CHAPTER

V

CHAPTER-V

THE DERIVATIVE OF THE TOPOLOGICAL CHARGE SUSCEPTIBILITY AT ZERO MOMENTUM AND AN ESTIMATE OF η' MASS IN THE CHIRAL LIMIT

5.1 Introduction

The axial vector current in QCD has an anomaly

$$\partial^{\mu}\bar{q}\gamma_{\mu}\gamma_5 q = 2im_q\bar{q}\gamma_5 q - \frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \quad (5.1)$$

$$\tilde{G}^{a\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}G^a_{\rho\sigma} \quad (5.2)$$

The topological susceptibility $\chi(q^2)$ defined by

$$\chi(q^2) = i\int d^4x e^{iqx}\langle 0|T\{Q(x),Q(0)\}|0\rangle \quad (5.3)$$

$$Q(x) = \frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \quad (5.4)$$

is of considerable theoretical interest and has been studied using a variety of theoretical tools like lattice gauge theory, QCD sum rules, chiral perturbation theory etc In particular the derivative of the susceptibility at $q^2 = 0$

$$\chi'(0) = \left.\frac{d\chi(q^2)}{dq^2}\right|_{q^2=0} \quad (5.5)$$

enters in the discussion of the proton-spin problem [1-5] As is well known the first moment of g_1^p can be expressed in terms of the axial charges of the proton

$$\int_0^1 dx g_1^p(x,Q^2) = \frac{1}{12}C_1^{NS}(\alpha_s(Q^2)(a^3+\frac{1}{3}a^8)+\frac{1}{9}C_1^s((\alpha_s(Q^2)a^0(Q^2)) \quad (5\ 6)$$

$$\begin{aligned}&\left\langle 0\middle|A_\mu^{(3)}\middle|p,s\right\rangle = \frac{1}{\sqrt{2}}a^3 s_u, \left\langle 0\middle|A_\mu^{(8)}\middle|p,s\right\rangle = \frac{1}{\sqrt{6}}a^8 s_\mu \\ &\left\langle p,s\middle|A_\mu^{(0)}\middle|p,s\right\rangle = \frac{1}{\sqrt{2}}a^0(Q^2)s_\mu\end{aligned} \quad (5\ 7)$$

In QCD parton model, the axial charges are represented in terms of moments of parton distribution as

$$\begin{aligned}&a^3 = \Delta u - \Delta d, a^8 = \Delta u + \Delta d - 2\Delta s \\ &a^0(Q^2) = \Delta u + \Delta d + \Delta s - n_f\frac{\alpha_s}{2\pi}\Delta g(Q^2)\end{aligned} \quad (5\ 8)$$

In naive parton model $a^0 = a^8$, the OZI prediction The 'proton spin' problem is a question of understanding the dynamical origin of the OZI violation $a^0(Q^2) < a^8$ Shore, Veneziano and Narison [1] have shown that

$a^0(Q^2) = \frac{1}{2m_N}6\sqrt{\chi'(0)}\hat{\Gamma}_{\eta^0 NN}$ where η^0 is the OZI Goldstone boson, the unphysical state which would become Goldstone boson for spontaneously broken $U_A(1)$ in the absence of anomaly Ioffe et al [2] have calculated the part of the proton spin carried by u,d,s quarks in the framework of the QCD sum rules in the external fields An important contribution comes from the operator, which in the limit of massless u,d,s quarks is equal to $\chi'(0)$

5.2 Calculation of the First Derivative of Topological Susceptibility

In the QCD sum rule approach, one can determine $\chi'(0)$ as follows Using dispersion relation one can write

$$\frac{\chi'(q^2)}{q^2}-\frac{\chi'(0)}{q^2}=\frac{1}{\pi}\int ds Im\chi(s)[\frac{1}{(s-q^2)^2 s}+\frac{1}{(s-q^2)s^2}]+subtractions \quad (5\ 9)$$

