless product of three quark fields $\eta(x) = \varepsilon_{abc} q^a q^b q^c$, q=u,d, the exact form of the current will be specified below.

Unlike mesons, in baryons there exist several currents with quantum numbers of a given baryon. The choice between them should be done from physical reasons in order to provide: (1) renormcovariance, (2) existence of nonrelativistic limit, (3) the above formulated requirement (for proton) for the functions f_1 and f_2 (given below in Eq. (3.2)) to be of the same order, (4) convergence of operator expansion series within accounted terms. Specifically for proton all these requirements are satisfied by the current ,

$\eta = (u^a C\gamma_\mu u^b)\gamma_\mu \gamma_5 d^c \varepsilon_{abc}$

The general structure of $\prod$(p) is

$$\prod(p) = \not{p} f_1(p^2) + f_2(p^2) \qquad (3.2)$$

For each of the function $f_i(p^2)$, i=1,2 the following operator expansion can be written:

$$f_i(p^2) = \sum_n C_n^i(p^2)\langle 0|O_n^{(i)}(0)|0\rangle \qquad (3.3)$$

where $\langle 0|O_n^i|0\rangle$ are vacuum expectation values of different operators (vacuum condensates) and $C_n^i(p^2)$ are functions calculable in QCD.

$$f_1(p^2) = C_0 p^4 \ln\frac{\Lambda_u^2}{-p^2} + \sum_k C_2^k \langle 0|\bar{q}\Gamma^{(k)} q\, \bar{q}\Gamma^{(k)} q|0\rangle \frac{1}{p^2} +$$
$$C_4 \langle 0|\frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a\mu\nu}|0\rangle \ln\frac{\Lambda_u^2}{-p^2} + C_6 \langle 0|\frac{\alpha_s}{\pi} \bar{q}q.\bar{q}g_s\sigma^{\mu\nu} G_{\mu\nu}^a \frac{\lambda^a}{2} q|0\rangle \frac{1}{p^4} \qquad (3.4)$$
$$+ polynomials$$

$$f_2(p^2)=C_1p^2\ln\frac{\Lambda_u^{\,2}}{-p^2}\langle 0|\bar{q}q|0\rangle+C_3\langle 0|\bar{q}q\frac{\alpha_s}{\pi}G^{a\mu\nu}G^a_{\mu\nu}|0\rangle\frac{1}{p^2}$$
$$C_5\sum_k\langle 0|\bar{q}q\,\bar{q}\Gamma^{(k)}q\,\bar{q}\Gamma^{(k)}q|0\rangle\frac{\alpha_s}{p^4}+polynomials \tag{3.5}$$

where C_i's are constants, Λ_u is the ultraviolet cut-off. The current u-and d-quark masses entering the Lagrangian of QCD are very small, of the order of several MeV, so they can be neglected with very good accuracy, i.e for the time being, we neglect the quark masses, then L_{QCD} is chiral-invariant. If this chiral symmetry would not be spontaneously broken, then $f_2(p^2)$ would remain identically zero. As explained in Chapter 1, the chiral symmetry is spontaneously broken. The first evidence of this is the existence of large baryon masses: $M_B \gg \Lambda_{QCD}$. Another signal is the fact that chiral symmetry violating quark condensate $\langle 0|\bar{q}q|0\rangle$ is non-zero, and is approximately equal to $-(240\text{MeV})^3$.

$$\langle 0|\bar{q}q|0\rangle=-\frac{1}{2}\frac{f_\pi^2m_\pi^2}{m_u+m_d}=-(240MeV)^3, \tag{3.6}$$

Since $\langle 0|\bar{q}q|0\rangle$ is the lowest dimensional chirality violating operator, the operator expansion for $f_2(p^2)$ starts from the term proportional to $\langle 0|\bar{q}q|0\rangle$.

For any colorless operator O_1 and O_2 at large N_c

$$\langle 0|O_1O_2|0\rangle=\langle 0|O_1|0\rangle\langle 0|O_2|0\rangle(1+O(\frac{1}{N_c})) \tag{3.7}$$

i.e in the limit $N_c\to\infty$ factorization becomes exact. By virtue of factorization and taking into account the relation

$$\langle 0|q^a_\alpha(0)\bar{q}^b_\beta(0)|0\rangle=-\frac{1}{12}\delta^{ab}\delta_{\alpha\beta}\langle 0|\bar{q}q|0\rangle \tag{3.8}$$

(a, b = 1,2,3 are color, α, β are Lorentz indices) all four-quark vacuum expectation values (v.e.v) reduce to the quark condensate square $\langle 0|\bar{q}q|0\rangle^2$. In order to improve and control the accuracy in the calculation of mass, other v.e.v's will also be taken into account: Gluonic condensate $\langle 0|\frac{\alpha}{4\pi}G^a_{\mu\nu}G^{a\mu\nu}|0\rangle$, mixed condensate $\langle 0|\bar{q}\sigma_{\mu\nu}(\lambda^n/2)G^n_{\mu\nu}q|0\rangle$ and higher dimensional v.e.v's $\langle 0|\bar{q}q|0\rangle\langle 0|\bar{q}\sigma_{\mu\nu}(\lambda^n/2)G^n_{\mu\nu}q|0\rangle$, $\alpha_s\langle 0|\bar{q}q|0\rangle^3$, $\langle 0|\bar{q}q|0\rangle\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}|0\rangle$. The gluonic condensate gives a contribution into chirality preserving structure $f_1(p^2)$.

