CHAPTER - III

MAXIMUM LIKELIHOOD ESTIMATES OF THE FRACTION DEFECTIVE IN CURTAILED SAMPLING PLANS

3.1 In this chapter we have obtained the maximum likelihood estimate of the fraction defective under curtailed sampling plans for both the situations related to the reporting of the data discussed in the previous chapter. We have further obtained the asymptotic variances of these estimates and compared their efficiencies.

3.2 Situation A:

3.2.1 As emphasized in Section 2.6, the estimation process is based on T observations on the random variable s from one of the populations $f_j(s)$ $(j=1,2,3)$. Recalling the particulars of these T observations, we have the following likelihood functions based on them:

Plan 1:

$$L_1 = \prod_{x=0}^{k-1} \left[\binom{n}{x} p^x q^{n-x} \right]^{a_{x,1}} \prod_{x=k}^{n} \left[\binom{n}{x} p^x q^{n-x} \right]^{r_{x,1}} ,$$

Plan 2:

$$L_2 = \prod_{x=0}^{k-1} \left[\binom{n}{x} p^x q^{n-x} \right]^{a_{x,2}} \prod_{y=k}^{n} \left[\binom{y-1}{k-1} p^k q^{y-k} \right]^{r_{y,2}},$$

Plan 3:

$$L_3 = \prod_{i=0}^{k-1} \left[\binom{n-k+i}{n-k} q^{n-k+1} p^i \right]^{a_{i,3}} \prod_{y=k}^{n} \left[\binom{y-1}{k-1} p^k q^{y-k} \right]^{r_{y,3}}$$

Differentiating the above likelihood functions with respect to p and equating the derivatives to zero, we get the following expressions for estimating p :

Plan 1:

$$p_1 = \frac{\sum_{x=0}^{k-1} x a_{x,1} + \sum_{x=k}^{n} x r_{x,1}}{n(T_{a,1} + T_{r,1})} \qquad \ldots(3.1)$$

Plan 2:

$$p_2 = \frac{\sum_{x=0}^{k-1} x a_{x,2} + kT_{r,2}}{nT_{a,2} + \sum_{y=k}^{n} y r_{y,2}} \qquad \ldots(3.2)$$

Plan 3:

$$p_3 = \frac{\sum_{i=0}^{k-1} i a_{i,3} + kT_{r,3}}{\sum_{i=0}^{k-1} i a_{i,3} + (n-k+1)T_{a,3} + \sum_{y=k}^{n} y r_{y,3}} \qquad \ldots(3.3)$$

Furthermore, since z= i+(n-k+1), equation (3.3) may be expressed as

$$p_3 = \frac{\sum_{z=n-k+1}^{n} za_{z,3} - (n-k+1)\, T_{a,3} + kT_{r,3}}{\sum_{z=n-k+1}^{n} za_{z,3} + \sum_{y=k}^{n} yr_{y,3}} \qquad \ldots(3.4)$$

Use (3.3) when the random variable i is observed and (3.4) when z is observed.

Now, it may be noted that

$\sum_{x=0}^{k-1} xa_{x,j}$ = total number of defectives observed in accepted lots under Plan j(j=1,2),

$\sum_{x=k}^{n} xr_{x,1}$ = total number of defectives observed in rejected lots under Plan 1,

$\sum_{y=k}^{n} yr_{y,j}$ = total number of articles inspected in rejected lots under Plan j(j=2,3),

$\sum_{i=0}^{k-1} ia_{i,3}$ = total number of defectives observed in accepted lots under Plan 3,

and $\sum_{z=n-k+1}^{n} za_{z,3} = ia_{i,3} + (n-k+1)\, T_{a,3}$

= total number of articles inspected in accepted lots under Plan 3.

The physical meaning of the expressions given above leads to an interesting observation that in all the three plans the estimate of p has one common feature, namely,

$$p = \frac{\text{Total number of defectives noted}}{\text{Total number of articles inspected}}$$

Now we have stated that (3.3) may be used when i is reported and (3.4) may be used when z is reported. The interesting property given above leads to the fact that the estimating equations (3.3) and (3.4) are basically same. However such is not the case when the fraction defective is estimated by the method of moments (vide equations (2.21) and (2.25) of Section 2.7.1).