Defining the Borel transform of a function $f(q^2)$ by

$$\hat{B}f(q^2)=\underset{\substack{-q^2,n\to\infty \\ -q^2/n=M^2 \text{ fixed}}}{Lim}\frac{(-q^2)^{n+1}}{n!}\frac{d}{dq^{2n}}f(q^2) \quad (5\ 10)$$

one gets from Eq (5 9)

$$\chi'(0)=\frac{1}{\pi}\int\frac{ds\,\mathrm{Im}\,\chi(s)}{s^2}(1+\frac{s}{M^2})e^{-s/M^2}-\hat{B}[\frac{\chi'(q^2)}{q^2}] \quad (5\ 11)$$

According to Eq (5 3) Im $\chi(s)$ receives contribution from all states $|n\rangle$ such that $\langle 0|Q|n\rangle \neq 0$ In particular we have [6]

$$\langle 0|Q|\pi^0\rangle = i f_\pi m_\pi{}^2(\frac{m_d-m_u}{m_d+m_u})\frac{1}{2\sqrt{2}} \quad (5\ 12)$$

The matrix elements, when $|n\rangle$ is $|\eta\rangle$ or $|\eta'\rangle$, can be determined as follows It is known from both theoretical considerations based on chiral perturbation theory as well as phenomenological analysis that one needs two mixing angles θ_8 and θ_0 to describe the coupling of the octet and singlet axial vector currents to η and η' [7-9] Introducing the definition

$$\langle 0|J^a_{\mu 5}|P(p)\rangle = i f^a_P p_\mu\ ,\quad a=0,8,\ P=\eta,\ \eta', \quad (5\ 13)$$

where $J^{8,0}_{\mu 5}$ are the octet and singlet axial currents

$$J^8_{\mu 5}=\frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d-2\bar{s}\gamma_\mu\gamma_5 s) \quad (5\ 14)$$

$$J^0_{\mu 5} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s) \qquad (5\ 15)$$

The $|P(p)\rangle$ represents either η or η' with momentum p_μ The couplings f_P^a can be equivalently represented by two couplings f_8, f_0 and two mixing angles θ_8 and θ_0 by the matrix identity

$$\begin{pmatrix} f^8_\eta & f^0_\eta \\ f^8_{\eta'} & f^0_{\eta'} \end{pmatrix} = \begin{pmatrix} f_8\cos\theta_8 & -f_0\sin\theta_0 \\ f_8\sin\theta_8 & f_0\cos\theta_0 \end{pmatrix} \qquad (5\ 16)$$

Phenomenological analysis of the various decays of η and η' to determine f_P^a has been carried out by a number of authors [7-9] In a recent analysis [9] Escribano and Frere find with

f_8=1 28 f_π (f_π=130 7MeV), (5 17)

the other three parameters to be

θ_8=(-22 2±1 8)°, θ_0=(-8 7±2 1)°, f_0=(-1 18±0 04)f_π (5 18)

The divergence of the axial currents are given by

$$\partial^\mu J^8_{\mu 5} = \frac{2}{\sqrt{6}}(m_u \bar{u}i\gamma_5 u + m_d \bar{d}i\gamma_5 d - 2m_s \bar{s}i\gamma_5 s) \qquad (5\ 19)$$

$$\partial^\mu J^0_{\mu 5} = \frac{2}{\sqrt{3}}(m_u \bar{u}i\gamma_5 u + m_d \bar{d}i\gamma_5 d + m_s \bar{s}i\gamma_5 s) - \frac{1}{\sqrt{3}}\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \qquad (5\ 20)$$

Since m_u, m_d <<m_s one can neglect them [10] to obtain

$$\langle 0|\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}|\eta\rangle = \sqrt{\frac{3}{2}}m^2_\eta(f_8\cos\theta_8 - \sqrt{2}f_0\sin\theta_0) \qquad (5\ 21)$$

$$\langle 0|\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}|\eta'\rangle = \sqrt{\frac{3}{2}}m^2_{\eta'}(f_8\sin\theta_8 + \sqrt{2}f_0\cos\theta_0) \qquad (5\ 22)$$