∏(p) may be expressed using the dispersion relations

$$f_i(s) = -\frac{1}{\pi}\int_0^\infty \frac{\operatorname{Im} f_i(p^2)}{p^2 - s}dp^2 + polynomial \tag{3.9}$$

In order to extract physical quantities of interest, it is useful, at this stage, to apply Borel transformation on both side of this equation. The Borel (Laplace) transformation is defined as

$$\hat{B}f_i(q^2) = \lim_{\substack{-q^2, n\to\infty \\ -q^2/n = M^2 \, fixed}} \frac{(-q^2)^{n+1}}{n!}\frac{d}{dq^{2n}}f_i(q^2). \tag{3.10}$$

This gives

$$B_{M^2}f_i(s) = \frac{1}{\pi}\int_0^\infty \exp(-\frac{p^2}{M^2})\operatorname{Im} f_i(p^2)dp^2,$$

where $f_i(s)$ is given by dispersion relation (3.9). In particular, we have

$$\mathrm{B}_{M^2}\frac{1}{s^n} = \frac{1}{(M^2)^{n-1}(n-1)!} \tag{3.11}$$

The Borel transform permits to attain three goals at once:

(1) to nullify subtraction term,

(2) to suppress the contribution of the higher excited states compared to the desired lowest state (proton)

(3) to suppress the contributions of high order terms in the operator expansion (owing to factor 1/(n-1)! in (3.11)) .

The lowest state (proton) contribution to imaginary part of $\prod$(p) has the form

$$\operatorname{Im}\Pi(p) = \pi\langle 0|\eta|p\rangle\langle p|\bar{\eta}|0\rangle\delta(p^2 - m^2) = \pi\lambda_N{}^2(\not{p} + m)\delta(p^2 - m^2), \tag{3.12}$$

where

$$\langle 0|\eta|p\rangle = \lambda_N u(p)$$

Here λ_N is the proton transition constant into quark current and u(p) is the proton spinor.

It is clear from above that proton contribution will dominate in some region of the Borel parameter M^2 only when QCD calculated functions f_1 and f_2 are of same order, and the spontaneous violation of chiral invariance characterized by the value of quark condensate has to explain the numerical value of the proton mass.

Contributions of higher mass states will also be taken into account in order to improve and control the accuraey in the dispersion representation as written above, by replacing Imf(p^2) by contribution of the simplest quark loops starting from some "continuum threshold" W. Taking into account all points stated above, the sum rule for the calculation of the proton mass is given as

$$M^6 E_2(\frac{W^2}{M^2})L^{-4/9} + \frac{4}{3}a^2 L^{4/9} + \frac{1}{4}bM^2 E_0(\frac{W^2}{M^2})L^{-4/9} - \frac{1}{3}a^2 \frac{m_o^2}{M^2}$$
$$= \tilde{\lambda}_n^2 \exp(-\frac{m^2}{M^2}) \quad , (3.13)$$

$$2aM^4 E_1(\frac{W^2}{M^2})L^{-4/9} + \frac{272}{81}\frac{a^3}{M^2}L^{4/9} + \frac{1}{4}bM^2 E_0(\frac{W^2}{M^2}).L^{-4/9} - \frac{1}{3}a^2 \frac{m_o^2}{M^2}$$
$$= \tilde{\lambda}_n^2 \exp(-\frac{m^2}{M^2})m \quad (3.14)$$

Here

$a_q = -(2\pi)^2 \langle 0|\bar{q}q|0\rangle$,

$b = (2\pi)^2 \langle 0|\frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu}|0\rangle \approx 0.5GeV^2$ and

$-g\langle 0|\bar{q}\sigma_{\alpha\beta}\frac{\lambda^n}{2}G_{\alpha\beta}{}^n q|0\rangle \equiv m_0^2 \langle 0|q\bar{q}|0\rangle$

$m_0^2 \approx 0.8GeV^2$

The factors

$E_0(x) = 1-e^{-x}$, $E_1(x) = 1-(1+x)e^{-x}$ and $E_2(x) = 1-(1+x+x^2/2)e^{-x}$ (3.15)

take into account the continuum contribution. We also have

$\tilde{\lambda}_N^2 = 32\pi^4 \lambda_N^2$,

the factors $L = \frac{\ln(M/\lambda)}{\ln(\mu/\lambda)}$ take into account the anomalous dimensions of the operators

(Λ is the QCD parameter, μ is the renormalization point, numerical values hereafter corresponds to μ=1.0GeV).

charge independence breaking is explained in terms of one-pion exchange together with a four-nucleon contact term [4]. Therefore, it is useful to constrain the isospin violation in the pion-nucleon couplings directly from QCD based non-perturbative methods such as QCD sum rule.

At the fundamental level, isospin violation takes place due to charge difference and mass difference of up- and down-quarks. At the phenomenological level, the effect of these differences may get augmented due to strong interaction, and in practice, this may appear in the form of isospin splitting of other phenomenological parameters such as quark condensates. QCD sum rules have been used in past to study pion-nucleon couplings and also their isospin breaking[5-11]. Three different methods have been used to investigate pion-nucleon coupling constant in the framework of the conventional QCD sum rule. In the three-point function method, one studies the vacuum-to-vacuum matrix element of the correlation function of the interpolating fields of the two nucleons and a meson[6]. However, it has been argued that the method suffers from contamination of higher resonance states [12].

