Chapter 2

Review of Electroweak Model

The relevant topics of the Standard Model are briefly reviewed in this chapter along with the discussions of $\mathcal{CP}$ violation in the neutral K and B meson systems This review provides the background for our work that will be reported in the next chapter.

2.1 The $SU(2) \otimes U(1)$ gauge theory

The electroweak model, also known as Glashow-Salam-Weinberg (GSW) model, is based on the gauge group $SU(2) \otimes U(1)$. The observation of weak neutral currents (1973), followed by the discovery of the gauge bosons themselves ($W^{\pm}$ and Z) (1983) constitute the major experimental support for the model, which has proved over the years to be very successful phenomenologically and is in detailed agreement with all observed electroweak phenomena so far.

The gauge sector of this model consists of 4 vector bosons, three denoted by $W_{\mu}^{i}, i = 1,2,3$ are associated with the adjoint representation of $SU(2)$ and one with $U(1)$ is denoted by B_{μ} The fermion sector of the model is such that the charged weak interactions couple the left-handed component of the charged lepton to the associated (left-handed) neutrino. Parity violation is incorporated by assigning all the left handed fermions to transform as doublets under $SU(2)$, while the right-handed fermions are singlets.

The Lagrangian is made gauge-invariant by replacing ∂_{μ} in the fermion kinetic energy terms by the gauge covariant derivative D_{μ} i e

$$\partial_{\mu} \longrightarrow D_{\mu} \equiv \partial_{\mu} + igT_{i}W_{\mu}^{i} + ig^{'}\frac{Y}{2}B_{\mu}, \tag{2.1}$$

where $g, g^{'}$ and $T_{i}, \frac{1}{2}Y$ are the $SU(2), U(1)$ couplings and group generators respectively. The T_{i}'s satisfy the $SU(2)$ algebra

$$[T_{i}, T_{j}] = i\epsilon_{ijk}T_{k} \tag{2.2}$$

and act on the fermion fields as follows

$$T_{i}\psi_{L} = \frac{1}{2}\tau_{i}\psi_{L}, \;\; T_{i}\psi_{R} = 0 \tag{2.3}$$

where the τ_i are the 2×2 Pauli matrices. The assignments for the other two generations are just replica of this.

Since the weak interaction involves electrically charged $W^\pm$ bosons it must be related to electromagnetism, and to incorporate QED in the model, some linear combination of the weak generators has to be identified with the electric charge operator Q corresponding to the group $U(1)_{em}$. The clue comes from the fact that the adjacent members of an isospin multiplet are eigenstates of T_3 with eigenvalues that differ by one unit of electric charge (in units of e) Therefore, we may write

$$Q = T_3 + \frac{Y}{2} \tag{2.4}$$

where T_i and Y are referred to as the weak isospin and weak hypercharge generators respectively. The above relation (called Gell-Mann–Nishijima relation) can be used to specify the eigenvalues of the $U(1)$ generator, $\frac{1}{2}Y$ where the factor $\frac{1}{2}$ is purely a matter of convention. Thus the fermions transform under the full symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ of the standard model as follows:

$$\begin{aligned}
leptons : l_{iL} &= \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L (1, 2, -1); \\
e_{iR} & \quad (1, 1, -2); \\
quarks : q_{iL} &= \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L (3, 2, 1/3); \\
u_{iR} & \quad (3, 1, 4/3); \\
d_{iR} & \quad (3, 1, -2/3)
\end{aligned} \tag{2.5}$$

where i denotes the fermion generation. The group structure permits an arbitrary hypercharge assignment for each left-handed doublet and each right-handed singlet, and so we have chosen Y to give the correct electric charges Apparently, charge quantization must be put by hand in $SU(2)_L \otimes U(1)_Y$ theory

Now, the $SU(2) \otimes U(1)$ invariant Lagrangian that consists of the kinetic energy terms for massless fermions and gauge bosons and the fermion-fermion-gauge boson couplings takes the form

$$\begin{aligned}
\mathcal{L} &= \sum_f [\overline{f_L} \gamma^\mu (i\partial_\mu - g\frac{\tau_i}{2} W^i_\mu - g'\frac{Y}{2} B_\mu) f_L + \\
& \overline{f_R} \gamma^\mu (i\partial_\mu - g'\frac{Y}{2} B_\mu) f_R] - \frac{1}{4} W^i_{\mu\nu} W^{\mu\nu}_i - \frac{1}{4} B_{\mu\nu} B^{\mu\nu},
\end{aligned} \tag{2.6}$$

where the sum is over all left- and right-handed fermion fields and the field strength tensors of the $SU(2)$ and $U(1)$ gauge fields are given by

$$\begin{aligned}
W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk} W^j_\mu W^k_\nu, \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu.
\end{aligned} \tag{2.7}$$

The term bilinear in $W_{\mu\nu}$ generates the trilinear and quadrilinear self-couplings of the W_μ fields that are a characteristic of non-Abelian gauge theory.

2.2 Giving mass to the particles

Since the gauge fields transform under the gauge group $SU(2) \otimes U(1)$ as

$$\begin{aligned} W_\mu \to W_\mu^{'} &= U^{-1} W_\mu U + \frac{i}{g} U^{-1} \partial_\mu U, \\ B_\mu \to B_\mu^{'} &= B_\mu + \frac{i}{g} U^{-1} \partial_\mu U; \end{aligned} \tag{2.8}$$

it is evident that an explicit gauge-boson mass term is not gauge-invariant. To see the possible mass terms for fermions, consider two left-handed spinors ψ_L and χ_L that transform as $(1/2, 0)$ under Lorentz transformation (LT) The quantity $\chi_L^T \sigma^2 \psi_L$ is invariant under LT. With $\chi_L = \sigma^2 \psi_R^*$ this invariant is

$$(\sigma^2 \psi_R^*)^T \sigma^2 \psi_L = -\psi_R{}^\dagger \psi_L \tag{2.9}$$

and in 4 component notation, is the Dirac mass term

$$m\overline{\psi}\psi \equiv m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L) \tag{2.10}$$

which is excluded because ψ_L, ψ_R transform under $SU(2)$ as doublet and singlet respectively, so that this term too manifestly breaks the gauge invariance.