Using Eqns (5 12), (5 21) and (5 22) we get the representation of $\chi(q^2)$ in terms of physical states as

$$\chi(q^2) = -\frac{m^4{}_\pi}{8(q^2-m^2{}_\pi)} f_\pi^2 (\frac{m_d - m_u}{m_d + m_u})^2 - \frac{m^4{}_\eta}{24(q^2-m^2{}_\eta)}(f_8\cos\theta_8 - \sqrt{2}f_0\sin\theta_0)^2 - \frac{m^4{}_{\eta'}}{24(q^2-m^2{}_{\eta'})}(f_8\sin\theta_8 + \sqrt{2}f_0\cos\theta_0)^2 \quad (5\ 23)$$

+ higher mass states

On the other hand $\chi(q^2)$ has an operator product expansion [11,12,1,5]

$$\chi(q^2)_{OPE} = -(\frac{\alpha_s}{8\pi})^2 \frac{2}{\pi^2} q^4 \ln(\frac{-q^2}{\mu^2})[1+\frac{\alpha_s}{\pi}(\frac{83}{4}-\frac{9}{4}\ln(-\frac{q^2}{\mu^2}))]$$
$$-\frac{1}{16}\frac{\alpha_s}{\pi}\left\langle 0\left|\frac{\alpha_s}{\pi}G^2\right|0\right\rangle(1-\frac{9}{4}\frac{\alpha_s}{\pi}\ln(\frac{q^2}{\mu^2})) + \frac{1}{8q^2}\frac{\alpha_s}{\pi}\left\langle 0\left|\frac{\alpha_s}{\pi}g_s G^3\right|0\right\rangle$$
$$-\frac{15}{128}\frac{\pi\alpha_s}{q^4}\left\langle 0\left|\frac{\alpha_s}{\pi}G^2\right|0\right\rangle^2 + 16(\frac{\alpha_s}{4\pi})^3\sum_{i=u,d,s} m_i\left\langle \overline{q}_i q_i\right\rangle[\ln(-\frac{q^2}{\mu^2})+\frac{1}{2}]$$ + screening correction to the
$$-\frac{1}{2}\int d\rho\, n(\rho)\rho^4 q^4 K_2^2(Q\rho)$$

direct instantons (5 24)

In Eqn (5 24), the first term arises from the perturbative gluon loop with radiative correction [12], the second, third and the fourth term are from the vacuum expectation values of G^2, G^3 and G^4 The $\langle 0|G^4|0\rangle$ term has been expressed as $\langle 0|G^2|0\rangle^2$ using factorization [11] The fifth term proportional to the quark mass has been computed by us and is indeed quite small compared to other terms numerically Finally, the last two terms represent the contribution to $\chi(q^2)$ from the direct instantons [11] n(ρ) is the density of instanton of size ρ, K_2 is the Mc Donald function and Q^2=-q^2 In a recent work [13] Forkel has emphasized the importance of screening correction which almost cancels the direct instanton contribution (cf especially Fig 8 and Secs V and VI of Ref [13] For this reason we shall disregard the direct instanton term and screening correction for the present and return to it later