3.2.2 Variance of the Estimates:

We make use of the following expressions to obtain the expectations of the second derivatives of the likelihood functions:

$$E\left(\sum_{x=0}^{k-1} xa_{x,1} + \sum_{x=k}^{n} xr_{x,1}\right) = np \quad , \qquad \ldots(3.5)$$

$$E\left(T_{a,1} + T_{r,1}\right) = 1 \ , \qquad \ldots(3.6)$$

$$E\left(\sum_{x=0}^{k-1} xa_{x,2} + kT_{r,2}\right)/T$$

$$= np\ B(p,n-1,k-2) + k\ [1-B(p,n,k-1)] = J_1, \qquad \ldots(3.7)$$

$$E\left(nT_{a,2} + \sum_{y=k}^{n} yr_{y,2}\right)/T$$

$$= np\ B(p,n,k-1) + k\ [1-B(p,n+1,k)] = J_2, \qquad \ldots(3.8)$$

$$E\left(\sum_{i=0}^{k-1} ia_{i,3} + kT_{r,3}\right)/T$$
$$= (n-k+1)\, B(p,n+1,k-1)/q$$
$$-(n-k+1)\, B(p,n,k-1)+k\,[1-B(p,n,k-1)]=J_3, \qquad \ldots(3.9)$$

and $$pE\left[\sum_{i=0}^{k-1} ia_{i,3} + (n-k+1)T_{a,3} + \sum_{y=k}^{n} yr_{y,3}\right]/T$$

$$= (n-k+1)p\, B(p,n+1,k-1)/q + k\,[1-B(p,n+1,k)] = J_4, \qquad \ldots(3.10)$$

where $B(p,n,k)$ has been defined in (2.15) and the definitions of J_1, J_2, J_3 and J_4 are clear from the expressions themselves.

Furthermore, using the recurrence relation

$$B(p,n+1,k) = pB(p,n,k-1)+q\,B(p,n,k), \qquad \ldots(3.11)$$

We find that $J_1 = J_2$ ...(3.12)

and $J_3 = J_4$ ...(3.13)

Now the asymptotic variance of the maximum likelihood estimate of p is given by

$$V(\hat{p}) = -1/E\,(\partial^2 \log L/\partial p^2) \qquad \ldots(3.14)$$

Therefore using the above expectations (3.5) through (3.10) and (3.12) and (3.13) the asymptotic variance of the maximum likelihood estimate of the fraction defective in the respective plans are :

<u>Plan 1</u>: $V(\hat{p}_1) = pq/Tn$ ...(3.15)

<u>Plan 2</u>: $V(\hat{p}_2) = p^2q/TJ_1$ ...(3.16)

<u>Plan 3</u>: $V(\hat{p}_3) = p^2q/TJ_3$ ...(3.17)

It may, however, be noted that expression (3.15) is true for any T since $nT\hat{p}_1$ has a binomial distribution with index nT, giving the exact results $E(nT\hat{p}_1)=nTp$ and $V(nT\hat{p}_1)=nTpq$.

3.2.3 <u>Efficiency of these estimates:</u>

Lemma 1: To prove that $J_1 \leq np$. ..(3.16)

Proof:

Now $$\sum_{y=k}^{n} y\binom{y-1}{k-1}p^k q^{y-k} = k\binom{k-1}{k-1}p^k q^{k-k} + (k+1)\binom{k+1-1}{k-1}p^k q^{k+1-k} + \cdots + n\binom{n-1}{k-1}p^k q^{n-k}$$

$$\leq n\binom{k-1}{k-1}p^k q^{k-1} + n\binom{k+1-1}{k-1}p^k q^{k+1-k} + \cdots + n\binom{n-1}{k-1}p^k q^{n-k}$$

$$\leq n \sum_{y=k}^{n} \binom{y-1}{k-1}p^k q^{y-k}$$

$$\leq n \sum_{x=k}^{n} \binom{n}{x}p^x q^{n-x} \quad \text{[recalling (2.6)]}$$

Therefore, adding $n \sum_{x=0}^{k-1} \binom{n}{x}p^x q^{n-x}$ to both the sides of above inequality we have,

$$n \sum_{x=0}^{k-1} \binom{n}{x}p^x q^{n-x} + \sum_{y=k}^{n} y\binom{y-1}{k-1}p^k q^{y-k} \leq n \qquad ...(3.17)$$

i.e. $n\,B(p,n,k-1) + k\,[1-B(p,n+1,k)]\,/p \leq n$

i.e. $J_2/p \leq n$

i.e. $J_1 \leq np$, since $J_1 = J_2$.

