CHAPTER

I

CHAPTER-I

NUCLEONIC PROPERTIES: AN INTRODUCTION AND OVERVIEW

1.1 Introduction

Dramatic progress has been made in particle physics during the past four decades [1]. A series of important experimental discoveries have firmly established the existence of sub-nuclear worlds of quarks and leptons. The nucleons, i.e, proton and neutron which form nuclei are no longer regarded as elementary particles but are found to be made up of quarks. Later on, the quark structure of the nucleon was directly observed in deep inelastic electron scattering experiments.

The dynamics of quarks and leptons can be described by an extension of the sort of quantum field theory (QFT) that proved successful in describing electromagnetic interaction of charged particles, QED. To be more precise, the fundamental interactions are widely believed to be described by QFT possessing local gauge symmetry [2], whereby the interaction between quarks and leptons are being discussed through the exchange of gauge field quanta, mainly photons, gluons and weak bosons. The short range attractive force is responsible for binding the nucleons inside the nucleus. The fact that the large variety of nuclei are constructed out of nucleons makes their study interesting. Hence the internal structure of the nucleon is of fundamental importance in nuclear and particle physics, to both experimentalists and theorists.

In 1933, Frisch and Stern performed the first experiment for measuring the magnetic moment of the proton. These measurements are the experimental evidence for the internal structure of the nucleon which says that nucleon is not a point-like particle. The anomalous magnetic moment of the proton was determined to be 2.5 times as large as one would expect for a spin 1/2 Dirac particle (the actual value is 2.793 nucleon magneton).

In 1935 Yukawa proposed, in analogy with quantum electrodynamics (QED), that the nuclear forces were due to the exchange of quanta of finite mass, a meson. The interaction between two nucleons would proceed via the exchange of a virtual intermediate meson. A simple calculation based on the uncertainty principle shows that for a range of 1.4×10^{-15} m for the strong force the exchanged meson must have a mass of about 140 MeV/c^2 in contrast to the infinite range of the electromagnetic force which is due to the fact that photon is massless.

As far as the fundamental constituents of matter were concerned, it appeared, by 1939, that the proton, neutron, electron and neutrino are the constituents of matter, supplemented by photon and the hypothesized Yukawa meson as the mediating particles of the electromagnetic and strong interaction respectively.

Low energy properties of nucleons can be studied in various approaches. There are various models of nucleons in terms of their elementary constituents which are suitable to study some aspects of their properties. The QCD sum rules have been extensively used to investigate nonperturbative regions of hadronic physics. Effective theories based on symmetries of QCD are fit to study the low energy interactions among hadrons. In the present thesis, we shall use some of these approaches to study some aspects of low energy properties of nucleons. In the following we shall give a brief introduction of relevant developments in the subject which will be useful in the course of investigating the problems discussed in the following chapters.

1.2 The Quark Model

In 1963 Gell-Mann and Zweig [3] proposed a model that explained the spectrum of strongly interacting particles (i.e hadrons) in terms of elementary constituents called quarks. The quark model was developed to account for the regularities observed in the hadron spectrum, with hadrons interpreted as bound states of localized but essentially non-interacting quarks. It provides us a simple picture of internal structure of hadrons and an effective way to describe their dynamics at high energy. Much of the success of the model lies in the circumstance that to a reasonably good approximation we can regard quarks as free or weakly interacting particles (except for the confining mechanism). Mesons were expected to be quark-antiquark bound states. Baryons were interpreted as bound states of three quarks. The quark constituents of the baryons are assigned to have spin ½ from the observed spins of low-lying baryons.

The low-lying baryons were interpreted in the quark model as symmetric states of space, spin and $SU(3)_f$ flavor degrees of freedom. However, Fermi-Dirac statistics requires a total antisymmetry of the wave function. The resolution of this dilemma come through the introduction of color degree of freedom. The baryon wave functions are totally anti-symmetric in the color degree of freedom. Of course, the introduction of another degree of freedom would lead to a proliferation of states, so the color degree of freedom had to be supplemented by a requirement that only color singlet states exist in nature. Hence proton would be a bound state of (uud) and neutron would be a bound state of (udd) quarks which makes them color singlet. This model had great success in predicting new hadronic states, and in explaining the strength of electromagnetic and weak interaction transitions among different hadrons. In particular, it naturally incorporates the most important symmetry relations among hadrons.

Once quark structure of hadrons got some acceptance, it became natural to look for the dynamics obeyed by the quark system responsible for the composition of hadrons as well as for hadronic reactions. In order to get experimental information on quark dynamics, the most sensible way, is to probe the inside of hadrons, (e.g., proton) by applying a beam of structureless particles such as leptons. We need much higher energies and larger momentum transfers for the study of hadronic structure to have higher resolutions. The electromagnetic form factors are key ingredients to the understanding of the internal structure of composite particles like the nucleon, since they contain the information about the distributions of charges and currents. The knowledge of hadron form factors, especially for the nucleons and the pions, represent an important source of information about their electromagnetic structure. By varying the momentum transfer, large as well as small distances can be explored, allowing one to learn about hadronic physics. De-Broglie wavelength of an electron becomes much shorter than the size of a typical nucleus at sufficiently high energies in GeV range. In such cases, the scattering result is dominated by the charge distributions within individual nucleons. The primary interest of scattering at these energies shifts to the structure of nucleon rather than that of nucleus.