In the two-point function external field method, one studies the correlation function of the interpolating fields of the two nucleons in the presence of an external pion field [7]. However, the induced condensates appearing in this method are not as reliably known, as the other more commonly used condensates. In the following we shall follow the third, the two-point function method [5, 8-10] in which one studies vacuum-to-pion matrix element of the correlation function of the interpolating fields of two nucleons:

$$\Pi(p,k)=i\int d^4x\, e^{ipx} \langle 0 | T\{\eta(x), \bar{\eta}(0)\} | \pi^0(k)\rangle \tag{3.16}$$

Here, η is the interpolating field of a nucleon and $|\pi^0(k)\rangle$ is the neutral pion state with momentum k. Isospin is suppressed for simplicity. For η, Ioffe's interpolating field [13] will be used; for proton, it is written as

$$\eta(x)=\varepsilon_{abc}\,(u_a^T(x)C\gamma_\mu\, u_b(x))\gamma_5\gamma^\mu d_c(x) \tag{3.17}$$

The correlation function is calculated, on the one hand, by Wilson's operator product expansion(OPE), and on the other hand it is evaluated using hadronic physical states. The two descriptions are matched in the deep Euclidean region via the dispersion relation and the physical quantity of interest is extracted.

The expression (3.16) is known to have four Dirac structures [11]. Among these, the coefficient of the double pole of $i\gamma_5\hat{p}$ structure on the mass shell vanishes, and the sum rule obtained at the Dirac structure $i\gamma_5$ substantially underestimates the ratio F/D compared to its value known in SU(3) symmetry limit [5]. The Dirac structure $i\gamma_5\hat{k}$ has been found not to be reliable for calculating the πNN coupling as it contains large contribution from the continuum [8]. The sum rules for the meson-baryon coupling constant at the structures $i\gamma_5\hat{k}$ and $\gamma_5\sigma_{\mu\nu}p^\mu k^\nu$ have been studied extensively in [5,8-10]. Kim et al. [8,9] have claimed to find nice features in the sum rule at the $\gamma_5\sigma_{\mu\nu}p^\mu k^\nu$ Dirac structure for calculation of πNN coupling constant. It was observed that for this sum rule the coupling constant comes out to be independent of the choice of the effective Lagrangian, i.e, independent of pseudoscalar and axial vector schemes [10], and is stable against the variation of the continuum parameter due to cancellation of contributions from higher resonances of different parities [8]. We use this sum rule to calculate isospin splitting in the diagonal pion-nucleon coupling constant $g_{\pi NN}$ In the existing result for the correlation function (3.16), we also include quark mass dependent terms and do renormalization group improvement. In addition, we also take into account the effect of π^0–η mixing and electromagnetic correction to the π^0– quark couplings.

In order to reduce the dependence of the splitting in the coupling on the isospin splitting in the quark condensate, which is rather poorly known, we take the ratio of the sum rule for the coupling $g_{\pi NN}$ to the corresponding chiral-odd sum rule for the nucleon mass, and then consider the difference and the average of this ratio for proton and neutron. This resulting sum rule is fitted to a straight line form, which directly gives the difference and the average of the couplings:

$$\delta g = g_{\pi^0 pp} - g_{\pi^0 nn}, \qquad g_{\pi NN} = (g_{\pi^0 pp} + g_{\pi^0 nn})/2 . \tag{3.18}$$

3.4 Sum Rules for Pion-Nucleon Couplings

As stated above, in order to construct sum rules for the coupling $g_{\pi NN}$ at the structure $\gamma_5\sigma_{\mu\nu}p^\mu k^\nu$, in addition to the results already derived in Ref.[5], we calculate contributions coming from the quark mass dependent terms of Figs.3.1(a) and 3.1(b). We enumerate below the Fourier transforms and the Borel transforms of the coefficients of $\gamma_5\sigma_{\mu\nu}p^\mu k^\nu$, of these contributions for the proton:

$$\text{Fig 3.1(a)} \xrightarrow{FT} -(1/2\pi^2)m_d f_\pi \gamma_5\sigma_{\mu\nu} p^\mu k^\nu \ln(-p^2)$$

$$\xrightarrow{BT} (M^2/2\pi^2)m_d f_\pi \gamma_5\sigma_{\mu\nu} p^\mu k^\nu \tag{3.19a}$$

$$\text{Fig 3.1(b)} \xrightarrow{FT} -(1/9f_\pi)(m_u/p^4)\langle \bar{u}u\rangle\langle \bar{d}d\rangle \gamma_5\sigma_{\mu\nu} p^\mu k^\nu$$

$$\xrightarrow{BT} (1/9f_\pi)(m_u/M^2)\langle \bar{u}u\rangle\langle \bar{d}d\rangle \gamma_5\sigma_{\mu\nu} p^\mu k^\nu \tag{3.19b}$$