Lemma 2: To prove that $J_3 \leq J_1$. ...(3.18)

Proof:

Replacing z, the coefficient of $\binom{z-1}{n-k} q^{n-k+1} p^{z-(n-k+1)}$,

by n in the summation $\sum_{z=n-k+1}^{n} z\binom{z-1}{n-k} q^{n-k+1} p^{z-(n-k+1)}$

we have

$$\sum_{z=n-k+1}^{n} z\binom{z-1}{n-k} q^{n-k+1} p^{z-(n-k+1)}$$

$$\leq n \sum_{z=n-k+1}^{n} \binom{z-1}{n-k} q^{n-k+1} p^{z-(n-k+1)}$$

$$\leq n \sum_{t=n-k+1}^{n} \binom{n}{t} q^{t} p^{n-t} \quad \text{referring to (2.6)}$$

$$\leq n \sum_{x=0}^{k-1} \binom{n}{x} p^{x} q^{n-x}$$

Then adding $\sum_{y=k}^{n} y\binom{y-1}{k-1} p^{k} q^{y-k}$ to each side of the above

inequality we have,

$$\sum_{z=n-k+1}^{n} z\binom{z-1}{n-k} q^{n-k+1} p^{z-(n-k+1)} + \sum_{y=k}^{n} y\binom{y-1}{k-1} p^{k} q^{y-k}$$

$$\leq n \sum_{x=0}^{k-1} \binom{n}{x} p^{x} q^{n-x} + \sum_{y=k}^{n} y\binom{y-1}{k-1} p^{k} q^{y-k} \quad \text{...(3.19)}$$

i.e. $J_4/p \leq J_2/p$

i.e. $J_4 \leq J_2$

Or $J_3 \leq J_1$ < since $J_1 = J_2$ and $J_3 = J_4$.

The above two lemma evidently prove the fact that

$$V(\hat{p}_1) \leqslant V(\hat{p}_2) \leqslant V(\hat{p}_3) \qquad \ldots(3.20)$$

Therefore the efficiency of the estimate of the fraction defective based on the results of Plan 2 with respect to those based on Plan 1 is given by

$$\frac{V(\hat{p}_1)}{V(\hat{p}_2)} = \frac{J_1}{np} \leqslant 1 \qquad \ldots(3.21)$$

and that based on the results of Plan 3 with respect to those based on Plan 1 is given by

$$\frac{V(\hat{p}_1)}{V(\hat{p}_3)} = \frac{J_3}{np} \leqslant 1 \qquad \ldots(3.22)$$

The results are not far from expectations. The purpose of these plans is not to estimate the fraction defective but a reduction in the inspection cost, as the name and nature of the plans suggest. One has always to pay a price for any reduction in testing and in this case, this price has been paid by an increase in the variance of the estimate.

3.2.4 A Numerical Example :

Consider all the plans with n=25, and k=3. In standard notations, these plans ensure the producer's and the consumer's risk as follows:

$p_1 = 3\%$, producer's risk $= \alpha = 8\%$

$p_2 = 10\%$, consumer's risk $= \beta = 9\%$.

Let all the plans be administered on [redacted] three sets of 50 lots. We assume that the inspector gives the complete information of the inspection results. It is tabulated in Tables 3.1, 3.2 and 3.3. From these tables we find that in

Plan 1: $T_{a,1} = 30$, $T_{r,1} = 20$, $\Sigma x a_{x,1} = 39$, $\Sigma x r_{x,1} = 75$,

Plan 2: $T_{a,2} = 31$, $T_{r,2} = 19$, $\Sigma x a_{x,2} = 40$, $\Sigma y r_{y,2} = 326$

Plan 3: $T_{a,3} = 33$, $T_{r,3} = 17$, $\Sigma i a_{i,3} = 41$, $\Sigma y r_{y,3} = 303$

Hence using equations (3.1), (3.2) and (3.3) we find the following maximum likelihood estimate of p :

