

## APPENDIX 4.1

To show that a SRGD design with parameters  $b = v - m + r$ ,  $v = 2k$ , where  $k$  is an odd integer, cannot exist.

Proof. As  $v = 2k$ , we have  $b = 2r$ . Hence substituting  $b = 2r$  in  $b = v - m + r$ , we get  $v = m + r$ . But for a SRGD design  $v = mn$ . Hence  $r = m(n - 1)$ . Now  $rk - v\lambda_2 = 0$ . Hence  $\lambda_2 = rk/v = m(n-1)k/2k = m(n-1)/2$ .

Now  $v = mn = 2k$ . As  $k$  is an odd integer, therefore  $m$  and  $n$  cannot both be even simultaneously. Hence  $m$  or  $n$  must be a multiple of 2, but not of  $2^\alpha$ , where  $\alpha \geq 2$ ; while the other must be odd. Thus, we have two alternatives:

- (i)  $m = 2t$ , where  $t$  is an odd positive integer and  $n$  is odd, or
- (ii)  $n = 2s$ , where  $s$  is an odd positive integer and  $m$  is odd.

Suppose (i) holds. Now  $k = cm = mn/2 = tn$ , so that  $c = n/2$ . Hence  $n$  must be an even integer, which contradicts the requirement in (i) that  $n$  be odd. Hence the given design cannot exist.

Next, suppose that (ii) holds. Substituting the value of  $n = 2s$  in  $\lambda_2 = m(n-1)/2$ , we have  $\lambda_2 = ms - (m/2)$ . As  $m$  is odd,  $\lambda_2$  is fractional which is impossible. Hence the given design cannot exist.

## APPENDIX 4.2

To show that a triangular design with parameters satisfying the relations  $rk - v\lambda_1 = n(r - \lambda_1)/2$ ,  $b = v - n + r$  and  $v = 2k$  cannot exist.

Proof. As  $v = 2k$ , we have  $b = 2r$ . Substituting  $b = 2r$  in  $b = v - n + r$ , we get  $r = v - n = n(n-1)/2 - n = n(n-3)/2$ . Also  $v = 2k = n(n-1)/2$  gives  $k = n(n-1)/4$ .

Putting the values of  $v$ ,  $r$  and  $k$  in terms of  $n$  in  $rk - v\lambda_1 = n(r - \lambda_1)/2$ , and solving it for  $\lambda_1$ , we get  $\lambda_1 = n(n-3)^2/4(n-2)$ .

Now from  $k = n(n-1)/4$ , we see that if  $n$  is even, it must be of the form  $4t$ , ( $t$  a positive integer); while if  $n$  is odd,  $n-1$  must be of the form  $4t$ , ( $t$  a positive integer). Thus, we consider two alternatives for  $k$ :

- (i)  $n$  is even and of form  $n = 4t$ , or
- (ii)  $n$  is odd and of form  $n = 4t + 1$ ,  
where  $t$  is a positive integer.

If (i) holds, then substituting  $n = 4t$ , in  $\lambda_1 = n(n-3)^2/4(n-2)$ , we get  $\lambda_1 = 4t^2 - 4t + t/(4t-2)$ , which is clearly fractional for all positive integral values of  $t$ . Hence the given design cannot exist.

Next, suppose (ii) holds. Then substituting  $n = 4t + 1$  in  $\lambda_1 = n(n-3)^2/4(n-2)$ , we get  $\lambda_1 = 4t^2 - 2t - (2t-1)/(4t-1)$ , which is again fractional for all positive integral values of  $t$ . Hence the given design cannot exist.

## APPENDIX 4.3

To show that a  $L_2$  design with parameters satisfying the relation  $v = 2k$ , where  $k$  is an odd integer, cannot exist.

Proof. As  $k = v/2 = s^2/2$ , it follows that  $s$  must be even. Then,  $k$  is also even, which contradicts the fact that  $k$  is an odd integer. Hence the given design cannot exist.

## APPENDIX 4.4

To show that a rectangular design with parameters satisfying relations  $\theta_1 = 0 = \theta_2$ ,  $b = p + r$  and  $v = 2k$ , where  $k$  is an odd integer, cannot exist.

Proof. As  $v = 2k$ , therefore  $b = 2r$ . But  $b = p + r$ , hence  $r = p$ . Also  $k = v/2 = v_1 v_2 / 2$ . Using this information, we have from Section 2.5 of Chapter 2,  $\lambda_1 = (v_1 - 1)(v_2 - 2)/2$ ,  $\lambda_2 = (v_1 - 2)(v_2 - 1)/2$  and  $\lambda_3 = (v_1 v_2 - v_1 - v_2 + 2)/2$ .

As  $k = v_1 v_2 / 2$  and  $k$  is an odd integer, it follows that  $v_1$  and  $v_2$  cannot both be even. Also for the same reason  $v_1$  nor  $v_2$  can contain a factor  $2^\alpha$  where  $\alpha \geq 2$ . Hence, the following two alternatives are possible:

- (i)  $v_1$  is even and  $v_1 = 2v'_1$ , where  $v'_1$  and  $v_2$  are both odd integers, or
- (ii)  $v_2$  is even and  $v_2 = 2v'_2$ , where  $v'_2$  and  $v_1$  are both odd integers.

Under alternative (i),  $\lambda_2$  is integral but  $\lambda_1$  and  $\lambda_3$  are fractional, which is an impossible situation; under alternative (ii),  $\lambda_1$  is integral but  $\lambda_2$  and  $\lambda_3$  are fractional, which is also an impossible situation. Hence, the given rectangular design cannot exist.

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