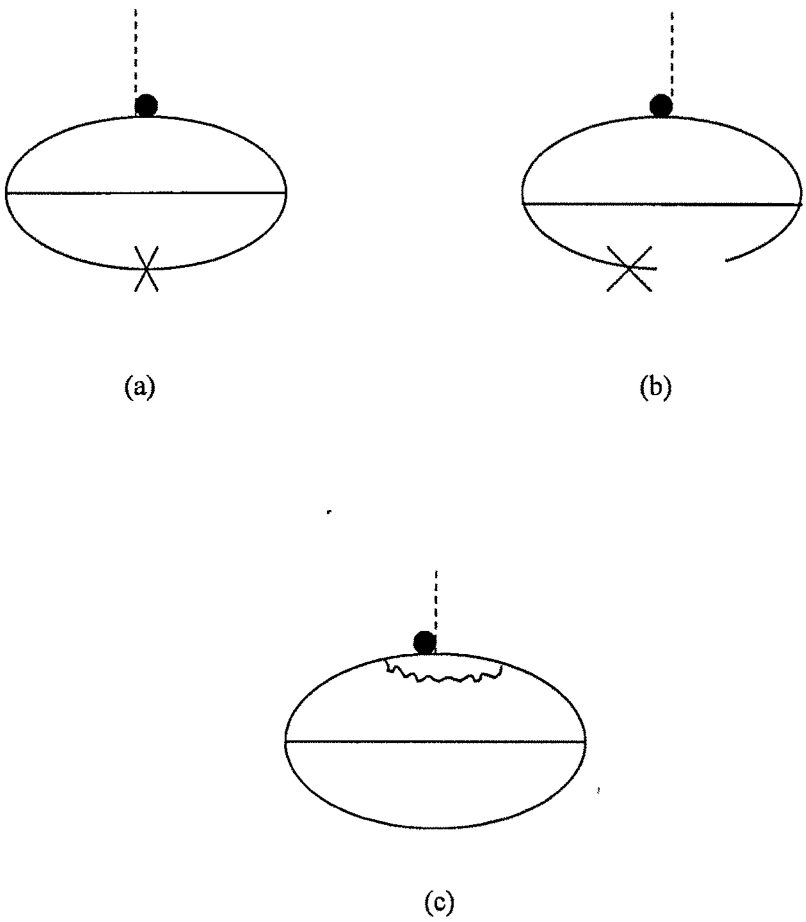

Figure 3.1: The additional diagrams considered in this work. The solid lines denote the quark propagator, dashed line is the pion propagator, and the blob denotes their interaction vertex. Cross denotes quark mass insertion.

We have checked that the coefficient of the operator m_q $(\langle \bar{u}u \rangle, \langle \bar{d}d \rangle)$ $\langle (\alpha_s/\pi)G^2 \rangle$ is zero. So far we have assumed that π^0 mass eigen state is a pure isovector state. However, it is well known that the mass eigenstates π^0 and η are not pure octet states [14], rather they are mixtures of flavor octet eigenstates π_3 and π_8 Denoting $\pi-\eta$ mixing angle by θ, the mass eigenstates may be written as :

$|\pi^0\rangle = |\pi_3\rangle + \theta|\pi_8\rangle$, $|\eta\rangle = |\pi_8\rangle - \theta|\pi_3\rangle$.

Since θ is small $\cong$ 0.01, this amounts to the replacement for the couplings:

$$g_{\pi}{}^{0}{}_{pp} = g_{\pi 3pp} + \theta\, g_{\pi 8pp},\quad g_{\pi}{}^{0}{}_{nn} = g_{\pi 3nn} - \theta\, g_{\pi 8nn}. \tag{3.20}$$

Here, we ignore any possible mixings of π^0 and η with η'. We use the sum rules for the couplings of pure octet states, $g_{\pi 3NN}$ and $g_{\pi 8NN}$ [5] in the above Eq. (3.20) to get the couplings of the physical state π^0 with nucleons.

It has been pointed out in [7] that the vertex corrections to π^0uu and π^0dd couplings, due to photon exchanges, can give rise to non negligible isospin breaking in $g_{\pi NN}$. Specifically, it has been found that in the minimum subtraction scheme the following electromagnetic corrections arise to the pion-quark couplings:

$$g_{\pi}{}^{0}{}_{uu} \to g_{\pi}{}^{0}{}_{uu}\{1+\frac{\alpha}{4\pi}(\frac{52}{9}-\frac{4}{3}\gamma_E)\},\quad g_{\pi}{}^{0}{}_{dd} \to g_{\pi}{}^{0}{}_{dd}\{1+\frac{\alpha}{4\pi}(\frac{13}{9}-\frac{1}{3}\gamma_E)\} \tag{3.21}$$

The most important correction arising due to these vertex corrections is shown in Fig.3.1(c) for perturbative contribution. This correction will be different for proton and neutron because π^0 couples to different quark lines for the two cases. Similar correction will also arise in other terms. In effect, the coefficient of each term in the OPE is multiplied with a factor which depends on the charge of the quark to which π^0 couples in the nucleon.

Combining the sum rule for the meson-nucleon couplings as obtained in Ref.[5], but with the specification of the flavor of the quark condensate, at the Dirac structure $\gamma_5\sigma_{\mu\nu}p^{\mu}k^{\nu}$ with the above three types of corrections, we get the following sum rules, after Borel transformation and renormalization group improvement, for the diagonal pion-nucleon couplings:

$$g_{\pi^0 pp}\lambda_p^2(1+D_{\pi p}M^2)$$

$$= -\ e^{(m_p^2/M^2)}\ \frac{\langle \bar{d}d\rangle}{f_\pi}\ [\{\ \frac{M^4 E_0(S_o^p/M^2)}{12\pi^2} + \frac{1}{216}\ \langle\ (\alpha_s/\pi)G^2\ \rangle + \frac{\langle \bar{u}u\rangle}{9}\ m_u\}(\ 1-\frac{\theta f_\pi}{\sqrt{3}f_\eta}\ ) \times$$

$$\{1+\frac{\alpha}{4\pi}\ (\ \frac{13}{9}\ -\ \frac{1}{3}\ \gamma_E)\}+f_\pi^2\ (\ \frac{4M^2}{3}\ -\ \frac{m_0^2 L^{-14/27}}{6} + \frac{26\delta^2}{27}\ -\ \frac{M^4 L^{-8/9} m_d E_0(S_o^p/M^2)}{2\pi^2\langle \bar{d}d\rangle}\ \times$$

$$\left(1+\frac{10\delta^2}{9M^2}\right)\ )(1+\frac{\theta f_\eta}{\sqrt{3}f_\pi})\ \{1+\frac{\alpha}{4\pi}(\frac{52}{9}-\frac{4}{3}\gamma_E\ )\}], \qquad (3.22a)$$