Plan 1: $\hat{p}_1 = 0.0912$

Plan 2: $\hat{p}_2 = 0.0881$

Plan 3: $\hat{p}_3 = 0.0834$

Now the hypothetical value of p used for obtaining the frequency distributions under discussion is 0.09. Using this hypothetical value of p the asymptotic variance can be calculated using (3.15), (3.16) and (3.17). The asymptotic variance, standard error and $|\hat{p}-p|/S.E.$ are listed below:

	$V(\hat{p})$	S.E.$(\hat{p})$	$\lvert\hat{p}-p\rvert$ /S.E.$(\hat{p})$
Plan 1	0.00006552	0.0081	0.15
Plan 2	0.00007482	0.0086	0.22
Plan 3	0.00007644	0.0087	0.76

The entries below $\lvert\hat{p}-p\rvert$ /SE$(\hat{p})$ lead to the conclusion that the difference between the estimate of p and the hypothetical value of p can be regarded as due to sampling fluctuations. In practice one may substitute the estimate of p in (3.15), (3.16) and (3.17) to compute the estimate of the asymptotic variance of $\hat{p}$.

Furthermore, we have computed the asymptotic variance for all the plans for different values of p ranging from 0.04 to 0.20. These variances are given in Table 3.4. It may be noted that T times the variance is presented. This may help in using conveniently the same table for any value of T.

In the same table one can find the efficiencies of the maximum likelihood estimates in columns (6) and (7). These are calculated using (3.21) and (3.22). It is revealed from this table that the efficiency of Plans 2 and 3 decreases as p increases. This is in accordance with expectations, since a greater fraction defective means a greater probability of rejection and thereby curtailing of the sampling in a greater number of cases.

Furthermore, it is worth noting that there is no appreciable loss in efficiency if one administers Plan 3 instead of Plan 2 for larger value of p, where the probability of acceptance decreases. This is due to the fact that there is no appreciable reduction in inspection of the items as one passes from Plan 2 to Plan 3 for larger value of p. This fact will be evident when the relation between ASN and the variance of the estimate and reduction in ASN will be discussed in Chapter V.

Table 3.4 has been prepared with the help of the Tables of the Cumulative Binomial Probability Distributions [55] .

Table 3.1

Observed Data of Plan 1

Number of defectives x	Number of accepted lots $a_{x,1}$	Number of defectives x	Number of rejected lots $r_{x,1}$
(1)	(2)	(3)	(4)
0	5	3	10
1	11	4	6
2	14	5	3
-	-	6	1
Total	30	-	20

Table 3.2

Observed Data of Plan 2

Number of articles inspected y	Number of rejected lots $r_{y,2}$	Number of articles inspected y	Number of rejected lots $r_{y,2}$	Number of defectives x	Number of accepted lots $a_{x,2}$
(1)	(2)	(1)	(2)	(3)	(4)
3	1	15	0	0	5
4	0	16	1	1	12
5	1	17	1	2	14
6	0	18	1		
7	0	19	1		
8	0	20	1		
9	1	21	1		
10	0	22	3		
11	1	23	2		
12	0	24	2		
13	1	25	0		
14	1	-	-		
Total	-	-	19	-	31

Table 3.3

Observed Data of Plan 3

Number of articles inspected y	Number of rejected lots $r_{y,3}$	Number of articles inspected y	Number of rejected lots $r_{y,3}$	Number of defectives i	Number of accepted lots $a_{i,3}$
(1)	(2)	(1)	(2)	(3)	(4)
3	0	15	1	0	6
4	0	16	0	1	13
5	0	17	1	2	14
6	1	18	0		
7	1	19	1		
8	0	20	1		
9	0	21	2		
10	0	22	2		
11	0	23	2		
12	1	24	2		
13	1	25	0		
14	1	-	-		
Total	-	-	17	-	33

Table 3.4

p	Probability of acceptance	$TV(\hat{p})$			Efficiency in % Plan 1=100	
		Plan 1	Plan 2	Plan 3	Plan 2	Plan 3
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.04	0.92352	0.001536	0.001567	0.001643	98.022	93.488
0.05	0.87289	0.001900	0.001967	0.002049	96.594	92.728
0.07	0.74656	0.002604	0.002811	0.002897	92.636	89.886
0.09	0.60630	0.003276	0.003741	0.003822	87.570	85.714
0.11	0.47087	0.003916	0.004780	0.004854	81.925	80.676
0.13	0.35171	0.004524	0.005948	0.006012	76.059	75.250
0.15	0.25374	0.005100	0.007255	0.007309	70.296	69.776
0.17	0.17739	0.005644	0.008706	0.008750	64.829	64.503
0.19	0.12045	0.006156	0.010300	0.010334	59.767	59.571
0.20	0.09823	0.006400	0.011147	0.011178	57.415	57.255