From Eq (5 11), we now obtain

$$
\begin{aligned}
\chi'(0) = {} & \frac{f_\pi^{\,2}}{8}\left(\frac{m_d - m_u}{m_d + m_u}\right)^2 \left(1+\frac{m_\pi^2}{M^2}\right) e^{-\frac{m_\pi^2}{M^2}} + \frac{1}{24}\left(f_8 \cos\theta_8 - \sqrt{2} f_0 \sin\theta_0\right)^2 \times \\
& \left(1+\frac{m_\eta^2}{M^2}\right) e^{-\frac{m_\eta^2}{M^2}} + \frac{1}{24}\left(f_8 \sin\theta_8 + \sqrt{2} f_0 \cos\theta_0\right)^2 \left(1+\frac{m_{\eta'}^2}{M^2}\right) e^{-\frac{m_{\eta'}^2}{M^2}} \\
& -\left(\frac{\alpha_s}{4\pi}\right)^2 \frac{1}{\pi^2} M^2 E_0\left(\frac{w^2}{M^2}\right)\left[1+\frac{\alpha_s}{\pi}\frac{74}{4}+\frac{\alpha_s}{\pi}\frac{9}{2}\left(\gamma - \ln\frac{M^2}{\mu^2}\right)\right] \\
& -16\left(\frac{\alpha_s}{4\pi}\right)^3 \frac{1}{M^2} \sum_{i=u,d,s} m_i \left\langle \bar{q}_i q_i \right\rangle - \frac{9}{64}\frac{1}{M^2}\left(\frac{\alpha_s}{\pi}\right)^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \\
& +\frac{1}{16}\frac{1}{M^4}\frac{\alpha_s}{\pi}\left\langle g_s \frac{\alpha_s}{\pi} G^3 \right\rangle - \frac{5}{128}\frac{\pi^2}{M^6}\frac{\alpha_s}{\pi}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle^2
\end{aligned}
\qquad (5.25)
$$

Here $E_0(x)$= 1-exp(-x) and takes into account the contribution of higher mass states, which has been summed using duality to the perturbative term in χ_{OPE}, and W is the effective continuum threshold We take W^2=2 3GeV^2, and in Fig 5 1 plot the r h s of Eq (5 25) as a function of M^2 We take α_s=0 5 for μ=1GeV and

$$\left\langle 0 \left| g_s^2 G^2 \right| 0 \right\rangle = 0\ 5\text{GeV}^4 \qquad (5\ 26)$$

$\left\langle 0 \left| s\bar{s} \right| 0 \right\rangle$ =0 8 $\left\langle 0 \left| u\bar{u} \right| 0 \right\rangle$=-0 8(240MeV)^3, m_s=150MeV and $m_u/m_d \approx$ 0 5 Writing

$$\left\langle 0 \left| g_s^3 G^3 \right| 0 \right\rangle = \frac{\varepsilon}{2} \left\langle 0 \left| g_s^2 G^2 \right| 0 \right\rangle, \qquad (5\ 27)$$

we take ε=1GeV^2 We also have PCAC relation

$$-2(m_u+m_d)\left\langle 0 \left| u\bar{u} \right| 0 \right\rangle = f_\pi^2\, m_\pi^2 \qquad (5\ 28)$$

For f_0, f_8, θ_8 and θ_0 we use the central values given in Eqs (5 17) and (5 18)