The quarks are classified as "light" or "heavy" depending on their entries in the mass matrix m of QCD Lagrangian equation. These masses are "running" as well: they depend on the scale μ at which they are determined. The masses of the lightest (u and d) quarks, $m_{u,d} < 10MeV$ (estimated at a renormalization scale $\mu \sim 1$ GeV) are very small compared to typical hadron masses of order 1 GeV, such as those of the ρ meson or the nucleon. The strange quark mass, $m_s \simeq (100 - 150)$ MeV is an order of magnitude larger than $m_{u,d}$ but still counted as "small" on hadronic scales. The charm quark mass $m_c \simeq (1.1\text{-}1.4)$ GeV takes an intermediate position while the b and t quarks $m_b \simeq (4.1\text{-}4.4)$GeV, $m_t=(174\pm5)$GeV) fall into the "heavy" category. These different quark masses set a hierarchy of scales, each of which is governed by distinct physics phenomena.

1.3 The Parton Model

The first series of experiments to study the structure of proton was initiated in 1960's at SLAC and the process was called electron-proton deep inelastic scattering (DIS). For DIS, the momentum transfer squared q^2 is so large so that the spatial resolution for observing the target nucleon (proton) by projectile electron is high. DIS experiments are of utmost importance since it helps in revealing the internal structure of the proton. The finite size of the proton was measured to be about 0.8 fm.

In 1969 Bjorken [4] reported the scaling property of structure function in electron-nucleon scattering which was expected in the deep inelastic region where momentum transfer squared q^2 and energy transfer v of electron are very large with the ratio q^2/v kept fixed. It is claimed that structure function in the deep inelastic region depend only on the ratio q^2/v rather than on two independent variable q^2 and v. Bjorken scaling is obtained by assumption of the existence of free independent point-like particles (partons) inside proton. Conversely, it suggests that the quark dynamics must have the property of asymptotic freedom, i.e, the coupling constant decreases at short distances, hence quark interaction gets weaker at short distances.

The correlation pattern of energy and angular distribution of the scattered leptons in the DIS can be described simply by Feynman's parton model [5]. The essence of the parton model is the assumption that, when a sufficiently high momentum transfer reaction takes place, the projectile, be it a lepton or a parton inside a hadron, sees the target as made up of almost free constituents, and is scattered by a single, free, effectively massless constituent. Moreover the scattering from individual constituents is incoherent. The picture thus looks much like the subnuclear version of the impulse approximation of high energy scattering of composite particles with weakly bound constituents. The inclusive scattering is viewed as due to incoherent elastic scattering from point-like constituents of the nucleons: partons. The final state partons then recombine somehow into hadronic states. These partons were later identified as quarks, since experimentally it was suggested that their quantum numbers such as charges and spins were practically the same as those of quarks.

1.4 Quantum Chromodynamics (QCD)

Quantum chromodynamics (QCD) is the theory of strong interaction with interacting quarks and gluons. It is well tested in the high energy regime where perturbative QCD is applicable. Understanding confinement and hadronic structure in the non-perturbative region of QCD remains a challenge. It describes the interactions of quarks, via their color quantum numbers. It is an unbroken gauge theory and the gauge bosons are gluons.

It is a consistent quantum field theory with a simple and elegant underlying Lagrangian, based entirely on the invariance under a local gauge group, $SU(3)_{color}$. Out of this Lagrangian emerges an enormously rich variety of physical phenomena, structures and phases. Exploring and understanding these phenomena is undoubtedly one of the most exciting challenges in modern science.

In QCD, which is to some extent similar to QED, the fundamental interactions are between spin ½ quarks and massless spin 1 gluons. The quarks and gluons carry a new quantum number called color. Each quark can exist in three different color states and each gluon in eight color states. Under an SU(3) group of transformations which mixes up colors, the quarks and gluons are said to transform as a triplet and an octet respectively. No physical particle with the attribute of color has ever been found, so it is believed that all particles are 'color neutral'. By this we mean that all physical states must be invariant, or singlets under color transformations.

The elementary spin- ½ particles of QCD, the quarks, come in six species, or flavors, grouped in a field $\psi(x) = (u(x), d(x), s(x), c(x), b(x), t(x))T$. Each of the u(x), d(x), etc., is a four-component Dirac spinor field. Quarks experience all three fundamental interactions of the Standard Model [2]: weak, electromagnetic and strong. Their strong interactions involve N_c = 3 "color" charges for each quark. These interactions are mediated by the gluons, the gauge bosons of the underlying gauge group of QCD, $SU(3)_{color}$.

The Lagrangian density of QCD [6] in terms of quark and gluon degrees of freedom for interacting quarks with masses m_i is given by the equation

$$L_{QCD} = -\frac{1}{4}F^A_{\mu\nu}F^{\mu\nu}_A + \sum_i^n \overline{q^a_i}(i\not{D} - m_i)_{ab} q^b_i - \frac{1}{2\lambda}(\partial^\mu A^A_\mu)^2 + L_{Ghost} \tag{1.1}$$

Here q^a_i are quark fields with mass m_i, A^A_μ is the gluon field and the covariant derivative is given by

$$(D_\mu)_{ab} = \delta_{ab}\partial_\mu + ig_s(t^c A^c_\mu)_{ab} \tag{1.2}$$

Under local gauge transformations they transform as ($t^a = \frac{\lambda^a}{2}$ are Gell-Mann matrices of SU(3) group).

$$q_a(x) \to q'_a(x) = \exp(it\,\theta(x))_{ab} q_b(x) = \Omega(x)_{ab} q_b(x), \quad t.\theta = t^c\theta^c \tag{1.3}$$

$$t.A_\mu \to t\,A'_\mu = \Omega(x) t\,A_\mu \Omega^{-1}(x) - \frac{1}{ig}(\partial_\mu \Omega(x))\Omega^{-1}(x) \tag{1.4}$$

$$D_\mu q(x) \to D'_\mu q'(x) = \Omega(x) D_\mu q(x) \tag{1.5}$$

The non-Abelian field strength tensor is given by

$$F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - g_s f^{ABC} A^B_\mu A^C_\nu \tag{1.6}$$

which transforms as

$$t.F_{\mu\nu} \to t.F'_{\mu\nu} = \Omega(x) t.F_{\mu\nu} \Omega^{-1}(x) \tag{1.7}$$

With the transformations (1.3), (1.5) and (1.7), it is easy to see that L_{QCD} remains invariant under local gauge transformations.