$$g_{\pi^0 nn}\lambda_n^2(1+D_{\pi n}M^2)$$

$$= -\ e^{(m_n^2/M^2)}\ \frac{\langle \bar{u}u\rangle}{f_\pi}\ [\{\frac{M^4 E_0(S_o^n/M^2)}{12\pi^2}+\frac{1}{216}\langle(\alpha_s/\pi)G^2\rangle+\frac{\langle \bar{d}d\rangle}{9}m_d\ \}(1+\frac{\theta f_\pi}{\sqrt{3}f_\eta})\ \times$$

$$\{1+\frac{\alpha}{4\pi}(\frac{52}{9}-\frac{4}{3}\gamma_E)\}+f_\pi^2\ (\frac{4M^2}{3}-\frac{m_0^2}{6}L^{-14/27}+\frac{26\delta^2}{27}-\frac{M^4 L^{-8/9} m_u E_0(S_o^n/M^2)}{2\pi^2\langle \bar{u}u\rangle})\times\left(1+\frac{10\delta^2}{9M^2}\right)$$

$$(1-\frac{\theta f_\eta}{\sqrt{3}f_\pi})\{1+\frac{\alpha}{4\pi}(\ \frac{13}{9}-\frac{1}{3}\gamma_E\ )\}] \qquad (3.22b)$$

Here, $L = \ln(M^2/\Lambda_{QCD}^2)/\ln(\mu^2/\Lambda_{QCD}^2)$, μ is the renormalization scale, and Λ_{QCD} is the QCD scale parameter. The anomalous dimensions of various operators have been taken into account through the appropriate powers of L [15]. It is understood that within this truncated series, v.e.v's of the quark and the gluon operators have to be taken at the scale μ. $D_{\pi N}$ are unknown constants arising from resonance states $\langle \bar{q}g\sigma.Gq\rangle = m_0^2\langle \bar{q}q\rangle$, γ_E is the Euler's constant. $S_o^{p,n}$ are the continuum thresholds which take care of contributions of excited states in a standard way, and

$E_0(x)=1-e^{-x}$. It is clear from the sum rules (3.22a) and (3.22b) that the isospin splitting in the diagonal coupling constant, δg, has a direct dependence on the isospin splitting of the light quark condensate $\langle \bar{q}q \rangle$ and on the same of the coupling of the nucleon interpolating field to the nucleon state, λ_N. Both these splittings are rather poorly known. However, if we divide these sum rules by the chiral-odd mass sum rules of the respective nucleons, then the λ_N dependence will get cancelled and the dependence of δg on the isospin splitting of the quark condensate will get minimized. We use the sum rule for the calculation of neutron-proton mass difference derived by Yang et al.[15]:

$$\lambda_p^2 = -(\frac{1}{m_p})\exp(\frac{m_p^2}{M^2}) < \bar{d}d > [\frac{1}{4\pi^2} M^4 E_1(\frac{S_{oN}^p}{M^2})(1+\frac{4\chi}{9}) - \frac{1}{18} < (\frac{\alpha_s}{\pi})G^2 > + \frac{544}{81}\pi\alpha_s < \bar{u}u >^2 \frac{L^{-1/9}}{M^2} - (\frac{M^6}{16\pi^2})(\frac{m_d}{<dd>})L^{-8/9} E_2(\frac{S_{oN}^p}{M^2}) + (\frac{M^2}{32\pi^2})(\frac{m_d}{<dd>}) \times < (\frac{\alpha_s}{\pi})G^2 > L^{-8/9} E_2(\frac{S_{oN}^p}{M^2}) - \frac{4}{3} m_d \frac{< \bar{u}u >^2}{< \bar{d}d >} - 2m_u < \bar{u}u > + (\frac{m_{em}^2 M^2}{24\pi^2}) E_0(\frac{S_{oN}^p}{M^2})] \tag{3.23a}$$

$$\lambda_n^2 = -(\frac{1}{m_n})\exp(\frac{m_n^2}{M^2}) < \bar{u}u > [\frac{1}{4\pi^2} M^4 E_1(\frac{S_{oN}^n}{M^2})(1+\frac{\chi}{9}) - \frac{1}{18} < (\frac{\alpha_s}{\pi})G^2 > + \frac{544}{81}\pi\alpha_s < \bar{d}d >^2 \frac{L^{-1/9}}{M^2} - (\frac{M^6}{16\pi^2})(\frac{m_u}{<uu>})L^{-8/9} E_2(\frac{S_{oN}^n}{M^2}) + (\frac{M^2}{32\pi^2})(\frac{m_u}{<uu>}) \times < (\frac{\alpha_s}{\pi})G^2 > L^{-8/9} E_2(\frac{S_{oN}^n}{M^2}) - \frac{4}{3} m_u \frac{< \bar{d}d >^2}{< \bar{u}u >} - 2m_d < \bar{d}d >] \tag{3.23b}$$

The terms with χ in Eqs.(3.23a) and (3.23b) take into account perturbative electromagnetic contribution, and $m_{em}^2 a_q \equiv (2\pi)^2 \langle e\bar{q}\sigma\, Fq\rangle$, and $\langle \bar{u}\sigma.Fu\rangle = \frac{2}{3}\langle \bar{q}\sigma.Fq\rangle$ with $\langle \bar{d}\sigma\, Fd\rangle = -\frac{1}{3}\langle \bar{q}\sigma\, Fq\rangle$, has been introduced ($F_{\mu\upsilon}$ is the electromagnetic-field strength tensor). In order to attain fit, $\chi = 0.0036$ and $m_{em}^2 = 0.048 GeV^2$ has been chosen [15]. Here, $S_{oN}^{p,n}$ are the continuum thresholds for the mass sum rules, and these may, in general be different from $S_o^{p,n}$. $E_1(x) = 1-(1+x)e^{-x}$ and $E_2(x) = 1-(1+x+x^2/2)e^{-x}$. Eliminating λ_p^2 of Eq.(3.22a) with the above λ_p^2 of Eq.(3.23a) and λ_n^2 of Eq.(3.22b) with the above λ_n^2 of Eq.(3.23b), we get the sum rules for $g_{\pi^0 pp}$ and $g_{\pi^0 nn}$. Finally, on taking the difference and the average of these two sum rules we get sum rules for δg and $g_{\pi NN}$, which we express as:

$$\delta g\,(1+ D^a_{\pi N} M^2) = F_a(M^2), \quad g_{\pi^0 NN}\,(1+D^s_{\pi N} M^2) = F_s(M^2), \qquad (3.24)$$

where $D^a_{\pi N}$ and $D^s_{\pi N}$ are constants. We shall study the sum rule for $g_{\pi^0 NN}$ also, in parallel with that for δg, and compare the result for $g_{\pi^0 NN}$ with that derived earlier [5] in a similar approach. Thus a straight line fit of $F_{a,s}(M^2)$ will directly give δg and $g_{\pi^0 NN}$ in the form of intercepts.

3.5 Analysis of Result and Discussion

Let us define $a_q = -(2\pi)^2 \langle \bar{q}q\rangle$, $b = \langle g_s^2 G^2\rangle$, $\gamma = \frac{\langle \bar{d}d\rangle}{\langle \bar{u}u\rangle} - 1$ and set $\langle \bar{q}q \rangle = \frac{1}{2}[\langle \bar{d}d\rangle + \langle \bar{u}u\rangle]$.

Normally, for the calculation of $g_{\pi NN}$, light quark mass dependent terms are not included. However, we find that the perturbative quark mass dependent term is numerically more important than the power corrections in quark mass independent terms. To get an idea of the errors involved in values of δg and $g_{\pi NN}$, we vary the values of condensates and the continuum threshold consistent with their values used in literature: the value of a_u has been varied from 0.45 GeV^3[17] to 0.55 GeV^3 [7,15,18], while that of b has been varied between 0.47 GeV^4[5,15] to 1.0 GeV^4[16,17]; δ^2 has been varied from 0.2 GeV^2[19] to 0.35 GeV^2 [20]. Most of our analysis is based on $\gamma = -0.01$ which is the upper limit from a range given in Ref.[6] based on various sources: $0.002 < -\gamma < 0.010$. For the sake of comparison, we have also given results obtained for $\gamma = -0.00657$[14] .The variation of the continuum threshold S_0 from 2.07GeV^2 to 2.57GeV^2 [5], for a given set of condensates, changes $g_{\pi NN}$ by a maximum of 3% and changes δg at most by 7%. The range of the Borel mass squared is 0.8 $GeV^2 \leq M^2 \leq 1.0$ GeV^2. This range is chosen so as to ensure that the contribution of excited states remains less than 50% and that of the operator of the highest dimension considered remains less than 10% of the total. Smaller range of Borel mass parameter, such as the above, is normally used whenever QCD sum rules are applied for calculating isospin splittings of nucleonic parameters [15]. Moreover, this range is within the ones used in Refs. [5,15].

We have looked for the values of $g_{\pi NN}$ and δg in the parameter space spanned by a_u, b, δ^2 and S_0 within the ranges stated above. The highest and the lowest values of δg and $g_{\pi NN}$ along with the values of the parameters for which they arise are displayed in Table 3.1. In all, we get $\delta g = -(4.92 \pm 1.90) \times 10^{-2}$ and $-(5.09 \pm 1.87) \times 10^{-2}$,

$g_{\pi NN}$=11.76±2.43 and 11.13±2.45 for Λ_{QCD}=0.1 GeV[15] and 0.15 GeV respectively. For a given set of values of a_u, b, δ^2 and S_0, the maximum variation occurring in δg due to change in Λ_{QCD} from 0.1 GeV to 0.15 GeV is 6.3% while that for $g_{\pi NN}$ is 7.5%. The values of $\delta g/g_{\pi NN}$ obtained are $-(4.17\pm1.42)\times10^{-3}$ and $-(4.55\pm1.42)\times10^{-3}$ for Λ_{QCD} = 0.1 GeV and 0.15 GeV respectively. The lowest numerical range of δg, for γ =-0.00657 in the same parameter space, gets pushed down to -1.13×10^{-2}. In Table 3.2, we have displayed a set of values of parameters (a_u, b, δ^2 and S_0) which give rise to central values of δg for the two values of Λ_{QCD} and γ.

Contributions to δg for its central value coming from various symmetry breaking parameters are displayed in Table 3.3. We observe that the contributions coming from the non vanishing values of each of γ, α, θ, Δm_q, and Δm_N individually add up almost linearly to give the final value of δg when all of these parameter are non zero. It is well known that $m_q<\bar{q}q>$ is renormalization group invariant quantity [15]. From the contributions of Δm_q and γ to δg , it is evident that δg will remain stable for a variation in Δm_q and the corresponding variation in γ in accordance with the renormalization group equation. The largest contribution to δg ($\delta g/g_{\pi NN}$ = -2.0×10^{-2}) comes from $\Delta m_N\neq0$ alone. Naively, one may expect its contribution to be ~ $\Delta m_N/m_N\sim10^{-3}$. However, r.h.s of Eqs (3.22a,b), when divided by r.h.s of mass sum rules of Eqs.(3.23a,b) contain electromagnetic contribution from phenomenological parameters χ and m_{em}. Moreover, $D_{\pi N}$ in eqs.(3.22a,b), which decide the slope of the straight line, arises due to the transitions N→N*, and depends on the nucleon mass nonlinearly due to its dependence on λ_N and $g_{\pi NN^*}$. The resulting fractional change in $D_{\pi N}$, due to change in m_N, is larger, and is in opposite direction ($D_{\pi p}$ = 7.35×10^{-2}, $D_{\pi n}$

