

APPENDIX 2.1

Let

$$\begin{aligned}
 w &= m, \text{ for a SRGD design,} \\
 &= n, \text{ for a triangular design,} \\
 &= 2s - 1, \text{ for a } L_2 \text{ design,} \\
 &= v - p, \text{ for a rectangular design.}
 \end{aligned}$$

It is now required to show that for the four classes of designs mentioned above

$$(v-k)(b-r) - (v-rk)(v-w) > 0.$$

Proof. Let $Q = (v-k)(b-r) - (v-rk)(v-w)$.

Now,

$$\begin{aligned}
 Q &= [(v-k)(b-r)(b-1) - (v-rk)(v-w)(b-1)] / (b-1) \\
 &= [(v-k)(b-r)(b-v+w-1) + (v-k)(b-r)(v-w) \\
 &\quad - (v-rk)(v-w)(b-1)] / (b-1) \\
 &= [(v-k)(b-r)(b-v+w-1) + v(v-w)(r-1)^2] / (b-1).
 \end{aligned}$$

Hence, if $b \geq v - w + 1$, then $Q > 0$. Now, for the four classes of designs mentioned above, $b \geq v - w + 1$. Hence $Q > 0$ for the four classes of designs mentioned above.