The extra term in $F_{\mu\nu}^{A}$ makes it invariant under non-Abelian gauge transformation. This extra term has profound consequences for the theory: it means that gluons are self-interacting through three- and four-point vertices. This will turn out to give rise to asymptotic freedom at high energies and strong interactions at low energies, among the most fundamental properties of QCD.

Finally, it turns out that in a non-Abelian gauge theory, it is necessary to add one extra term to the Lagrangian density, related to the need for ghost particles. Basically they arise because when a non-Abelian gauge theory is renormalized it is possible for unphysical degrees of freedom to propagate freely. These are cancelled off by introducing into the theory an unphysical set of fields, the ghosts, which are scalars but have Fermi statistics. For practical purposes it is enough to know that there exist Feynman rules for ghosts and that in every diagram with a closed loop of internal gluons, we must add a diagram with them replaced by ghosts. It is worth noting that in physical gauges, as the name suggests, ghost contributions always vanish and they can be ignored.

QCD has similar structure as QED, but with one important difference; the gauge group is non-Abelian SU(3), and gluons are self interacting. The non-linear three- and four-point couplings of the gluon fields A_{μ}^{A} with each other are at the origin of the very special phenomena encountered in QCD and strong interaction physics. Hence the theory is asymptotically free (i.e coupling constant decreases at short distances) at high-energy and grows strong at low energies. These interactions are confining and dictates that quarks must be confined within a region of about ~1 Fermi in radius to give a hadron, so one would expect that as two or more nucleons approach each other within a nucleus, quarks and gluons should take over the dynamics and show up in observables. The only stable color singlets are quark-antiquark pairs, mesons, and three quark states, baryons.

There exist two limiting situations in which QCD is accessible with "controlled" approximations. At momentum scales exceeding several GeV (corresponding to short distances, r<0.1 fm), QCD is a theory of weakly interacting quarks and gluons (perturbative QCD). At low momentum scales considerably smaller than 1GeV (corresponding to long distances, r>1 fm), QCD is characterized by confinement and a non-trivial vacuum (ground state) with strong condensates of quarks and gluons. Confinement is believed to be behind the spontaneous breaking of a symmetry which is exact in the limit of massless quarks: chiral symmetry. Spontaneous chiral symmetry breaking in turn implies the existence of pseudoscalar Goldstone bosons. For two flavors ($N_f = 2$) they are identified with the isotriplet of pions (π^+, π^0, π^-). For $N_f = 3$, with inclusion of the strange quark, this is generalized to the pseudoscalar meson octet. Low-energy QCD is thus realized as an Effective Field Theory (EFT) in which these Goldstone bosons are the active, light degrees of freedom.

1.5 Asymptotic Freedom and Confinement

The property of QCD that led directly to its discovery as a candidate theory of the strong interaction is asymptotic freedom, i.e., coupling strength decreases at short distance [7]. This property is due to the presence of gluons which carry color charge and have spin one. It can either be explained as a dielectric or a paramagnetic effect. In first case, one calculates the dielectric properties of the vacuum and ascribes the asymptotic freedom of the theory to the self interaction of the gluon field. In the later one, asymptotic freedom is explained as a paramagnetic effect due to the spin of the gluons.

The success of QCD in describing the strong interactions is summarized by two terms i.e asymptotic freedom and confinement and their importance can be better understood by recalling certain facts about strong interaction. Asymptotic freedom refers to the weakness of short distance interaction, while the confinement of quarks follows from its strength at long distances.

Confinement has a relatively simple interpretation for heavy quarks and the "string" of (static) gluonic field strength that holds them together, expressed in terms of a static potential. When light quarks are involved, the situation is different. Color singlet quark antiquark pairs prop out of the vacuum as the gluon fields propagate over larger distances. Light quarks are fast movers: they do not act as static sources. In this case the potential picture is not applicable. The common features of the confinement phenomenon can nevertheless be phrased as follows: non-linear gluon dynamics in QCD does not permit the propagation of colored objects over distances of more than a fraction of a Fermi. Beyond the one-Fermi scale, the only remaining relevant degrees of freedom are color-singlet composites (quasiparticles) of quarks, antiquarks and gluons.

Hadron spectra are very well described by the quark model, but quarks have never been seen in isolation. Any effort in scattering experiment leads only to the production of the familiar mesons and baryons. Evidently, the forces between quarks are strong. In QFT, when higher order effects in perturbation theory are taken into account, then couplings acquire momentum dependence. An isolated charge in vacuum polarizes the surrounding medium in virtual electron-positron pairs, which, in turn, screen its charge. Hence, when the charge of such a particle is measured by scattering another charged particle on it, the charge depends on the distance between these particles· the smaller the distance, larger is the charge since then the test charge can penetrate inside the charge cloud. In quantum theory, separation is inversely proportional to the momentum transferred. Thus, the result of scattering experiment can be summarized as:

$$\frac{d}{dt_0}\alpha(t_0) > 0, \tag{1.8}$$

where α is the fine structure constant and $t_0 = -\vec{k}^2$ is the momentum transferred. For QED, however, the charge is so small that $\alpha(t_0)$ does not become large until t_0 is of astronomical scale. In QCD, in addition to the processes which are already there in QED, we also have to include the processes arising out of three-gluon couplings. This makes a very important difference. The emission of a gluon "leaks away" the color charge of the heavy particle into the cloud of virtual particles. Thus, for small t_0, when the two heavy particles stay far apart, they are actually more likely to see each other's true charge. As t_0 increases, they penetrate further and further into each other's charge cloud and are less and less likely to measure the true charge. For this reason, we expect "antiscreening" for QCD:

$$\frac{d}{dt_0}\alpha_s(t_o) < 0. \tag{1.9}$$

To be more quantitative, let us define ($\mu^2 = -t_0$)

$$\mu \frac{dg_s(\mu)}{d\mu} = \beta(g_s(\mu)), \tag{1.10}$$

where g_s is the strong coupling constant and μ is the renormalization scale. It has been found that

$$\beta(g_s) = -g_s[\frac{\alpha_s}{4\pi}\beta_1 + (\frac{\alpha_s}{4\pi})^2\beta_2 + ...], \tag{1.11}$$

β_1=11-2n_f/3.

Here n_f is the number of quark flavors. For n_f =6, β_1 is positive and β negative. Differential equation (1.10) can be solved and, to the lowest-order, $\alpha_s(\mu^2)$can be written in terms of a single variable as

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_1 \ln(\frac{\mu^2}{\Lambda^2})} \tag{1.12}$$

where $\Lambda = \Lambda_{QCD}$, is a free parameter which sets the scale for the running coupling. The QCD scale parameter Λ is determined empirically ($\Lambda \simeq$ 0.2 GeV for N_f = 4). The fact that α_s decreases with increasing μ leads to the property known as "asymptotic freedom" in the domain $\mu \gg 1$ GeV in which QCD can indeed be treated as a perturbative theory of quarks and gluons. The theoretical discovery of asymptotic freedom was honored with the 2004 Nobel Prize in Physics. At the scale of the Z-boson mass, $\alpha_s(M_Z) \simeq 0.12$ [8] . So while α_s is small at large μ, it is of order one at $\mu <$ 1 GeV. At low energies and momenta, an expansion in powers of α_s is therefore no longer justified: we are entering the region commonly referred to as non-perturbative QCD.

1.6 Operator Product Expansion

The operator-product expansion is a technique in which the singularities of the operator products are expressed as a sum of nonsingular operators with the coefficients being singular c-number functions [9]. The physical basis for this expansion is that a product of local operators at distances small compared to the characteristic length of the system should look like a local operator. In theories like QCD, the functions describing the singularities in this expansion have a momentum dependence governed by renormalization group equations; hence due to the asymptotic freedom, they can be calculated at large momenta using perturbation theory. Secondly, these functions exhibit the full symmetry of the underlying theory by possible spontaneous symmetry breaking.

It also enables us to extract a short distance piece in the scattering cross sections, which is calculable through the QCD Lagrangian by using renormalization group method. OPE can be defined using proper renormalization scale μ which is used to separate hard and soft momenta.

Wilson [9] hypothesized that the singular part as x→y of the product A(x)B(y) of two operators is given by a sum over other local operators O_i:

$$A(x)B(y) \to \sum C_i(x-y)O_i(\frac{1}{2}(x+y)) \qquad (1.13)$$

where $C_i(x-y)$ are singular c-number functions called Wilson coefficients. It has been proven for renormalizable theories that such expansions are valid as $x \to y$ to any finite order of perturbation theory. The short distance behaviour of the Wilson coefficients is expected to be that obtained, up to a logarithmic multiplicative factor, by dimensional counting ($x \ll 1/m$)

$$C_i(x) \to x^{d_i - d_A - d_B}(\ln xm)^p[1+O(xm)] \quad (1.14)$$

where d_A, d_B and d_i are the dimensions (in units of mass) of A, B and O_i respectively. The higher the dimension of O_i the less singular are the coefficients $C_i(x)$; hence the dominant operators at a short distance are those with the smallest dimensions.

The usefulness of this expansion derives from its universality: the Wilson coefficients are independent of the process under considerations. Process dependence is exhibited in the matrix element of the local operator O_i which is nonsingular at short distances. Another advantage is that in a given theory the expansion usually involves a rather small number of operators. Hence the ensuing calculation is relatively simple.

1.7 Chiral Symmetry

Chiral symmetry is an internal symmetry of right and left handed spinors. It has importance in low energy hadronic physics, since its spontaneous breaking generates Goldstone bosons with negative parity, zero spin, unit isospin and zero baryon number called pions. Thus a broken approximate chiral symmetry entails the existence of pions where u and d quarks have small but non-zero masses whereby spontaneous breaking of a symmetry is expressed as the non-vanishing of the vacuum when operated by the charge Q. The transition from the fundamental to the effective level occurs via a phase transition due to spontaneous symmetry breaking generating (pseudo) Goldstone boson. A spontaneously broken symmetry relates processes with different numbers of Goldstone bosons. Since the masses of light quarks are small compared to Λ_{QCD}, let us set these parameters equal to zero in the first approximation and moreover, make the masses of heavy quarks, m_c, m_b and m_t to be infinity. In this limit QCD Lagrangian L_{QCD} becomes invariant under the following group of (space-time independent) transformations which act on the three flavor indices (u, d, s):

$$q = q_L + q_R \rightarrow q' = g_R q_R + g_L q_L$$
$$q_R = \frac{1}{2}(1+\gamma_5)q, \tag{1.15}$$

$$q_L = \frac{1}{2}(1-\gamma_5)q$$
$$g_I g_I^{\dagger} = 1, \det g_I = 1, I = L, R \tag{1.16}$$