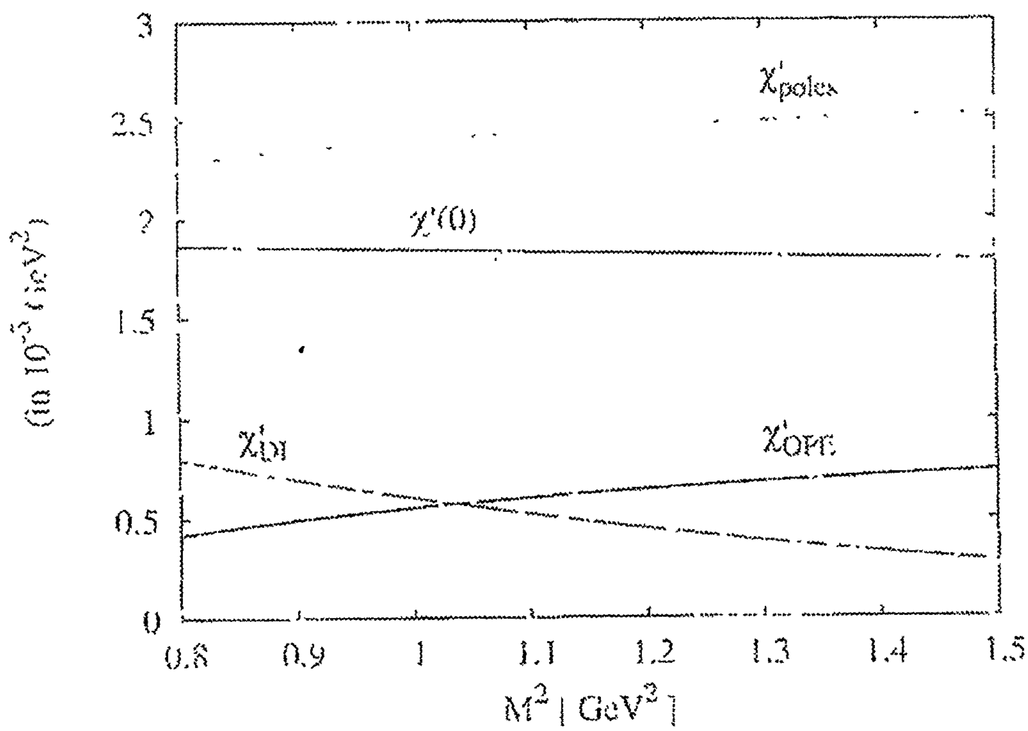

Figure 5.1: Various terms contributing to $\chi'(0)$, Eq (5 25) The value of $\chi'(0)$ is the one obtained without the direct instantons The latter, see Eq (5 34), is given by χ'_{DI}, which is larger than χ'_{OPE} and also has the wrong behaviour suggesting that screening corrections are important

Let us now examine how the various terms in the r h s of Eq (5 25) add up to remain a constant The pion term is small and has little variation because of the low mass, η and η' are significantly larger and η is even larger than η' In Fig 5 1 the upper line gives the combined contribution of π, η, and η' which we denote as χ'_{poles} and it is seen that it has gentle increase with M^2 The OPE terms given by the last three lines in Eq (5 25), which we denote by χ'_{OPE}, so that

$$\chi'(0) = \chi'_{poles} - \chi'_{OPE}$$

is also plotted in Fig 5 1 It is seen that χ'_{OPE} is roughly about 25% of χ'_{poles} also increases with M^2, with the result that $\chi'(0)$ is nearly constant w r t M^2

We expect this trend of compensating variation in χ'_{poles} and χ'_{OPE} to be maintained when variation in χ'_{poles} due to uncertainties in θ_8, θ_0, f_8, f_0 [see Eqs (5 17) and (5 18)] and the variations in χ'_{OPE} due to uncertainties in the estimates of the vacuum condensates are taken into account We can then obtain from Fig 5 1 the value

$$\chi'(0) \approx 1\ 82\times10^{-3}\ \mathrm{GeV}^2 \qquad (5\ 29)$$

We note that the determination, Eq (5 29) is in agreement with an entirely different calculation by two of us from the study of the correlator of isoscalar axial vector currents

$$\pi_{\mu\nu}{}^{I=0} = \frac{i}{2}\int d^4x e^{iqx}\left\langle 0\left|\{\bar{u}\gamma_\mu\gamma_5 u(x)+\bar{d}\gamma_\mu\gamma_5 d(x), \bar{u}\gamma_\mu\gamma_5 u(0)+\bar{d}\gamma_\mu\gamma_5 d(0)\}\right|0\right\rangle$$

$$\pi_{\mu\nu}^{I=0} = -\pi_1^{I=0}(q)^2 g_{\mu\nu} + \pi_2^{I=0}(q^2)q_\mu q_\nu \qquad (5\ 30)$$