3.3 Situation B:

3.3.1 Recalling the particulars of T observations on the random variables t,u,v and w given in section 2.6.2, we construct the following likelihood functions based on them:

Case I:

$$L_1 = \prod_{t=0}^{k-1} \left[\binom{n}{t} p^t q^{n-t}\right]^{a_{t,2}} \left[\sum_{y=k}^{n} \binom{y-1}{k-1} p^k q^{y-k}\right]^{T_{r,2}} \quad ..(3.23)$$

Case II:

$$L_2 = \left[\sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}\right]^{T_{a,2}} \prod_{u=k}^{n} \left[\binom{u-1}{k-1} p^k q^{u-k}\right]^{r_{u,2}} \quad ..(3.24)$$

Case III:

$$L_3 = \prod_{v=0}^{k-1} \left[\binom{n-k+v}{n-k} q^{n-k+1} p^v\right]^{a_{v,3}} \left[\sum_{y=k}^{n} \binom{y-1}{k-1} p^k q^{y-k}\right]^{T_{r,3}} \quad ...(3.25)$$

Case IV:

$$L_4 = \left[\sum_{i=0}^{k-1} \binom{n-k+i}{n-k} q^{n-k+1} p^i\right]^{T_{a,3}} \prod_{w=0}^{n-k} \left[\binom{w+k-1}{k-1} p^k q^w\right]^{r_{w,3}} \quad ..(3.26)$$

Then equating to zero the first derivative of the likelihood functions with respect to p we get the following maximum likelihood equations to estimate p :

Case I :

$$p = \frac{\sum_{t=0}^{k-1} t a_{t,2}}{nT_{a,2} - (n-k+1)\psi T_{r,2}} \quad ...(3.27)$$

Case II :

$$p=\frac{kT_{r,2}}{\sum_{u=k}^{n} ur_{u,2}+(n-k+1)\phi\ T_{a,2}} \qquad \ldots(3.28)$$

Case III:

$$p=\frac{\sum_{v=0}^{k-1} va_{v,3}}{(n-k+1)\ T_{a,3}+\sum_{v=0}^{k-1} va_{v,3}-(n-k+1)\psi T_{r,3}} \qquad \ldots(3.29)$$

Case IV:

$$p=\frac{kT_{r,3}}{\sum_{w=0}^{n-k} wr_{w,3}+kT_{r,3}+(n-k+1)\phi T_{a,3}} \qquad \ldots(3.30)$$

where $\psi = \binom{n}{k-1} p^{k-1}q^{n-k+1} / [1-B(p,n,k-1)]$ ...(3.31)

$$= p^{k-1}q^{n-k+1}/\ (n-k+1)\int_q^1 x^{n-k}(1-x)^{k-1}\,dx \qquad \ldots(3.32)$$

$$\phi = \binom{n}{k-1} p^{k-1}q^{n-k+1}/B(p,n,k-1) \qquad \ldots(3.33)$$

$$= p^{k-1}q^{n-k+1}/(n-k+1)\int_0^q x^{n-k}(1-x)^{k-1}\,dx \qquad \ldots(3.34)$$

Maximum likelihood equations (3.27), (3.28),(3.29) and (3.30) can be solved by iteration and do not present any difficulty, since the binomial distribution is extensively tabulated in [17] and [55] . The tables in [17] give both the individual probabilities and cumulative probabilities of the binomial distribution. However in this era of computers, it may not be necessary to use these tables. Furthermore, $T_{r,j}/T$ and $T_{a,j}/T$ (j=2,3) which estimate respectively the denominator of the r.h.s of equations (3.31) and (3.33) may help in assessing the initial value of p and thereby the initial value of Ψ and $\emptyset$, to start the iteration.