The above group of transformations (1.15) and (1.16) is $SU(3)_R \times SU(3)_L$ and the resulting symmetry of the QCD Lagrangian is called chiral symmetry of QCD. According to the Noether's theorem, there are $2\times(3^2\text{-}1) = 16$ conserved currents associated with this symmetry.

$$J_I^{\mu a} = \overline{q_I}\gamma_\mu T^a q_I$$

$$\partial_\mu J_I^{\mu a} = 0; I = L, R \ ; \ a = 1, \ldots\ldots. 8 \tag{1.17}$$

The associated conserved charges

$$Q_I^a = \int_{x^0 = const} J^{oa}{}_I d^3x$$

$$\frac{dQ_I^a}{dt} = 0 \tag{1.18}$$

generate the algebra of G= $SU(3)_R \times SU(3)_L$[10]

$$[Q_I^a, Q_I^b] = i f^{abc} Q_I^c$$

$$[Q_L^a, Q_R^b] = 0 \tag{1.19}$$

Vector and axial charges can be defined as

$$Q_V^a = Q_R^a + Q_L^a, \ \ Q_A^a = Q_R^a - Q_L^a \tag{1.20}$$

It can be shown that the state of lowest energy is necessarily invariant under the vector charges: $Q_V^a|0\rangle=0$. For axial charges, however the Wigner-Weyl realization of G, in which $Q_A^a|0\rangle=0$ is not true, since that would imply that this spectrum contains degenerate parity partners forming multiplets of G. The real word has no parity doublets. For instance, the lightest meson, π(140) and the lowest state with the same spin and flavor, but of opposite parity $a_0(980)$ have large mass difference; so is the case with N(940) and N*(1520). Hence it is believed that the alternative possibility called Nambu-Goldstone mode of G, in which $Q_a^A|0\rangle \neq 0$ is realized. In this case, the spectrum contains 8 Goldstone bosons, one for each broken generator, and they form degenerate multiplets of $SU(3) \subset$ G. The eight lightest hadrons pions, kaons and η have desired quantum numbers of the Goldstone bosons, but they are not massless as required by Goldstone's theorem[11]. Using commutation relations of the vector charge with scalar currents and axial charges with pseudoscalar currents and using the fact that $Q_a^A|0\rangle \neq 0$ it can be shown that

$$\langle 0|u\bar{u}|0\rangle = \langle 0|d\bar{d}|0\rangle = \langle 0|s\bar{s}|0\rangle \neq 0 \quad (1.21)$$

In the theory of superconductivity, a small electron-electron attraction leads to the appearance of a condensate of electron pairs in the ground state of a metal. In QCD, quark and antiquark have strong attractive interaction, and, if these quarks are massless, the energy cost of creating an extra quark-antiquark pair is small. Thus we expect that the vacuum of QCD will contain a condensate of quark-antiquark pairs. These fermion pairs must have zero total momentum and angular momentum. They must contain net chiral charge, pairing left-handed quarks with the antiparticles of right-handed quarks. The vacuum state with a quark pair condensate is characterized by a nonzero vacuum expectation value for the scalar operator

$$\langle 0|\bar{q}q|0\rangle = \langle 0|\bar{q}_R q_L + \bar{q}_L q_R|0\rangle \neq 0 \tag{1.22}$$

and hence is noninvariant with $g_L \neq g_R$. The expectation value signals that the vacuum mixes the two quark helicities. This allows the u and d quarks to acquire effective masses as they move through the vacuum.

Chiral symmetry breaking (CSB) is a nonperturbative phenomenon, which is known to govern the low energy properties of hadrons. The effective chiral Lagrangians have been proposed before the advent of QCD and the phenomenon of CSB and Nambu-Goldstone theorem was established more than 40 years ago.

1.8 PCAC

Let $|\pi_a(p)\rangle$ be the state vectors of the Goldstone bosons associated with the spontaneous breakdown of chiral symmetry. We choose the standard normalization $\langle \pi_a(p)|\pi_b(p')\rangle = 2E_p\delta_{ab}(2\pi)^3\delta^3(\vec{p}-\vec{p}')$. Goldstone's theorem, implies non-vanishing matrix elements of the axial current which connect $|\pi_a(p)\rangle$ with the vacuum:

$$\langle 0|A_a^\mu(x)|\pi_b(p)\rangle = ip^\mu F_0 \delta_{ab} e^{-ip\cdot x} \quad . \tag{1.23}$$

The constant F_0 is called the pion decay constant (taken here in the chiral limit, i.e., for vanishing quark mass). Its physical value $f_\pi = (92.4 \pm 0.3)$ MeV is determined from the decay $\pi^+ \rightarrow \mu^+\nu_\mu + \mu^+\nu_\mu\gamma$. The difference between F_0 and f_π is a correction linear in the quark mass m_q. Non-zero quark masses $m_{u,d}$ shift the mass of the Goldstone boson from zero to the observed value of the physical pion mass, m_π. The relationship between m_π and the u and d quark masses is derived as follows. We start by observing that the divergence of the axial current is

$$\partial_\mu A_a^\mu = i\overline{\psi}\{m, \frac{\tau_a}{2}\}\gamma_5\psi \,, \tag{1.24}$$

where m is the quark mass matrix and {,} denotes the anti-commutator. This is the microscopic basis for PCAC, the Partially Conserved Axial Current (exactly conserved in the limit m→0) which plays a key role in the weak interactions of hadrons and the low-energy dynamics involving pions [10]. Consider for example the a = 1 component of the axial current:

$$\partial_\mu A_1^\mu = (m_u + m_d)\overline{\psi} i\gamma_5 \frac{\tau_1}{2}\psi$$

$$\langle \pi, p | \partial_\mu A_1^\mu(0) | 0 \rangle = +m_\pi^2 f_\pi \tag{1.25}$$

and combine this with $[Q_a^A, P_b] = -\delta_{ab}\overline{\psi}\psi$ where $P_a(x) = \overline{\psi}(x)\gamma_5\tau_a\psi(x)$ the pseudoscalar quantity, to obtain

$$\langle 0 | [Q_1^A, \partial A_1^\mu] | 0 \rangle = \frac{i}{2}(m_u + m_d)\langle \overline{u}u + \overline{d}d \rangle \tag{1.26}$$