The other possibility for a mass term is given by the chioce $\chi_L = \psi_L$. Then we have

$$\frac{m}{2}(\psi_L^T \sigma^2 \psi_L + \psi_L^\dagger \sigma^2 \psi_L^*). \tag{2.11}$$

This mass term is called Majorana mass term and it is not invariant under $U(1)$. Consequently, any additive quantum number carried by ψ_L, such as charge, lepton number, etc. is not conserved if ψ_L has a Majorana mass. Since the gauge structure of the SM conserves the lepton number such a mass term for the fermions is not allowed

Now we discuss how to generate gauge-boson and fermion masses without destroying the renormalizability of the theory, which depends so critically on the gauge symmetry of the interactions. As the low energy symmetry observed in nature is $SU(3)_C \otimes U(1)_{em}$, the gauge symmetry $SU(2)_L \otimes U(1)_Y$ must break down to $U(1)_{em}$. This is achieved through the spontaneous symmetry breaking of the gauge symmetry $SU(2)_L \otimes U(1)_Y$ by introducing a complex scalar field (**higgs sector**) ϕ, which couples gauge invariantly to the gauge bosons through the covariant derivative

$$\partial_\mu \phi \partial^\mu \phi^\dagger = |\partial_\mu \phi|^2 \longrightarrow |(\partial_\mu + igT_i W_\mu^i + ig' \frac{Y}{2} B_\mu)\phi|^2, \tag{2.12}$$

and to the fermions through so-called "Yukawa" couplings of the form

$$-h_f[(\overline{\psi_L}\phi)\psi_R + \overline{\psi_R}(\phi^\dagger \psi_L)] \tag{2.13}$$

Evidently, the field ϕ should transform as $(1, 2, 1)$ under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ to preserve the gauge invariance. Hence, we write

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \text{where} \quad \begin{pmatrix} \phi^+ \equiv (\phi_1 + i\phi_2)/\sqrt{2} \\ \phi^0 \equiv (\phi_3 + i\phi_4)/\sqrt{2} \end{pmatrix} \tag{2.14}$$

with ϕ_i real, while the Hermitian conjugate doublet $\phi^\dagger$ describes the antiparticles ϕ^- and ϕ^{0*}.

Besides the above interactions, there can be a self-interaction between the higgs fields. The most general $SU(2)$ invariant and renormalizable form (with dimension ≤ 4) for such a self-interaction term in the Lagrangian is

$$V(\phi) = -\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2, \tag{2.15}$$

where λ must be positive to keep the potential bounded below. For $\mu^2 > 0$, $V(\phi)$ is at its minimum when $|(\phi^\dagger\phi)| = \mu^2/2\lambda$ The minima that has the vacuum expectation values (*vevs*)

$$< 0|\phi_i|0 >= 0, i = 1,2,4; \ \ < 0|\phi_3|0 >\equiv \frac{v}{\sqrt{2}} \equiv \sqrt{\mu^2/\lambda} \tag{2.16}$$

is chosen and then the field ϕ is expanded about this minimum such that the particle quanta of the theory (i.e. the physical higgs) correspond to quantum fluctuations of $\phi_3(x)$ about the value $\phi_3 = v$ rather than to $\phi_3(x)$ itself, that is, to

$$h(x) \equiv \phi_3(x) - v. \tag{2.17}$$

The choice of the non-zero *vev* for the neutral field ϕ_3 ensures that the vacuum is invariant under $U(1)_{em}$ of QED, and the photon remains massless.

When the relevant term in the Lagrangian is rewritten[6] in terms of $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$, the mass term for gauge bosons read

$$\begin{aligned} |(\partial_\mu + igT_iW^i_\mu + ig'\frac{Y}{2}B_\mu) < \phi > |^2 &= (\frac{1}{2}vg)^2 W^+_\mu W^{-\mu} + \\ & \frac{1}{8}v^2(gW^3_\mu - g'B_\mu)^2 + 0(g'W^3_\mu - gB_\mu)^2, \end{aligned} \tag{2.18}$$

where

$$W^\pm \equiv (W^1 \pm iW^2)/\sqrt{2}. \tag{2.19}$$

The mass matrix of the neutral fields is off-diagonal in the (W^3, B) basis. As expected, one of the mass eigenvalues is zero and, thus the normalised neutral mass eigenstates are

$$\begin{aligned} Z_\mu &= \frac{(gW^3_\mu - g'B_\mu)}{\sqrt{g^2+g'^2}} \equiv W^3_\mu \cos\theta_W - B_\mu \sin\theta_W, \\ A_\mu &= \frac{(g'W^3_\mu + gB_\mu)}{\sqrt{g^2+g'^2}} \equiv W^3_\mu \sin\theta_W + B_\mu \cos\theta_W; \end{aligned} \tag{2.20}$$

where θ_W is the Weinberg angle defined by

$$\theta_W = \arctan\frac{g'}{g}. \tag{2.21}$$

Comparision of eqn (2 18) with the mass terms in the Lagrangian of the physical $W^\pm_\mu, Z_\mu$ and photon A_μ fields, namely,

$$M^2_W W^+_\mu W^{-\mu} + \frac{1}{2}M^2_Z Z^2_\mu + \frac{1}{2}M^2_\gamma A^2_\mu \tag{2.22}$$

yields

$$M_W = \frac{1}{2}vg, \quad M_Z = \frac{1}{2}v\sqrt{(g^2 + g'^2)}, \quad M_\gamma = 0, \tag{2.23}$$

and so

$$\cos\theta_W = \frac{M_W}{M_Z}. \tag{2.24}$$

It is easy to see that the higgs mass comes out to be

$$M_H^2 = 2\lambda v^2 = 2\mu^2 \tag{2.25}$$

which cannot be predicted since neither μ^2 nor λ is determined, only their ratio v^2 is.

The $\rho - parameter$ that specifies the relative strength of the neutral and charged current weak interactions is defined as

$$\rho \equiv \frac{M_W^2}{M_Z^2 cos^2\theta_W}. \tag{2.26}$$

The GSW model with a single higgs doublet has $\rho = 1$, which is in excellent agreement with experiment. If the higgs sector is such that there are several representations (i=1,...,N) of higgs scalars whose neutral members acquire *vevs* v_i, then

$$\rho = \frac{\sum v_i^2[T_i(T_i+1) - \frac{1}{4}Y_i^2]}{\sum \frac{1}{2}v_i^2 Y_i^2}, \tag{2.27}$$

where T_i and Y_i are, respectively, the weak isospin and hypercharge of representation i.