$\pi_1^{I=0}(q^2=0)$ can be computed from the spectrum of axial vector meson In Ref [14] a value

$$\pi_1^{I=0}(q^2=0) = -0\ 0152\mathrm{GeV}^2 \qquad (5\ 31)$$

was obtained It is not difficult to see that when $m_u=m_d=0$

$$\pi_1^{I=0}(q^2=0) = -8\,\chi'(0) \qquad (5\ 32)$$

which shows consistency between Eqs (5 29) Let us now return to Eq (5 24) and consider the effect of incorporating the direct instanton term Eq (5 25) in the spike approach [5]

$$n(\rho) = n_0\delta\ (\rho-\rho_c) \qquad (5\ 33)$$

with n_0=0 75×10^{-3} GeV^4 and ρ_c=1 5GeV^{-1} The contribution of the instanton to $\hat{B}\,[\frac{\chi'(q^2)}{q^2}]$ can be found using the asymptotic expansion for $K_2(z)$ and $K'_2(z)$ and we find it to be

$$\chi'_{DI} = \frac{n_0}{4}\sqrt{\pi}\rho_c{}^4 M^2 [M\rho_c + \frac{9}{4}\frac{1}{M\rho_c} + \frac{45}{32}\frac{1}{M^3\rho_c{}^3}]e^{-M^2\rho_c^2} \qquad (5\ 34)$$

We have plotted this term separately in Fig 5 1 We note that unlike χ'_{poles} and χ'_{OPE}, which increase with M^2 and therefore compensate each other, the contribution of χ'_{DI}, Eq (5 34), decreases rapidly with M^2 It is not difficult to see that $\chi'(0)$ will no longer remain constant This strongly suggests that screening corrections to $[\frac{\chi'(q^2)}{q^2}]$ are important just as they are for $[\frac{\chi(q^2)}{q^2}]$ as found by Forkel [13]

5.3 η'-Mass in the Chiral Limit

We now turn to an estimate of η' mass in the chiral limit m_u=m_d= m_s=0 In this limit SU(3) flavor symmetry is exact and , we have m_π=m_η=0 while η' is a singlet Let us denote by η_χ = η'(m_s=0) and m_χ = $m_{\eta'}$(ms=0) the singlet particle and its mass in the chiral limit Returning to Eq (5 24), we first note that the explicitly quark mass dependent term in χ_{OPE}

$$-16(\frac{\alpha_s}{4\pi})^3 \sum_{i=u,d,s} m_i \langle \bar{q}_i q_i \rangle \approx 1\ 85\times10^{-6}(\text{GeV})^4$$

is numerically much smaller than for example

$$\frac{9}{64}(\frac{\alpha_s}{\pi})^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle \approx 4\ 5\times10^{-5}(\text{GeV})^4$$

which is itself much smaller than the perturbative term. In the chiral limit $\langle 0|Q|\pi\rangle = \langle 0|Q|\eta\rangle = 0$ If we assume that the quark mass dependence of $\chi'(0)$ is negligible then $\chi'(0)$ in Eq.(5.25) can also be expressed in terms of f_{η_χ} and m_χ

$$\chi'(0) = \frac{1}{12} f_{\eta_\chi}{}^2 (1 + \frac{m^2{}_\chi}{M^2}) e^{-\frac{m^2{}_\chi}{M^2}} - \hat{B}[\frac{\chi'_{OPE}(q^2)}{q^2}] \tag{5.35a}$$

We may then write from Eqs.(5.25) and (5.35a) for 0.8GeV^2<M^2<1.2GeV^2

$$\frac{1}{12} f_{\eta_\chi}{}^2 (1 + \frac{m^2{}_\chi}{M^2}) e^{-\frac{m^2{}_\chi}{M^2}} \approx \frac{f_\pi{}^2}{8} (1 + \frac{m_\pi{}^2}{M^2}) e^{-\frac{m_\pi{}^2}{M^2}} + \frac{1}{24} (f_8 \cos\theta_8 - \sqrt{2} f_0 \sin\theta_0)^2 (1 + \frac{m^2{}_\eta}{M^2}) e^{-\frac{m^2{}_\eta}{M^2}}$$

$$+ \frac{1}{24} (f_8 \sin\theta_8 + \sqrt{2} f_0 \cos\theta_0)^2 (1 + \frac{m^2{}_{\eta'}}{M^2}) e^{-\frac{m^2{}_{\eta}}{M^2}} \tag{5.35b}$$

In Fig.5.2 we have plotted the l.h s and r.h.s of Eq.(5.35b) in the interval 0.8 GeV^2< M^2<1.2GeV^2 for $m_\chi \approx 723$ MeV. From this we obtain f_{η_χ} =178MeV which is of the same order as physical decay constants f_8 and f_0.