Now insert a complete set of (pseudoscalar) states $|\pi, p\rangle\langle\pi, p|$ in the comutator on the left. Assume, in the spirit of PCAC, that this spectrum of states is saturated by the pion. Then use Eq.(1.23) to evaluate $\langle 0|Q_1^A|\pi\rangle$ and $\langle 0|\partial A^\mu|\pi\rangle$ at time t = 0, with E_p = m_π at $\vec{p}$=0. Since

$$\int \frac{d^3p}{2E_p}\langle 0|Q_1^A|\pi, p\rangle\langle \pi, p|\partial_\mu A_1^\mu|0\rangle = +i\int \frac{d^3p}{2E_p} E_p f_\pi \delta^3(\vec{p}) m_\pi^2 f_\pi = \frac{i}{2} m_\pi^2 f_\pi^{\ 2} \,, \tag{1.27}$$

we arrive at the Gell-Mann, Oakes, Renner (GOR) relation [12]:

$$m_\pi^{\ 2} f_\pi^2 = -(m_u + m_d)\langle \overline{q}q \rangle + O(m_{u,d}^2)$$

We have set $\langle \bar{q}q \rangle = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle$ making use of isospin symmetry which is valid to a good approximation. Neglecting terms of order $m^2{}_{u,d}$ (identifying $F_0 = f_\pi = 92.4$ MeV to this order) and inserting $m_u + m_d \simeq 14$ MeV [13] at a renormalization scale of order 1GeV, one obtains $\langle \bar{q}q \rangle \equiv (0.23 \pm 0.03\text{GeV})^3 \simeq 1.6\ \text{fm}^{-3}$ .This condensate (or correspondingly, the pion decay constant f_π) is a measure of spontaneous chiral symmetry breaking. The non-zero pion mass, on the other hand, reflects the explicit symmetry breaking by the small quark masses, with $m_\pi^2 \sim m_q$. It is important to note that m_q and $\langle \bar{q}q \rangle$ are both scale dependent quantities. Only their product $m_q\langle \bar{q}q \rangle$ is scale independent, i.e., invariant under the renormalization group.

1.9 OZI Rule and its Violation

The phenomenologically-inspired OZI rule [14] states that "disconnected quark diagrams are suppressed relative to connected ones", and it has served as an excellent guiding principle in the development of strong interaction theory and exception to the OZI rule, which are rare usually signify that some significant new physics is involved.

The OZI rule was originally invented to explain why dominant decay mode of vector meson ϕ [$\phi = \bar{s}s$] is kaon decay, (i.e. $\phi \to K^+K^-$), whereas the dominant decay mode of the vector ω meson [$\omega = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$] is pion decay(i.e., $\omega \to 3\pi$), even though the phase space for pion decay mode of the more massive ϕ meson is greater than that for the ω meson. Fig.1.1 (a, b, c) show how the OZI rule explains this experimental fact when the ω and ϕ mesons are represented in terms of their constituents quarks. Thus Fig1.1a allows the $\omega \rightarrow 3\pi$ decay process via "connected" quark diagrams whereas Fig.1.1b shows why the decay $\phi \rightarrow 3\pi$, involving as it does "disconnected" quark diagrams, cannot occur; on the other hand, using a quark diagram of the type shown in Fig.1.1c, $\phi \rightarrow K^+K^-$ can take place. It should be pointed out that there is a small width for $\phi \rightarrow 3\pi$ decay because, in accordance with the QCD there are gluon lines between the s and u and d quarks Fig.1.1b and such diagram gives rise to reduced pion decay (and the deviation from an absolute OZI rule). As an another example, the preferential decay of the heavy quarkonia Ψ and Υ into c-quark and b-quark containing mesons respectively can be explained with the same type of "OZI rule" argument. Here, J/Ψ = ($\bar{c}c$) is the analog of $\phi(\bar{s}s)$ and can decay into $D^0(c\bar{u})+\overline{D}^0(\bar{c}u)$ provided its mass is sufficient (which is true for the second excited state of J/Ψ and all higher ones), however, J/Ψ can not decay into mesons from which a c quark is absent. Since the ground state of charmonium J/Ψ is not sufficiently massive to allow $D^0+\overline{D}^0$ decay, its decay width(arising from gluon-induced diagrams.) is of the order of tens of KeV rather than MeV's so that the observed metastability of J/Ψ strongly supports the OZI rule. The OZI rule is rigorous in the large N_c limit. This follows from the fact that an OZI-forbidden process involves at least two closed loops and hence is suppressed (completely suppressed in the large N_c limit) compared to an OZI allowed process which receives contribution from one closed loop, and since we have no way of estimating the degree of accuracy of the OZI rule for finite $N_c = 3$, we must be prepared for violations of the OZI rule.