To see how the charged leptons acquire mass consider the Yukawa coupling term for the electron doublet :

$$\mathcal{L}_Y^e = -h_e[(\overline{\nu_e}, e)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \overline{e_R}(\phi^-, \phi^{0*}) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L]. \tag{2.28}$$

After the symmetry breaking

$$\mathcal{L}_Y^e = -\frac{h_e}{\sqrt{2}}(v+h)(\overline{e_R}e_L + \overline{e_L}e_R) \tag{2.29}$$

from which we read out the electrons mass and couplings to be

$$m_e = \frac{h_e v}{\sqrt{2}}, \quad g(hee) = \frac{gm_e}{2M_W}. \tag{2.30}$$

Although the electron's coupling to higgs is well specified, the actual mass of the electron is not predicted as h_e is arbitrary Similarly the most general $SU(2) \otimes U(1)$ invariant Yukawa terms for the quark doublet (u, d) are

$$\mathcal{L}_Y^q = -h_d(\overline{u}, \overline{d})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R + h_u(\overline{u}, \overline{d})_L \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} u_R + h.c. \tag{2.31}$$

here $\begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$ is $i\tau_2\phi^*$ that has a neutral upper member. Due to special proporties of $SU(2)$ it has $Y = -1$ After the symmetry breaking

$$\mathcal{L}_Y^q = -(m_d\overline{d}d + m_u\overline{u}u)(1 + \frac{h}{v}) \tag{2.32}$$

where mass of the quarks are given by

$$m_q = \frac{h_q v}{\sqrt{2}}. \tag{2.33}$$

2.3 Current mass and Physical mass

Unlike the electroweak part of the SM, the renormalised masses in QCD cannot be naturally defined by on-shell renormalisation due to the confinement of quarks. The quark mass parameters of the Lagrangian can be simply considered as additional coupling constants. Hence, like the running coupling constants, their measurement first requires a careful mention of the conventions needed for the unique definition of a renormalized running quark mass of the theory. We discuss within the domain of the $\overline{MS}$ scheme which has the advantage that renormalization group equations are flavour diagonal. The evolution of the quark masses and the strong coupling constant with the renormalisation scale μ is governed by the RG equations

$$\mu\frac{dg}{d\mu} = \beta(g)$$
$$\mu\frac{dm_i}{d\mu} = -\gamma_{m_i}(g)m_i. \tag{2.34}$$

In the modified minimal subtraction ($\overline{MS}$) scheme, the beta function and the anomalous dimension are respectively given[7] by

$$\beta(g) = -\frac{\beta_0}{(4\pi)^2}g^3 - \frac{\beta_1}{(4\pi)^2}g^5 + O(g^7), \tag{2.35}$$

and

$$\gamma_m(g) = \frac{\gamma_0}{(4\pi)^2}g^2 - \frac{\gamma_1}{(4\pi)^2}g^4 + O(g^6), \tag{2.36}$$

with

$$\begin{aligned}
\beta_0 &= (11C_G - 4T_R N_f)/3, \\
\beta_1 &= [(34C_G^2 - 45C_G + 3C_F)T_R N_f]/3, \\
\gamma_0 &= 6C_F, \\
\gamma_1 &= C_F[9C_F + 97C_G - 20T_R N_f]/3,
\end{aligned} \tag{2.37}$$

where N_f= number of quark flavours, T_R is given by the normalization of the generators $[Tr(T^aT^b) = N_f T_R]$ and C_G, C_F are the values of the quadratic Casimir operator on the gluons and quarks respectively Following the convention for $SU(3)$ i.e. $T_R = 1/2, C_G = 3$, and $C_F = 4/3$ we have

$$\begin{aligned}
\beta_0 &= (11 - \frac{2}{3}N_f), \\
\beta_1 &= 102 - \frac{38}{3}N_f, \\
\gamma_0 &= 8, \\
\gamma_1 &= \frac{4}{3}(101 - \frac{10}{3}N_f).
\end{aligned} \tag{2.38}$$

The solution to the differential equations are

$$\alpha_s(\mu) \equiv \frac{g^2(\mu)}{4\pi} = \frac{4\pi}{\beta_0}L[1 - \frac{\beta_1}{\beta_0^2}\frac{lnL}{L} + O((\frac{lnL}{L})^2)], \tag{2.39}$$

and

$$m_i(\mu) = \overline{m_i}(\frac{L}{2})^{-\gamma_0/2\beta_0}[1 - \frac{\beta_1\gamma_0}{2\beta_0^2}\frac{1+lnL}{L} + \frac{\gamma_1}{2\beta_0^2 L} + O((\frac{lnL}{L})^2)], \tag{2.40}$$

where $L = ln(\mu^2/\Lambda^2)$. Here Λ and $\overline{m_i}$ are the RG invariant scale parameter and masses, respectively defined through

$$e^{-\beta_0 g^2(0)} = \frac{\lambda^2}{\Lambda^2}(ln\frac{\lambda^2}{\Lambda^2})^{\beta_1/\beta_0^2}, \tag{2.41}$$

and

$$m_i(0) = \overline{m_i}(ln\frac{\lambda^2}{\Lambda^2})^{\gamma_0/2\beta_0}, \tag{2.42}$$

λ being the momentum cut-off

The physical mass of a quark is its value calculated at the same scale. Thus to one loop order, the physical mass of the ith quark is given by

$$m_i^{phy} = m_i(m_i)[1 + \frac{4}{3\pi}\alpha_s(m_i)]. \tag{2.43}$$

Although the determination of the light quark masses involves larger errors, still they are best estimated by the use of Chiral QCD perturbation theory as well as meson and baryon spectroscopy [7] :

$$\begin{aligned} m_u &= 5.1 \pm 1.5 \quad MeV \\ m_d &= 8.9 \pm 1.5 \quad MeV \\ m_s &= 175 \pm 55 \quad MeV \end{aligned} \tag{2.44}$$

Similarly the physical masses of the charm and bottom-quarks are obtained from e^+e^- data by using QCD sum rules for the vacuum polarisation amplitude. The running masses at $1GeV$ and $\Lambda_{QCD} = 100MeV$ [7] are

$$\begin{aligned} m_c(1GeV) &= 1.35 \pm 0.5 \quad GeV \\ m_b(1GeV) &= 5.3 \pm 0.1 \quad GeV \end{aligned} \tag{2.45}$$

While non-observation of the top-quark puts a lower limit to its mass

$$m_t^{phy} \geq 103 \quad GeV, \tag{2.46}$$

experimental consistency with the radiative corrections[9] in the SM requires

$$m_t \leq 180 \quad GeV. \tag{2.47}$$

2.4 The quark mixing matrix

In the early sixties, the LH quark states that take part in weak interactions were LH doublets, and a lonely strange singlet; and all the RH quarks were singlets :