3.3.2 Variance of the Estimate :

Using the formula for asymptotic variance of the maximum likelihood estimate, namely $v(\hat{p}) = -1/E(\partial^2 \log L/\partial p^2)$ the asymptotic variance of the maximum likelihood estimate of the fraction defective in the respective Cases will be as follows:

Case I $\quad v(\hat{p}) = (pq)^2/TH_1$, ...(3.35)

Cases II&IV $\quad v(\hat{p}) = (pq)^2/TH_2$, ...(3.36)

Case III $\quad v(\hat{p}) = (pq)^2/TH_3$, ...(3.37)

where

$$H_1 = np(q-p)B(p,n-1,k-2)+np^2B(p,n,k-1)-(n-k+1)\Psi p \cdot \left[(k-1)q+p\left\{1-(n-k+1)(1+\Psi)\right\}\right]\left[1-B(p,n,k-1)\right] \quad ...(3.38)$$

$$H_2 = (n-k+1)p\emptyset\left[(k-1)q+p\left\{1-(n-k+1)(1-\emptyset)\right\}\right]B(p,n,k-1) +k(q-p)\left[1-B(p,n,k-1)\right]+kp\left[1-B(p,n+1,k)\right] \quad ...(3.39)$$

$$H_3 = (n-k+1)q\left[B(p,n+1,k-1) - qB(p,n,k-1)\right]$$
$$+(n-k+1)p^2B(p,n,k-1)-(n-k+1)p\Psi$$
$$\cdot\left[(k-1)q+p\left\{1-(n-k+1)(1+\Psi)\right\}\right]\left[1-B(p,n,k-1)\right] \quad \ldots(3.40)$$

The expression for the asymptotic variance for Cases II and IV is the same. It may further be noted that the maximum likelihood equations (3.28) and (3.30) are the same except for the second suffix, since $w=u-k$ for $u=k,k+1,\ldots,n$.

3.4 Comparison Between Situations A and B with Respect to Censored Sampling*

3.4.1 All the cases considered in Situation B can be regarded as particular cases of censored sampling of Type I as defined by Gupta [32] . For instance T observations on the random variable v of Case III can be regarded as a censored sample on the random variable s of the population $f_3(s)$ defined by (2.4) in Chapter II, right hand tail of the sample being censored. In other words the exact value of s is available if $s \leq k-1$ and the exact value of s is not available if $k \leq s \leq n$; for in the latter case it is only reported that s lies in that

* Definition of censored sampling and the distinction between Type I and Type II censoring is explained in Section 3 of Chapter VII.

range. Had the exact value of s in the range $k \leq s \leq n$ been known, the reporting would belong to Situation A. Similar discussion holds for the remaining Cases. Case I can be regarded as a censored sample from $f_2(s)$ defined by (2.3) with the right hand tail being censored. Case II can be regarded as a censored sample from $f_2(s)$ defined by (2.3) with the left hand tail being censored. Lastly Case IV can be regarded as a censored sample from $f_3(s)$ defined by (2.4) with left hand tail being censored.

3.4.2 Comparison of Variances :

Since it is expected from realistic point of view that the estimate based on censored sample should be less efficient than that based on complete sample, the asymptotic variance of Cases I and II respectively given by (3.35) and (3.36) will be greater than the asymptotic variance of Plan 2 under Situation A, given by (3.16), namely p^2q/TJ_1. Similarly the asymptotic variance of Cases III and IV respectively given by (3.37) and (3.36) will be greater than the asymptotic variance of Plan 3 under Situation A given by (3.17), namely p^2q/TJ_3. Furthermore, since it has been established that $J_3 < J_1$ in Section 3.2.3 and that the asymptotic variances of Cases II and IV are equal, we have the result that the asymptotic variance of Case II is greater than p^2q/TJ_3.

Thus the lower bound of the asymptotic variance of Case II is increased. These results are well exhibited in Table 3.6.

3.5 Method of Moments and MLE :

Circumstances do occur in practice where the estimates by the method of maximum likelihood and those by the method of moments are identical. In case they differ one may prefer the maximum likelihood estimates as they are more efficient in the class of such estimates even if they involve computational difficulties. Unlike that in the usual situation we find here that the estimates by the method of maximum likelihood are simpler than those by the method of moments, so far the point of computation is concerned. In Situation A it is found that the maximum likelihood estimate is just the ratio of two statistics whereas one needs iteration if one desires to estimate it by the method of moments. In Situation B both the methods require iteration but in case of the method of maximum likelihood the estimating equation is somewhat simpler. The purpose of introducing Section 2.7 is certainly not to encourage the method of moments so far the question of estimating the fraction defective under curtailed sampling plans is concerned, but the purpose is just to study the nature of these estimates under this method.