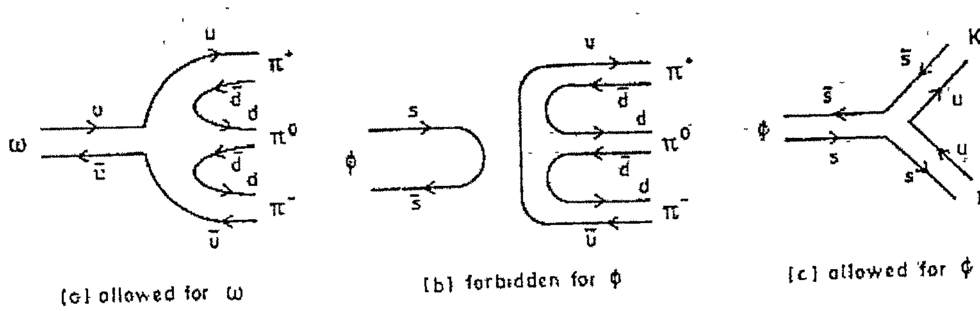

Figure 1.1: OZI connected and disconnected quark diagrams for ω and φ.

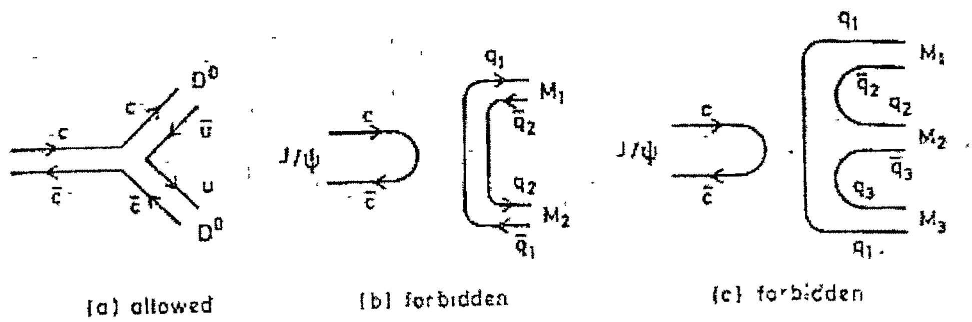

Figure 1.2: OZI connected and disconnected quark diagrams for J/ψ and ψ''

1.10 Effective Field Theory

Effective field theory (EFT) is a technique for describing the low energy limit of a theory. It is an effective description because it uses the degrees of freedom and interaction which are relevant at low energy. The basic idea of an effective theory is to introduce the active light particles as collective degrees of freedom, while the heavy particles are frozen and treated as (almost) static sources. The dynamics is described by an effective Lagrangian which incorporates all relevant symmetries of the underlying fundamental theory.

Effective field theories (EFT'S) have long proven to be a powerful tool in particle physics. EFT approach has the promise to establish a relationship of QCD, the theory of strong interaction, to various successful phenomenological models, and has a systematic expansion in a small parameter. Using these interactions one treats the low energy dynamics in a complete field theoretic description. With such treatment one encounters loop diagrams, in which the integration over the momenta includes both low and high energy components. The heavier modes do not appear explicitly, their contribution is somehow included through some parameters in the effective theory. The role of the small parameter is played by the ratio of the typical momentum scale Q in the problem to the scale associated with the physics left out of the effective theory. In the case of nuclear interaction up to momenta of the order of 300 MeV, one can build on effective theory containing nucleons and pions (and delta isobars). However, in those nuclear processes where the typical momentum scale is small compared to the pions mass, one is allowed to use an effective theory without explicit pions, only contact force remains.

For long distances the effective field theory is fully correct since it treats baryons and pions as point particles, but this convention does not provide an accurate representation of physics at distances less than the separation scale. The use of effective field theory technique is an ever growing approach in various fields of theoretical physics. For example, we do not need quantum gravity to understand the hydrogen atom nor does chemistry depend upon the structure of the electromagnetic interaction of quarks.

EFT's are approximate by their very nature. Once the relevant degrees of freedom for the problem at hand have been established, the corresponding EFT is usually treated perturbatively. It does not make much sense to search for an exact solution of the Fermi theory of weak interactions. In the same spirit, convergence of the perturbative expansion in the mathematical sense is not an issue. The asymptotic nature of the expansion becomes apparent once the accuracy is reached where effects of the underlying "fundamental" theory cannot be neglected any longer. The range of applicability of the perturbative expansion depends on the separation of energy scales that define the EFT.

Historically, effective Lagrangians were formulated so as to reproduce the results of current algebra and PCAC at tree level. Basically, the effective Lagrangians were used as convenient alterations to comutator algebra. In 1979, Weinberg [15] extended the scope of the effective Lagrangian formulation by postulating that the use of effective Lagrangians can go beyond current algebra. This assertion was based on the observation that for soft pion processes, chiral Lagrangians offer a powerful parameterization of the S-matrix based on chiral counting arguments and general principles such as symmetries, analyticity, unitarity etc. Weinberg's program has been systematized and extended by Gasser and Leutwyler [16]. The use of effective Lagrangians beyond tree level as a way to understand the hadronic S-matrix in the soft-pion limit, however, side-steps the basic issues of confinement and broken chiral symmetry.

1.11 Anomaly

Anomaly arises in quantum field theory when the symmetries of classical field theory are broken by quantum fluctuations inherent in a quantum field theory. Here a symmetry of classical action is not a true symmetry of full quantum theory. Classical Lagrangian in abelian QED with massless fermion or non-abelian QCD with massless fermion posses the property of scale invariance because the gauge field are massless and coupling constants are dimensionless. However, quantum mechanical renormalization introduces a finite renormalization scale for both unbroken QED and QCD and breaks the scale invariance in the process. The "quantum fluctuations" resulting from loop corrections in the renormalization process breaks down the classical chirality invariance and leads to the so called chiral gauge anomalies, where as the axial anomaly follows from the conflicts between gauge invariance and chiral invariance in the process of regulating the theory of quantum level.