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, s_L; u_R, d_R, s_R, \tag{2.48}$$

where d' is a mixed state of d and s states (Cabibbo's hypothesis)[10],

$$d' = V_{ud}d + V_{us}s, \tag{2.49}$$

when expressed in terms of the Cabibbo angle θ_C, $V_{ud} = \cos\theta_C, V_{us} = \sin\theta_C$. Cabibbo's hypothesis acccounted for relative coupling strength of stangness-conserving and strangness-violating baryon semi-leptonic decays, for the ratio of leptonic decay rates of pions and kaons, and for many other important features of the charged weak interactions. However, there is the strangness changing neutral current $(\overline{d}s + \overline{s}d)$ term arising from $\overline{d'}d' = \cos^2\theta_C\overline{d}d + \sin^2\theta_C\overline{s}s + \cos\theta_C\sin\theta_C(\overline{d}s + \overline{s}d)$. The too large rate of $K_L \to \mu^+\mu_-$ given by such a neutral strangeness changing current compared to the measured value of the branching ratio

$$Br(K_L \longrightarrow \mu^+\mu^-) = (9.1 \pm 1.8) \times 10^{-9}, \tag{2.50}$$

leads to the introduction of the 'charm' quark to form another left-handed doublet (GIM scheme[11]) (1970)

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L; u_R, d_R, c_R, s_R, \tag{2.51}$$

where $(d', s') = (d, s)V^T$, and V^T is the transposed matrix of

$$V = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}, \tag{2.52}$$

which represents a rotation by an angle θ_C in the two dimensional abstract space and all RH quarks are still singlets. The orthogonality of the mixing matrix guarrantees the absence of strangness-flavour changing neutral current terms. Kobayashi and Maskawa (1973) extended[5] the quark sector to

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L; u_R, d_R, c_R, s_R, t_R, b_R, \tag{2.53}$$

by introducing two more quarks, namely the top (t) and the bottom (b) on the basis of the theoretical observation that the 'reality' of the matrix V would not allow $\mathcal{CP}$ violation via the intermediate bosons $W^\pm$ coupling in the SM The matrix V that express (d', s', b') in terms of (d, s, b) is

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \tag{2.54}$$

and is called the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The V_{ij}'s characterize the modification of the charged current vertices for the physical quark fields induced by quark mixing. The unitarity of the V matrix i e $VV^\dagger = \mathrm{I}$ ensures the absence of FCNC.

Ever since it was noted that the quark flavour states (states that take part in the weak interactions) are mixed states of the physical states (states that have well-defined mass), attempts have been made to comprehend the dynamical origins of the mixing angles. The observation that the Cabibbo angle θ_C is very close to the mass ratios,

$$\theta_C \sim m_\pi/m_K \sim m_d/m_s, \tag{2.55}$$

initiated many studies around expressing the elements of CKM matrix in terms of the quark masses

To see how the CKM matrix is related to the procedure of quark mass matrix diagonalisation, consider the part of the Lagrangian containing the most general quark mass terms i.e.

$$\mathcal{L}_Y = h_{ij}^{(u)}\overline{u'}_{iL}u'_{jR}\phi^{0*} + h_{ij}^{(d)}\overline{d'}_{iL}d'_{jR}\phi^0 + h.c. \tag{2.56}$$

After symmetry breaking it reduces to

$$\mathcal{L}_Y = \frac{v}{\sqrt{2}}[\overline{u'}_{iL}h_{ij}^{(u)}u'_{jR} + \overline{d'}_{iL}h_{ij}^{(d)}d'_{jR}] + h.c. \tag{2.57}$$

with the generation index $i, j = 1, ..., N$ and $u'_{iL,iR} = \frac{1}{2}(1 \mp \gamma_5)u'_i$. The complex Yukawa couplings h_{ij} constitute $N \times N$ matrices which are generally neither Hermitian nor diagonal and $v(246\ GeV)$ is the vev of the neutral higgs field. Thus the quark mass matrices are given in the flavour basis as

$$M_{ij}^{(u)} = -(v/\sqrt{2})h_{ij}^{(u)}, M_{ij}^{(d)} = -(v/\sqrt{2})h_{ij}^{(d)}, \tag{2.58}$$

where the matrices $M^{(u)}$ and $M^{(d)}$ denote the quark mass matrices for charge 2/3 (up-type) and $-1/3$ (down-type) quarks respectively. In order to find the physical fields, the quark mass matrices $M^{(u)}$ and $M^{(d)}$ must be diagonalised. As it is well-known from the theory of matrices, any square matrix (hermitian or not) can be diagonalised by a bi-unitary transformation. Since the mutually exclusive left and right-handed fields in the standard model can be rotated differently i.e.

$$\begin{aligned} u_{L,R} &= U_{L,R}u'_{L,R}, \\ d_{L,R} &= D_{L,R}d'_{L,R}; \end{aligned} \tag{2.59}$$

we can find four matrices such that for three generations

$$\begin{aligned} U_L M^{(u)} U_R{}^\dagger &= \mathcal{M}^{(u)} = diag(m_u, m_c, m_t, ...), \\ D_L M^{(d)} D_R{}^\dagger &= \mathcal{M}^{(d)} = diag(m_d, m_s, m_b, ...). \end{aligned} \tag{2.60}$$

This defines the basis of the physical quarks, and we have

$$\mathcal{L}_Y = -(\overline{u_{iL}}\mathcal{M}_{ij}^{(u)}u_{jR} + \overline{d_{iL}}\mathcal{M}_{ij}^{(d)}d_{jR}) + h.c. \tag{2.61}$$

It should be noted that the matrix $S^{(u)} = (M^{(u)}M^{(u)\dagger})$ and $S^{(d)} = (M^{(d)}M^{(d)\dagger})$ are hermitian and can thus be diagonalised by a single unitary transformation i.e.

$$\begin{aligned} U_L M^{(u)} M^{(u)\dagger} U_L^\dagger &= (\mathcal{M}^{(u)})^2 \equiv (m_u{}^2, m_c{}^2, m_t{}^2, ...) \\ D_L M^{(d)} M^{(d)\dagger} D_L^\dagger &= (\mathcal{M}^{(d)})^2 \equiv (m_d{}^2, m_s{}^2, m_b{}^2, ...). \end{aligned} \tag{2.62}$$