$= 6.86\times10^{-2}$) compared to that in $g_{\pi NN}$ ($g_{\pi}{}^{0}{}_{pp}$ =11.602, $g_{\pi}{}^{0}{}_{nn}$=11.810) in the region of interest M^2~m_N^2. Finally, it should be kept in mind that the separation of contributions to δg , as shown in Table 3.3, is not very clear cut. As is evident from Eqs.(3.23a,b), Δm_N itself arises due to γ and Δm_q, in addition to its dependence on purely phenomenological parameters χ and m_{em}. The smallest contribution to δg ($\delta g/g_{\pi NN}$~$\pm10^{-4}$) comes from $\Delta S_0\neq0$ and $\Delta S_{0N}\neq0$. The continuum for the proton may come from a combination of $p\pi^0$ and $n\pi^+$, while that for the neutron may come from a combination of $n\pi^0$ and $p\pi^-$. This is well supported by the fact that the first ½$^+$ state [N(1440)] decays (60-70)% of the time to Nπ. Hence, in an average sense we expect $S_0^p = S_0^n$ for the sum rules (3.22a) and (3.22b), and $S_{0N}^p = S_{0N}^n$ for the sum rules (3.23a) and (3.23b). To get an idea of the effect of the difference of the above continuum thresholds for proton and neutron on δg, in view of the above argument, we consider this difference to be typically in the range of 0.1% [15].

The resulting value of δg, for the choice $(S_0^p-S_0^n)/\overline{S_0} = (S_{0N}^p-S_{0N}^n)/\overline{S}_{0N} = \pm0.1\%$, has been displayed in Table 3.3. In view of the very small contribution of ΔS_0 and ΔS_{0N} to δg , we set them zero in our further analysis.

To sum up, taking into account uncertainties in the quark condensate, the gluon condensate, the twist-4 parameter δ^2, the continuum threshold S_0 and the QCD scale parameter, Λ_{QCD}, we obtain for γ= -0.01 the following estimate of δg and $g_{\pi NN}$:

$$\delta g = -(4.99\pm1.97)\times10^{-2},$$

$$g_{\pi NN} = 11.44\pm2.76,$$

$$\delta g/g_{\pi NN} = -(4.36\pm1.62)\times10^{-3}. \quad (3.25)$$

TABLE 3.1: The maximum and minimum values obtained for δg and $g_{\pi NN}$ in the parameter space spanned by a_u = (0.45 − 0.55) GeV^3, b= (0.47 − 1.0) GeV^4, δ^2 = (0.2 − 0.35) GeV^2, $S_0^{p,n}$ = (2.07 − 2.57) GeV^2, Λ_{QCD}=(0.1 − 0.15) GeV and M^2= (0 8 − 1.0)GeV^2 The fixed parameters are $S_{0N}^{p,n}$ (the continuum threshold in the mass sum rule)=2 25[15], m_u=0.0051, m_d=0.0089, m_0^2=0.8, μ=0.5, m_p=0.93827, m_n=0.93957, f_π=0.093 (all in GeV units), f_η/f_π =1.1[21].

a_u GeV^3	b GeV^4	δ^2 GeV^2	S_0 GeV^2	Λ_{QCD}=0.1GeV				Λ_{QCD} =0.15 GeV			
				$\delta g\times10^2$		$g_{\pi NN}$		$\delta g\times10^2$		$g_{\pi NN}$	
				$\gamma=-0.01$	$\gamma=-0.00657$	$\gamma=-0.01$	$\gamma=-0\ 00657$	$\gamma=-0.01$	$\gamma=-0\ 00657$	$\gamma=-0.01$	$\gamma=-0.00657$
0.55	0.47	0.35	2.57	-3.02	-3 35	11 03	11.02	-3.23	-1.98	10.32	10.30
0.45	1.00	0.20	2 07	-6.82	-4.66	12.20	12.19	-6 96	-4.49	11.66	11.64
0 55	0 47	0 20	2 57	-3.80	-1.36	9.33	9.31	-3.88	-1.13	8.68	8.66
0.45	1.00	0 35	2.07	-5.79	-3.41	14.19	14.18	-6.06	-3.35	13.58	13.56

TABLE 3.2 : Values of parameters (in GeV units) used for determining central values of δg for different values of Λ_{QCD} and γ.

Central values	Λ_{QCD}=0.1GeV		Λ_{QCD} =0 15 GeV	
	$\gamma=-0.01$	$\gamma=-0.00657$	$\gamma=-0.01$	$\gamma=-0.00657$
Parameters	δg=-4.92×10^{-2} $g_{\pi NN}$=11.80	δg=-3.01×10^{-2} $g_{\pi NN}$=12.81	δg=-5.09×10^{-2} $g_{\pi NN}$=11.15	δg=-2.81×10^{-2} $g_{\pi NN}$=12.25
a_u	0.543	0.461	0.534	0.470
b	0.914	0.900	0 912	0.850
δ^2	0.310	0.300	0.310	0.310
S_0	2.520	2.520	2.560	2.160

TABLE 3.3: Contribution to δg from various symmetry breaking parameters (SBP's) taken to be non zero, one at a time, and also when all SBP's are non-zero The values of a_u, b, δ^2, and S_0 have been taken from Table 3.2, so as to give central values of δg and $g_{\pi NN}$ as obtained there. $\Delta m_q = 0.0$ means, $m_u = m_d = 0.007$, $\Delta m_N = 0.0$ means $m_p = m_n = 0.93892$ (average nucleon mass) along with the coefficients of χ in the Eqs. (3.23a) and (3.23b) being 5/18 and $m_{em}^2=0$. In the row with $\Delta S_0 \neq 0$ and $\Delta S_{0N} \neq 0$, the two results are for the two signs of ΔS_0 and ΔS_{0N} respectively.