Due to this anomaly, Noether current is no longer divergenceless but receives a contribution arising from quantum corrections, and hence is not valid at quantum level after consideration of quantum structures in the corresponding perturbation series. When anomaly arises, the Ward identities relating matrix element, no longer hold, but rather are replaced by a set of anomalous ward identities which take into account the correct current divergences. The QCD anomaly equation for a single flavor can be written as[17]

$$\partial^{\mu}(\bar{q}\gamma_{\mu}\gamma_{5}q) = 2im(\bar{q}\gamma_{5}q) - \frac{\alpha_s}{4\pi}G^{a}_{\mu\nu}\tilde{G}^{a\mu\nu} \tag{1.28}$$

Where $G^a_{\mu\nu}$ is gluon field tensor and $\tilde{G}^{a\mu\nu}$ is its dual. Three- flavor QCD anomaly can be utilized to explain "$U_A(1)$ problem" in QCD, namely the large mass of the η' meson and also the proton spin problem.

It is clear from Eq.(1.28) that the $U_A(1)$ chiral symmetry is explicitly broken by the QCD anomaly. In reality, there is competition between the spontaneous and explicit chiral symmetry breaking because of the anomaly. Since $U_A(1)$ symmetry is broken not spontaneously but explicitly by the anomaly, ή cannot be regarded as a nearly massless Nambu Goldstone boson like the other pseudoscalar mesons. In fact ή mass is as large as the nucleonic mass, i.e., m_η = 958MeV. This is called $U_A(1)$ problem. It can be shown that without the QCD anomaly, the mass of the "non strange" pseudoscalar, η_{ns} can only be slightly larger than the mass of the pion and $m_{\eta_{ns}} \leq \sqrt{3} m_\pi$. This inequality becomes a part of the $U_A(1)$ problem in QCD[1]. The resolution to the problem came when it was realized that the anomaly term has been neglected. Denoting η_s as the strange pseudoscalar, we can use the relation:

$$m_{\eta'}^2 + m_\eta^2 = m_{\eta_s}^2 + m_{\eta_{ns}}^2 \tag{1.29}$$

Using the current algebra manipulation and SU(3) symmetry in decay constants, it can be shown that

$$m_{\eta'}^2 = 2m_k^2 - m_\eta^2 + A^2 \tag{1.30}$$

where A^2 is the anomaly contribution and A can be expressed in terms of matrix elements of the axial anomaly between vacuum and, η_{ns} and η_s states. Numerically $A^2 \geq 0.37\text{GeV}^2$. Thus in strong interaction process in which the coupling of the quark current to gluon fields is involved, the three flavor QCD anomaly has a significant role in correcting the deficiencies in current algebra calculations. Also, anomaly makes it possible for spin carried by the gluons to mix the spin by quarks, thus modifying the structure of quark sea.

1.12 Proton Spin Problem

According to the nonrelativistic constituent quark model, the whole of the proton spin arises from the quarks. In relativistic quark model, the sum of the z-component of quark spins account for ¾ of the proton spin while rest of the proton spin arises from the quark orbital angular momentum. The 1987 EMC experiment[18] indicated that the first moment of the proton spin structure function $\Gamma_1^P = 0.126 \pm 0.018$ leading to the stunning implication that very little(15%) of the proton spin is carried by the quarks, contrary to the naïve quark model picture. The EMC data implied a substantial sea quark polarization in the region $x < 0.1$, a range not probed by earlier SLAC experiments[19]. In the naïve parton model, the data also implied a large and negative strange sea polarization which is contrary to the basic assumption of the Ellis-Jafe sum rule[20], namely $\Delta s = 0$.

Anomalous gluon effect originating from the axial anomaly provides a plausible and simple solution to the proton spin puzzle. A polarized gluon is preferred to split into a quark-antiquark pair with helicites antiparallel to the gluon spin. Thus a positive gluon spin component ΔG can give rise to negative sea quark polarization. The lattice calculation indicates that sea polarization is almost independent of light quark flavors. This empirical SU(3) flavor symmetry implies that it is indeed the axial anomaly, which is independent of light quark masses, that accounts for the bulk of helicity contribution of sea quarks. Hence anomaly makes it possible for spin carried by the gluons to mix with the spin carried by quarks, thus modifying the structure of quark sea and explicable for the smallness of the apparent quark contribution to the proton spin.

In chapter II, using a statistical model, in which a nucleon is taken as an ensemble of quark-gluon Fock states, we have calculated the quark contributions to the spin of the nucleon, the ratio of the magnetic moments of nucleons, their weak decay constant, and the ratio of SU(3) reduced matrix elements for the axial current. This has been done neglecting the contribution of s-quark and other heavy quarks, and covering only ~ 86% of the total Fock states. Two modifications of this model has also been worked out with a view to reduce the contributions of the sea components with higher multiplicities.

In chapter III, using the framework of the conventional QCD sum rule, we have studied the isospin splitting in the diagonal pion-nucleon coupling constant by including the quark mass dependent terms, π^0-η mixing and electromagnetic corrections to meson-quark vertices. Some of the implications of the isospin splitting have also been discussed.

In chapter IV, gluonic contributions to the self-energy of a nucleon has been investigated in an effective theory. The couplings of the topological charge density to nucleons give rise to OZI violating η-nucleon and η'-nucleon interactions. The one-loop self-energy of a nucleon arising due to these interactions has been calculated using a heavy baryon chiral perturbation theory. The divergences have been regularized using form factors. The nontrivial structure of the QCD vacuum has also been taken into account.

In chapter V, we calculate the first derivative of the topological susceptibility at zero momentum, $\chi'(0)$ using QCD sum rules. $\chi'(0)$ is useful, among others, in the discussion of the proton spin problem. The mass of η' and the singlet pseudoscalar decay constant in the chiral limit have also been found as a bonus.

Finally in the last chapter VI, we give summary and concluding remarks.

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