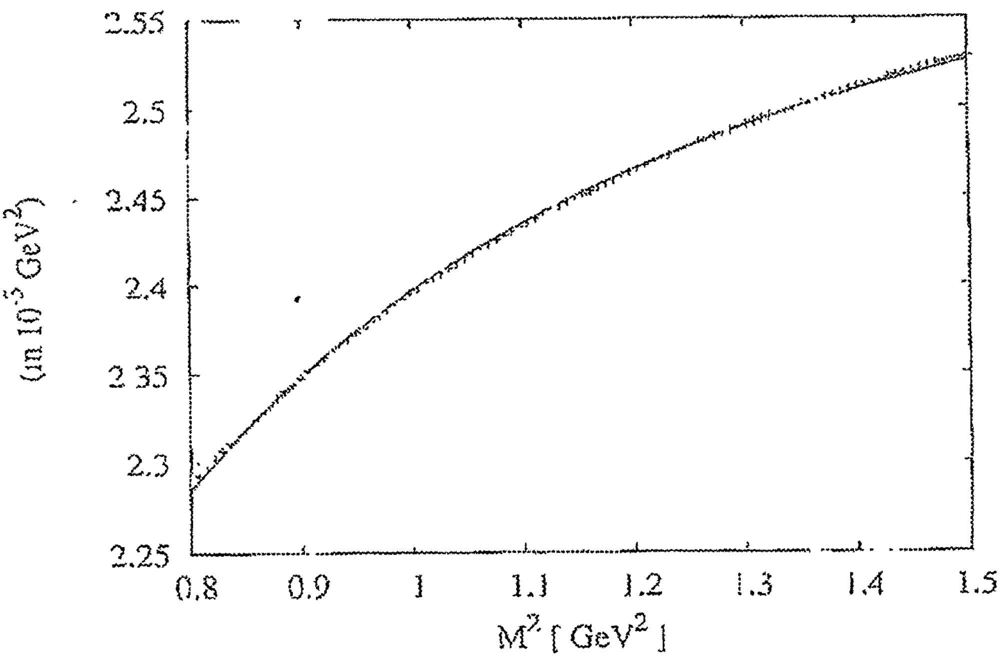

Figure 5.2 Estimate of η' mass and coupling in the chiral limit, see Eq (5 35b) The continuous curve corresponds to m_χ=723 MeV The continuous line is for l h s of Eq (5 35b) and line with croses is for r h s of Eq (5 35b)

5.4 Result and Discussion

We now compare our result for $\chi'(0)$ with some earlier results In Ref [1] Narison et al obtained a value for $\chi'(0)\approx 0\ 7\times10^{-3}(\mathrm{GeV})^2$ substantially different from the value derived here Since the expression for χ_{OPE} used by us is identical to theirs, albeit the estimate used for the gluon condensate is slightly different, we need to explain the difference in $\chi'(0)$ The most important difference is in expression of $\chi(q^2)$ in terms of physical intermediate states We have seen that both η and η' contribute, and in fact η makes a larger contribution than η' In Ref [1] only η'(950) state is taken into account We have also seen that if we were to take the chiral limit then η and η' contribution to $\chi(q^2)$ is representable by η_χ with mass $m_\chi \approx 723$MeV which is substantially different from the physical η' mass This also explains why Narison et al find stability in the sum rule for rather larger W^2=6GeV^2 instead of W^2=2 3GeV^2 We must also add that while our Eq (5 11) involves only $[\frac{\chi'(q^2)}{q^2}]$, Narision et al use the linear combination of two sum rules (cf Eq (6 22) of Ref[1]) Comparing with Ref [5] we note the following The radiative corrections to the perturbative loop given in Eq (5 25) viz $\frac{\alpha_s}{\pi}\frac{74}{4}$, which is large, is ignored in Ref [5] We also note that the coefficient of the $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle$ arises from radiative corrections, which is also ignored in Ref [5] As already remarked, they use physical η' mass even when m_s =0, the chiral limit Since in the sum rules squares of the masses exp [-(723)^2/M^2] as against exp[-(958)^2/M^2] occur, this is a serious error both in Ref [5] and [1] Even disregarding all the drawbacks, the sum rule in Ref [5] for $\tilde{f}_{\eta'}{}^2$ works rather poorly It is easy to read off from Fig 1 of Ref [5] that $\tilde{f}_{\eta'}{}^2 = 12\,\chi'(0)$ varies from 0 019 GeV^2 at M^2=1 5GeV^2 to 0 034GeV^2 at M^2=1 1 GeV^2, and grows even faster at lower M^2, hardly a constant This is to be contrasted $\chi'(0)$ as computed here, where it changes barely by 2% within the same range of M^2