3.6 A Numerical Example :

We shall illustrate Case II by a numerical example. We consider the same value of n and k as considered in the numerical example of 3.2.4, namely, n=25 and k=3. Suppose an inspector administers Plan 2 with n=25 and k=3 and reports only the number of articles inspected and the information regarding the acceptance or rejection of the lot. Table 3.5 gives the distribution of 50 observations associated with the inspection of 50 lots.

Table 3.5

Number of articles inspected	Number of rejected lots	Number of articles inspected	Number of rejected lots	Number of articles inspected	Number of accepted lots
(1)	(2)	(1)	(2)	(3)	(4)
3	0	15	2	25*	35
4	0	16	0		
5	0	17	0		
6	1	18	0		
7	0	19	0		
8	1	20	0		
9	0	21	2		
10	1	22	1		
11	1	23	1		
12	0	24	1		
13	2	25	2		
14	0	-	-		
Total	-	-	15	-	35

* Acceptance of a lot under Plan 2 implies that the number of articles inspected is n.

From the above table we find that

$$T_{a,2} = 35, \quad T_{r,2} = 15, \quad \sum_{u=3}^{25} u r_{u,2} = 252 \quad \text{and} \quad T_{a,2}/T = 0.7.$$

Referring to the tables of Cumulative Binomial Probability Distribution [55] for n=25 and k=3 we find the following by inspection:

p	0.07	0.08	0.09
$1-B(p,n,k-1)$	0.25344	0.32317	0.39370
$B(p,n,k-1)$	0.74656	0.67683	0.60630

Therefore we take the initial value of p as 0.08 to start the iteration. The following are the results of iteration:

Initial value of p	Value of p from equation (3.28)
0.08	0.0766592
0.07659	0.078381
0.07838	0.077446
.	.
.	.
.	.
0.07776	0.077770
0.07777	0.077766

Hence we take the estimate of p as 0.07777. The hypothetical value of p used for obtaining the above frequency distribution is 0.09. For this hypothetical value of p the asymptotic variance of the estimate of p using equation (3.36) is 0.00009088. Therefore, S.E.$(\hat{p})$=0.0095. Absolute difference between the estimate of p and the hypothetical value of p i.e.

$|0.07777-0.09| = 0.01223$ may be attributed due to sampling fluctuations since it is merely 1.29 times the S.E.$(\hat{p})$. Furthermore, in practice, one may substitute the estimate of p in (3.36) to compute the estimate of the asymptotic variance.

Furthermore, we have calculated the asymptotic variance for all the cases for different values of p ranging from 0.04 to 0.20. These results along with the asymptotic variance under complete information (i.e. p^2q/TJ_1 and p^2q/TJ_3) are presented in the Table 3.6. It may be noted that here also T times the variance is presented for the same reason explained against Table 3.4. For this purpose also we have used the Tables of the Cumulative Binomial Probability Distribution [55].

Table 3.6

p	Probability of acceptance	TV($\hat{p}$)					
		Plan 2			Plan 3		
		Under No censoring	Under Censoring Case 1	Under Censoring Case 2	Under No censoring	Under Censoring Case 3	Under Censoring Case 4
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0.04	0.92352	0.001567	0.001572	0.003467	0.001643	0.001649	0.003467
0.05	0.87289	0.001967	0.001981	0.003517	0.002049	0.002065	0.003517
0.07	0.74656	0.002811	0.002876	0.003906	0.002897	0.002965	0.003906
0.09	0.60630	0.003741	0.003943	0.004544	0.003822	0.004033	0.004544
0.11	0.47087	0.004780	0.005286	0.005378	0.004854	0.005376	0.005378
0.13	0.35171	0.005948	0.007052	0.006392	0.006012	0.007142	0.006392
0.15	0.25374	0.007255	0.009454	0.007583	0.007309	0.009546	0.007583
0.17	0.17739	0.008706	0.012819	0.008944	0.008750	0.012914	0.008944
0.19	0.12045	0.010300	0.017652	0.010469	0.010334	0.017754	0.010469
0.20	0.09823	0.011147	0.020855	0.011289	0.011178	0.020962	0.011289