Parameters		Λ_{QCD} =0.1 GeV		Λ_{QCD}=0.15 GeV	
		$\delta g\times 10^2$	$g_{\pi NN}$	$\delta g\times 10^2$	$g_{\pi NN}$
$\alpha = \theta = \Delta m_q = \Delta S_0 = \Delta S_{0N} = \Delta m_N = 0$, $\gamma = -0\ 01$		-8.48	11.87	-9.36	11.22
$\alpha = \theta = \Delta m_q = \Delta S_0 = \Delta S_{0N} = \Delta m_N = 0$, $\gamma = -0\ 00657$		-4.47	12.89	-5.18	12.32
$\alpha = 1/137, \gamma = \theta = \Delta S_0 = \Delta S_{0N} = \Delta m_q = \Delta m_N = 0$		1.90	11.82	1.86	11.17
$\theta = 0.01, \gamma = \alpha = \Delta S_0 = \Delta S_{0N} = \Delta m_q = \Delta m_N = 0$		11.42	11.80	11.11	11.15
$\Delta m_q \neq 0, \gamma = \alpha = \Delta S_0 = \Delta S_{0N} = \theta = \Delta m_N = 0$		11.10	11.80	10.35	11.15
$\Delta m_N \neq 0, \gamma = \alpha = \Delta S_0 = \Delta S_{0N} = \theta = \Delta m_q = 0$		-20.81	11.71	-19.01	11.07
$\alpha = \theta = \Delta m_q = \Delta m_N = \gamma = 0.0$ $(S_0^p - S_0^n)/\overline{S_0} = \pm 0.1\%$ $(S_{0N}^p - S_{0N}^n)/\overline{S}_{0N} = \pm 0.1\%$		0.26 -0.25	11.83 11.83	0.30 -0.28	11.17 11.17
All symmetry breaking parameters are non zero	$\gamma = -0\ 01$	-4.92	11.80	-5.09	11 15
	$\gamma = -0.00657$	-3.01	12 81	-2.81	12 25

Using QCD sum rules, in which pion field has been treated as the external field, authors of Ref.[7] have found $\delta g/g_{\pi NN}$ as –0.008, and in the cloudy bag model [22] it is –0.006. As already stated, bulk of the contribution to δg comes from the nucleon mass difference Δm_N. The quark mass difference Δm_q, and π-η mixing angle θ contribute to δg in the opposite direction, as obtained in[23] also; but these are almost cancelled by the contribution coming from δm_N The sign of our result for δg differs from that of the three-point function method [6], the chiral bag model [24] and quark-gluon model [25]. In chiral effective field theory, the underlying effective Lagrangian has been extended to include strong isospin-violating and electromagnetic four fermion contact interactions [4]. In these works there is no direct derivation of δg or $g_{\pi NN}$, but isospin violation in N-N scattering has been worked out. These authors find that the leading charge symmetry breaking (CSB) effects are four nucleon contact terms of order α and order Δm_q, while the contribution due to Δm_N is rather small. Since this formulation is based on a two-nucleon problem, a direct comparison with our result is difficult. In contrast to chiral effective field theory, in QCD sum rule approach, results based on QCD dynamics are matched to those obtained from effective field theory in an appropriate Borel window and the quantity of the interest is extracted. Our result of $g_{\pi NN}$ is consistent with that of Ref.[5], and the results of recent measurements [26]: $g_{\pi NN} \sim 13 - 13.5$.

Finally, we will discuss some of the implications of the isospin breaking in the diagonal pion-nucleon coupling constant. Obviously it will contribute to the long range part of the charge asymmetric nuclear potential $V_{CA}=V_{nn}-V_{pp}$ for the $^1S_0$ state. In order to calculate its effect on the difference of pp and nn scattering lengths, we use the phenomenological Argonne v_{18} potential [27] disregarding the electromagnetic potential part. With this potential, using $g_{\pi}{}^{0}{}_{nn}$ and $g_{\pi}{}^{0}{}_{pp}$, obtained for δg from $-6.0 \times 10^{-2} \leq \delta g \leq -3.8 \times 10^{-2}$ and the corresponding $g_{\pi NN}$ from $8.7 \leq g_{\pi NN} \leq 13.5$, in the OPEP part of v_{18}, we find using the standard method [28] that

$$0.8 \text{ fm} \leq |a_{nn}| - |a_{pp}| \leq 2.3 \text{ fm} \tag{3.26}$$

as against the experimental result [29]:

$$|a_{nn}| - |a_{pp}| = (1.6 \pm 0.6) \text{ fm}. \tag{3.27}$$

Earlier, we had observed that the nucleon mass difference gives the dominant contribution to δg Reversing the problem, one may ask how much of the nucleon mass difference arises due to δg? Analysis of the effect of pion loops on nucleon mass has been done by several authors in effective theories of meson-nucleon interaction [30]. Hecht et al. have concluded that the $\pi N-$loop reduces the nucleon's mass by ~$(10-20)\%$. Assuming that half of this is due to π^0-loop, we find that δg will give rise to a mass difference $\delta m_n - \delta m_p \approx -(0.25 - 0.5)$ MeV, which is a shift in opposite direction to the actual mass difference of the nucleons. Obviously in this case, we cannot neglect the effect of other heavier meson exchanges, and what we have got is far from the end of the story.

3.6 References

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