In Ref [3], Ioffe and Khodzhamiryan's claim that the OPE for $\chi(q^2)$ does not converge is based on the following They computed the correlators

$$q_\mu q_\nu i \int d^4x e^{iqx} \left\langle 0 \left| T\{J^0_{\mu 5}(x), J^q_{\nu 5}(0)\} \right| 0 \right\rangle \tag{5 36}$$

where $J^q_{\mu 5} = \bar{q}\gamma_\mu \gamma_5 q$ with $m_u=m_d=0$ but $m_s \neq 0$

and $J^0_{\mu 5}$ is flavor singlet current Introducing the definition

$$\left\langle 0 \left| J^q_{\mu 5}(x) \right| \eta'(p) \right\rangle = i\, p_\mu g_{\eta'}{}^q$$

they estimated $\frac{g_{\eta'}{}^s}{g_{\eta'}{}^u} \approx 2\ 5$ (5 37)

If SU(3) symmetry was exact this ratio would be unity Insisting that the ratio in Eq (5 37) should be close to unity even when $m_s \neq 0$, they concluded that their result signals a breakdown of OPE[3] As discussed earlier, $\left\langle 0 \left| J^8_{\mu 5} \right| \eta' \right\rangle \neq 0$ In fact using the phenomenological values given in Eqs (5 17) and (5 18), it is easy to obtain

$$\frac{g_{\eta'}{}^s}{g_{\eta'}{}^u} = \frac{\sqrt{2}[f_0 \cos\theta_0 - \sqrt{2} f_8 \sin\theta_8]}{[f_8 \sin\theta_8 + \sqrt{2} f_0 \cos\theta_0]} \approx 2\ 24 \quad (5\ 38)$$

which is enough close to the estimate of Ref [3] In Ref [5] θ_8 was estimated to be -18 8° assuming $\frac{f_8}{f_0} = 1\ 12$ and $\theta_0 = -2\ 7^\circ$ using QCD sum rules With these values one will still find that the ratio $\frac{g^s_{\eta'}}{g^u_{\eta'}} = 1\ 96$, far different from unity as may be naively expected As in the case of Narison et al [1], Ioffe and Samsonov [5] and, Forkel [13] also do not take into account the π, η matrix element of the anomaly in their sum rules involving $\chi(q^2)$ We also note that $\chi'(0)$ was estimated in Refs [2,4] to be $\chi'(0) = (2\ 3 \pm 0\ 6) \times 10^{-3}$ by fitting the QCD sum rule for singlet axial vector matrix element of the proton We must add, $\chi'(0)$ coincides with the longitudinal part of the SU(3) singlet axial vector current correlator only in the limit of zero strange quark mass

In conclusion we find a value of $\chi'(0)\approx 1$ $82\times10^{-3}GeV^2$ without incorporating direct instantons Screening corrections to the latter appears to be significant We also obtained an estimate m_χ=723MeV and $f_{\eta\chi}$=178MeV

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