On the other hand, the transformations that relate the flavour basis to the physical basis introduces non-diagonal coupling into the charged currents, when they are expressed in terms of the physical quark basis,

$$j_\mu{}^\dagger = \overline{u_{iL}}\gamma_\mu V_{ij} d_{jL}, \tag{2.63}$$

where V is the unitary $N \times N$ flavour mixing matrix (CKM matrix) and is given by

$$V = U_L{}^{\dagger} D_L. \tag{2.64}$$

The unitarity of the $N \times N$ complex matrix V reduces the number of real parameters from $2N^2$ to N^2. An orthogonal matrix in N dimensions can be parametrized by $N(N-1)/2$ rotation angles. Thus, out of N^2 real parameters of V, $N(N-1)/2$ are rotation angles and $N(N+1)/2$ are phase angles. Since under rephasing of the up and down quark fields the non-physical individual phases γ_j and β_i of V_{ij} transform as:

$$V_{ij} \rightarrow (V'_{ij}) = V_{ij} exp(\gamma_j - \beta_i), \tag{2.65}$$

$(2N-1)$ of these phase angles can be absorbed into the definition of the quark field phases without loss of generality. So an $N \times N$ flavour mixing matrix V can be parametrised by $\mathbf{N(N-1)/2}$ rotation angles and $\mathbf{(N-1)(N-2)/2}$ phase angles. Since there are many ways to absorb the phases as relative phases between quark fields and to introduce the rotation angles in a particular way, there exists no unique parametrization of V. Physical quantities do not depend upon the particular choice of the parametrization. However, some non-measurable parameters (like the phase of a transition amplitude) are sensitive to the phase convention accepted for the quark fields. The chioce of a particular parametrization for the CKM matrix always implies the adoption of a definite phase choice. Now we consider three frequently used parametrization of V for three generations. For three generations, V can be parametrised in terms of 3 Euler angles and one phase (since five phases of the quark fields can be rotated away).

Kobayashi and Maskawa were the first to point out the matrix V for three generations cannot be transformed into a real form Then, they suggested a parametrization where quark phases are so choosen that the first row and column of V are real,

$$V = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 c_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \tag{2.66}$$

where $c_i = \cos\theta_i$; $s_i = \sin\theta_i$ with $i = 1, 2, 3$. Without loss of generality θ_i can be chosen to lie in the first quadrant $i.e. 0 \leq \theta_i \leq \pi/2$ provided we allow the phase angle δ to take values in its full period, $i.e. -\pi \leq \delta \leq \pi$

An alternative parametrization proposed by Maiani[12] and advocated by PDG to be the standard one is given as.

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{2.67}$$

where the standard notation $c_{ij} = \cos\theta_{ij}$; $s_{ij} = \sin\theta_{ij}$ with $i, j = 1, 2, 3$ is used. It has the advantage that it makes it easy to incorporate the experimental results on B - meson deacy.

A third parametrization was introduced by Wolfenstein[14] in which he expanded the elements of the matrix V in terms of a small parameter $\lambda = \sin\theta_C$, exploiting the experimental information about the smallness of the mixing angles. The remaining structure is then determined by the unitarity constraint. This parametrization reads

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta + i\eta\frac{1}{2}\lambda^2) \\ -\lambda & 1-\frac{1}{2}\lambda^2 - i\eta A^2\lambda^4 & A\lambda^2(1+i\eta\lambda^2) \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{2.68}$$

This matrix is approximately unitary, the imaginary part of the unitarity relation is satisfied to order λ^5 and the real part to order λ^3 The coefficients A, ρ and η are of order one or even smaller

Now we discuss the measured values of these mixing angles. While θ_{12} is very accurately determined from K_{e3} and hyperon decays[15]

$$s_{12} = .221 \pm 0.002, \tag{2.69}$$

θ_{23} and θ_{13} are rather poorly determined. The value of s_{23} may be extracted from a determination of V_{cb} (since $s_{23} \approx |V_{cb}|$ to a very good approximation) from the semileptonic B-meson partial width, under the assumption that it is given by the W-mediated process to be

$$\Gamma(b \to c\, l\, \overline{\nu}_l) = (\frac{G_F^2 m_b^5}{192\pi^3}) F(m_c^2/m_b^2) |V_{cb}|^2 \tag{2.70}$$

where $F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln(x)$ is a phase space factor. Thus

$$s_{23}^2 = (\frac{192\pi^3}{G_F^2}) \frac{\mathrm{Br}(b \to c\, l\, \overline{\nu}_l)}{\tau_b m_b^5 F(m_c^2/m_b^2)} \quad . \tag{2.71}$$

Using the experimental results for the branching ratio and the B-meson lifetime[16]

$$\mathrm{Br}(b \to c\, l\, \overline{\nu}_l) = .121 \pm 0.008 \qquad \tau_b = (1.16 \pm 0.16) 10^{-12} \mathrm{sec} \tag{2.72}$$

and the estimation for the quark masses :

$$m_c = 1.5 \pm 0.2\ GeV \qquad m_b = 5.0 \pm 0.3\ GeV \tag{2.73}$$

we get

$$s_{23} = 0.044 \pm .009 \tag{2.74}$$

The charmless B-meson decay width imposes the limit[17]

$$0.05 \leq s_{13}/s_{23} \leq 0.13 \tag{2.75}$$

The $\mathcal{CP}$-violating phase δ is allowed to adopt any value in the range $[0, \pi]$ by these current experimental results

2.5 $\mathcal{CP}$ violation and quark mixing

Apart from the continuous symmetries that leads to conservation laws through the Noether currents the interactions that the particles undergo also respect certain discrete symmetries. Each discrete symmetry corresponds to a definite inversion and can be described in terms of a single operator.

Parity ($\mathcal{P}$) . Space-inversion. Invariance under $\mathcal{P}$ means that the LH frame obtained from RH frame by changing the signs of all spatial coordinates is an equally valid frame for expressing the laws of physics. In otherwords, the mirror image of an experiment would yield the same result in the reflected frame of reference as the original would do in the initial frame. Under $\mathcal{P}$ operation the 3-momenta are reversed. Interactions can be classified according to their transformations under the $\mathcal{P}$ operation. Since particles can be created or absorbed, intrinsic parity can also be assigned to particles. The over-all parity of a state is its parity under space inversion times the intrinsic parities of the particles in the state.

Charge-Conjugation ($\mathcal{C}$) : It transforms particles into anti-particles (i.e. reverses all additive quantum numbers) while spins and momenta are preserved. Invariance under $\mathcal{C}$ means that by turning all particles in a process into their anti-particles, we would get another process that would happen with equal probability Under this operation, a spinor field transforms as :

$$\mathcal{C} : \psi \rightarrow \psi^c \equiv C\overline{\psi}^T , \tag{2.76}$$

where C is a matrix in the Dirac space satisfying

$$C\gamma_\mu^T C^{-1} = -\gamma_\mu, \quad C^\dagger C = 1, \quad C^T = -C. \tag{2.77}$$

A look at the Dirac equation tells the way the chiral components transform under $\mathcal{C}$:

$$\begin{aligned} \mathcal{C} : \quad \psi_L \rightarrow \quad (\psi_R)^c = C\overline{\psi_R}^T = P_L\psi^c \\ \mathcal{C} : \quad \psi_R \rightarrow \quad (\psi_L)^c = C\overline{\psi_L}^T = P_R\psi^c , \end{aligned} \tag{2.78}$$

where $P_{L,R}$ are the left– and right–projection operators respectively.

Time-Reversal ($\mathcal{T}$) : It refers to the reversal of the flow of time. Under this anti-unitary transformation, the initial and the final states are interchanged and spins and momenta are reversed

Luders and Pauli had proved[18] that any Lorentz-invariant unitary local field theory is invariant under the combined transformation $\mathcal{CPT}$ (in any order) But, the invariance under any individual discrete symmetry is not assured by any deep-rooted theoretical motivations Three out of four basic interactions i.e. gravitation, electromagnetism, and the strong nuclear interactions respect each of these discrete symmetries to quite a great extent whereas weak interactions violate both $\mathcal{C}$ and $\mathcal{P}$ invariance maximally.

Since the two terms (having same strength) present in the V-A structure of the Noether currents for the weak interactions can completely interfere, a system of quarks or leptons can change its parity and it is said that $\mathcal{P}$ is violated maximally. Similarly, SM also violates $\mathcal{C}$

invariance maximally since (for example) processes occur involving LH neutrinos, but never LH anti-neutrinos. However, under the combined operation $\mathcal{CP}$, the LH neutrino is transformed into a RH anti-neutrino and in SM, we do have electroweak interactions of LH neutrinos as well as the RH anti-neutrinos. Thus it was believed that (mid 1950's) that even though SM violates $\mathcal{C}$ and $\mathcal{P}$ separately it conserves $\mathcal{CP}$ in the sense that if a process occurs, so does the $\mathcal{CP}$ transformed process. But in the mid-1960's, it was found[19] out that although processes and their $\mathcal{CP}$ conjugate processes occur in SM, their probabilities to occur are not identical but differ by a small amount, about a one part in a thousand (see next section for detailed discussion). This small difference in the probabilities is called the $\mathcal{CP}$ violation.

$\mathcal{CP}$ violation was incorporated into SM by noticing that $\mathcal{CP}$ violation implies a violation of T or vice versa since $\mathcal{CPT}$ is a good symmetry for all quantum field theories. It is well known that if T is a good symmetry, then the quantum mechanical transformation gives $<\psi'|H|\psi>=<T\psi|THT^{-1}|T\psi'>$ Since the anti-unitary time-reversal operation T involves complex conjugation, the T (and $\mathcal{CP}$) is violated if the Hamiltonian H is not real, as the complex conjugation will, then, mean that $THT^{-1} \neq H$. From the structure of charged-current in SM, it is evident that $\mathcal{L}$ (hence H) is complex if the phase angle in the CKM matrix is non-zero. Thus the $\mathcal{CP}$ violation in SM is attributed to the non-zero CKM phase.

It should be noted that for less than three generations of quark flavours, there is no $\mathcal{CP}$ violation in SM as CKM matrix could, with full generality, be made real in such a case. However, for the $\mathcal{CP}$ violating character of V not only the phase δ is important. If any of the mixing angles is zero, the theory can be made $\mathcal{CP}$ invariant by reabsorbing the $\mathcal{CP}$ violation phase δ into a redefinition of the quark field phases. As for the leptonic sector, $\mathcal{CP}$ violation is identically zero in the minimal SM, but could arise if neutrinos are made massive.

Another concept that has been advocated is whether $\mathcal{CP}$ symmetry might be violated in a 'maximal' way as $\mathcal{P}$ and $\mathcal{C}$ are separately. But, it is not easy to find a resonable definition of "maximal $\mathcal{CP}$ violation" because the definition should be invariant under a change of phase convention or a parametriztion The condition that the $\mathcal{CP}$ phase angle is equal to $\pi/2$ for maximal $\mathcal{CP}$ violation in some parametrization does not meet this standard. In fact, the rephasing invariant quantitative measure of $\mathcal{CP}$ violation was given in terms of elements of flavour mixing matrix elements as

$$J_{i\alpha} \equiv Im(V_{j\beta}V_{k\gamma}V^*_{j\gamma}V^*_{k\beta}) \tag{2.79}$$

where i,j,k and α,β,γ are cyclic. There are nine of these invariants for three generations case and they all are the same. this invariant $J_{i\alpha}$ is a small quantity bounded from above as

$$J_{i\alpha} < |V_{us}||V_{ub}||V_{cb}||V_{cs}| < 1.8 \times 10^{-4}, \tag{2.80}$$

and is given explicitly in various parametrizations as follows

$$\begin{aligned} KM : J_{i\alpha} &= c_1c_2c_3s_1^2s_2s_3\sin\delta \\ Maiani : J_{i\alpha} &= c_1c_2c_3s_1^2s_2s_3\sin\delta \\ Wolfenstein : J_{i\alpha} &= \lambda^6A^2\eta(1-\frac{1}{2}\lambda^2) \end{aligned} \tag{2.81}$$

Then the "minimal $\mathcal{CP}$ violation" can be defined in terms of $J_{i\alpha}$. $J_{i\alpha}$ is maximal if

$$\cos\theta_1 = 1/\sqrt{3},\ \sin\theta_2 = \sin\theta_3 = 1/\sqrt{2},\ |\sin\delta| = 1. \tag{2.82}$$

Obviously, $\mathcal{CP}$ in nature is not violated maximally according to this definition.

Jarlskog[20] has defined a convention independent measure of $\mathcal{CP}$ violation in terms of quark mass matrices as

$$[M^{(u)}M^{(u)\dagger}, M^{(d)}M^{(d)\dagger}] = iC, \tag{2.83}$$

where C is a traceless hermitian matrix whose determinant is a convention independent measure of $\mathcal{CP}$ violation,

$$detC = -2FF'J_{11}, \tag{2.84}$$

with

$$\begin{aligned} F &= (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \\ F' &= (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \end{aligned} \tag{2.85}$$

and it vanishes if two of the quark masses with charge $(2/3)e$ or any two masses with charges $(-1/3)e$ are degenerate.

2.6 $\mathcal{CP}$ violation in neutral meson systems

As we have seen how SM incorporates $\mathcal{CP}$ violation, the next thing we would discuss is the extent to which SM can account for observed $\mathcal{CP}$ violating effects through the phase in CKM quark mixing matrix. The system of neutral Kaons is still the only experimentally established system having $\mathcal{CP}$ violation since its discovery by Cronin and Fitch (1964). In this section we introduce a general formalism that describes the $\mathcal{CP}$ properties of neutral mesons like $K^0 - \overline{K^0}, D^0 - \overline{D^0}, B^0 - \overline{B^0}$ etc. Although the $K^0 - \overline{K^0}$ system is taken as the prototype , the results hold for other mesons as well.

2.6.1 The Neutral Kaons system

Since both the strong and electromagnetic interactions conserve strangeness, the neutral Kaons K^0 and $\overline{K^0}$ (characterised by definite strangeness) form the basis for the Hamiltonian $H_S + H_{em}$. But they do not possess well defined masses or life-times as a mixing between K^0 and $\overline{K^0}$ is caused by strangeness violating weak interactions. Instead there exist two independent linear combinations of these states, namely K_L and K_S (having no definite strangeness but having definite mass and decay rates) that are characterized by the differences in the mass and life-times. The short-lived state K_S decays primarily through the 2π channel (with $\mathcal{CP}$ eigenvalue +1), the long-lived state K_L has many decay channels mostly going to final states with $\mathcal{CP}$ eigenvalue -1 i.e. 3π or $\pi^{\pm}l^{\mp}\overline{\nu}$ mode If $\mathcal{CP}$ is respected in the above decays then it would follow that K_S and K_L are eigenstates of $\mathcal{CP}$ with eigenvalues +1 and -1 respectively.

With a convenient phase choice

$$\mathcal{CP}|K^0> = -|\overline{K^0}> \quad \text{and } \mathcal{CP}|\overline{K^0}> = -|K^0> \tag{2.86}$$

we define two $\mathcal{CP}$ eigenstates as follows :

$$|K^0_{1,2}> \equiv \frac{1}{\sqrt{2}}[|K^0> \pm|\overline{K^0}>], \; \mathcal{CP} \equiv \mp 1 \tag{2.87}$$

Then $\mathcal{CP}$ invariance would imply that

$$|K_L> \equiv |K^0_1> \quad \text{and } |K_S> \equiv |K^0_2> . \tag{2.88}$$

But it was observed by Cronin et al that $|K_L>$ does decay into the $\pi^+\pi^-$ mode ($\mathcal{CP} \equiv +1$) with a small branching ratio 2×10^{-3}. Hence, the states $|K_L>$ and $|K_S>$ should be a more general superposition of K^0 and $\overline{K^0}$ as

$$|K_{L,S}> \equiv N_{L,S}[|K^0> \pm e^{\imath\xi_{L,S}}|\overline{K^0}>], \tag{2.89}$$

where $\xi_{L,S}$ are complex numbers and $N_{L,S}$ the wavefunction normalization constants.

The mixing and decay of $|K^0>$ and $|\overline{K^0}>$ are governed by an effective Hamiltonian (non-hermitian) $H = H_S + H_{em} + H_{wk} = M - i\Gamma$ where M and Γ are 2×2 hermitian matrices called the mass and decay matrices respectively. In order to study the time evolution of the states we write them as a two-component vector which satisfies the Schrodinger's equation,

$$\imath h\frac{\partial}{\partial t}\begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix} = H\begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix} \equiv (M - \imath\Gamma/2)\begin{pmatrix} K^0 \\ \overline{K^0} \end{pmatrix}. \tag{2.90}$$

The eigenvalues of H are

$$E_{L,S} \equiv m_{L,S} - \imath\gamma_{L,S}/2 = \frac{1}{2}[H_{11} + H_{22} \pm \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}] \tag{2.91}$$

and the difference is given by

$$E_L - E_S = \Delta m - \imath\Delta\gamma/2 = \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}]. \tag{2.92}$$

For $K_{L,S}$ to be the eigenstate of H we must have

$$e^{\imath\xi_L} = \frac{E_L - H_{11}}{H_{12}} \text{ and } e^{\imath\xi_S} = \frac{E_S - H_{22}}{H_{21}}. \tag{2.93}$$

Now we use both $\mathcal{CPT}$ and $\mathcal{CP}$ invariances to relate the elements of H as follows :

$$\begin{aligned}
\mathcal{CPT} : H_{11} &\equiv <K^0|H|K^0> = <K^0|(\mathcal{CPT})^{-1}H(\mathcal{CPT})|K^0> \\
&= <\overline{K^0}|H|\overline{K^0}> \equiv H_{22} \\
&= M_{11} \doteq M_{22}, \; \Gamma_{11} = \Gamma_{22} \\
\mathcal{CP} : H_{12} &\equiv <K^0|H|\overline{K^0}> = <K^0|(\mathcal{CP})^{-1}H(\mathcal{CP})|\overline{K^0}> \\
&= <\overline{K^0}|H|K^0> \equiv H_{21} \\
&= M_{12} = M_{21}, \; \Gamma_{12} = \Gamma_{21}
\end{aligned} \tag{2.94}$$

Using the above relations, we get

$$\begin{aligned} \mathcal{CPT} \cdot \xi_L &= \xi_S = \xi = -\frac{i}{2}\ln(\frac{H_{21}}{H_{12}}) \\ \mathcal{CP} : e^{i\xi} &= 1. \end{aligned} \tag{2.95}$$

However the relation $e^{i\xi} = 1$ depends on the phase convention To see this let us define the phase rotation on Kaon wavefunctions as

$$\begin{aligned} |K^0> \longrightarrow |K^0>' &= e^{i\alpha}|K^0>, \\ |\overline{K^0}> \longrightarrow |\overline{K^0}>' &= e^{-i\alpha}|\overline{K^0}>. \end{aligned} \tag{2.96}$$

Under this rephasing of the Kaon fields the diagonal elements of any operator $\mathcal{O}$ remain unchanged whereas the off-diagonal elements pick up phases

$$\mathcal{O}_{12} \longrightarrow \mathcal{O}'_{12} = e^{-2i\alpha}\mathcal{O}_{12} \text{ and } \mathcal{O}_{21} \longrightarrow \mathcal{O}'_{21} = e^{2i\alpha}\mathcal{O}_{21}, \tag{2.97}$$

and, hence

$$\xi \longrightarrow \xi' = \xi + 2\alpha. \tag{2.98}$$

thus the basis independent condition for $\mathcal{CP}$ invariance is that ξ be real There exists a phase convention dependant parameter ϵ that is often used as a measure of $\mathcal{CP}$ violation as follows :

$$\epsilon \equiv \frac{1 - e^{i\xi}}{1 + e^{i\xi}}. \tag{2.99}$$

Next we consider the decay of neutral Kaons to 2π mode Bose-Einstein statistics demands that the 2π state be in either I=0 or I=2 where I denotes the total isospin. Parametrising the $K^0 \rightarrow 2\pi$ amplitudes as

$$< n|H_{wk}|\overline{K^0}> = \overline{a_n}e^{i\delta_n}; \; n = 0, 2, \tag{2.100}$$

where $|n> = |2\pi; I = n>$ and δ_n is the 2π s-wave phase shift in the I=n channel, we found out that $\mathcal{CPT}$ invariance implies $\overline{a_n} = -a_n^*$ whereas $\mathcal{CP}$ invariance demands that the ratio a_2/a_0 be real Under the phase rotation

$$a_n \longrightarrow a'_n = a_n e^{i\alpha} \tag{2.101}$$

and hence the following combinations are independent of phase choice convention :

$$\begin{aligned} \epsilon_0 &\equiv \frac{< 0|H_{wk}|K_L >}{< 0|H_{wk}|K_S >} = \frac{a_0 - a_0^* e^{i\xi}}{a_0 + a_0^* e^{i\xi}} \\ \epsilon_2 &\equiv \frac{1}{\sqrt{2}}\frac{< 2|H_{wk}|K_L >}{< 0|H_{wk}|K_S >} = \frac{1}{\sqrt{2}}\frac{a_2 - a_2^* e^{i\xi}}{a_0 + a_0^* e^{i\xi}} e^{i(\delta_2 - \delta_0)} \\ \omega &\equiv \frac{< 2|H_{wk}|K_S >}{< 0|H_{wk}|K_S >} = \frac{a_2 - a_2^* e^{i\xi}}{a_0 + a_0^* e^{i\xi}} \end{aligned} \tag{2 102}$$

For the neutral Kaons system the $\mathcal{CP}$ violating quantities, which are directly related to physical observables, are

$$\begin{aligned} \eta^{+-} &\equiv \frac{<\pi^+\pi^-|H_{wk}|K_L>}{<\pi^+\pi^-|H_{wk}|K_S>} = \frac{\epsilon_0+\epsilon_2}{1+\omega/\sqrt{2}} = \epsilon_0 + \frac{\epsilon'}{1+\omega/\sqrt{2}} \\ \eta^{00} &\equiv \frac{<\pi^0\pi^0|H_{wk}|K_L>}{<\pi^0\pi^0|H_{wk}|K_S>} = \frac{\epsilon_0-2\epsilon_2}{1-\sqrt{2}\omega} = \epsilon_0 - \frac{2\epsilon'}{1-\sqrt{2}\omega}. \end{aligned} \quad (2.103)$$

where

$$\epsilon' \equiv \epsilon_2 - \frac{\omega\epsilon_0}{\sqrt{2}}. \quad (2.104)$$

In terms of the matrix elements of M and Γ then

$$\begin{aligned} \epsilon_0 &= i\frac{Im(M_{12}a_0^2) - iIm(\Gamma_{12}a_0^2)}{Re(a_0^2M_{12}) - \frac{1}{2}Re(a_0^2\Gamma_{12}) + \frac{|a_0^2|}{2}(\Delta m - \frac{1}{2}\Delta\gamma)} \\ \epsilon' &= \frac{i}{\sqrt{2}}\frac{Im(a_2a_0^*)(\Delta m - \frac{1}{2}\Delta\gamma)e^{i(\delta_2-\delta_0)}}{Re(a_0^2M_{12}) - \frac{1}{2}Re(a_0^2\Gamma_{12}) + \frac{|a_0^2|}{2}(\Delta m - \frac{1}{2}\Delta\gamma)}. \end{aligned} \quad (2.105)$$

Now we will simplify these general expessions to the special case of neutral Kaons and use some experimental results to obtain the approximate but easy to handle expressions. Experimentally we have

$$\begin{aligned} m_K &= 0.498\ GeV \\ \Delta m_K &= 3.5\times 10^{-15}\ GeV \\ \Delta\gamma_K &\approx -\gamma_{K_S} = -7.3\times 10^{-15}\ GeV \\ |\eta^{+-}| &= (2.275\pm 0.021)\times 10^{-3} \\ |\eta^{00}| &= (2.299\pm 0.036)\times 10^{-3}. \end{aligned} \quad (2.106)$$

The dominant contribution to Γ_{12} comes from the 2π states and more specifically the I=0 state. Thus

$$\Gamma_{12} \sim < K^0|H_{wk}^{\Delta S=1}|0><0|H_{wk}^{\Delta S=1}|\overline{K^0}>, \quad (2.107)$$

and hence

$$\frac{Im\Gamma_{12}}{Re\Gamma_{12}} \approx \frac{Im(a_0^*)^2}{Re(a_0^*)^2} \quad (2.108)$$

The $\Delta I = \frac{1}{2}$ rule for neutral Kaon decays manifests itself through a small suppression factor

$$\omega \approx 0.045 \quad (2.109)$$

Using the value of ω along with the experimental numbers for η^{+-} and η^{00} we get

$$|\epsilon_0| = 2.3\times 10^{-3}, \quad (2.110)$$

and the phase of ϵ_0 is approximately $\pi/2$ this small value of $|\epsilon_0|$ gives the inequalities

$$\begin{aligned} ImM_{12}/ReM_{12} &\ll 1, \\ Im\Gamma_{12}/Re\Gamma_{12} &\ll 1. \end{aligned} \quad (2.111)$$

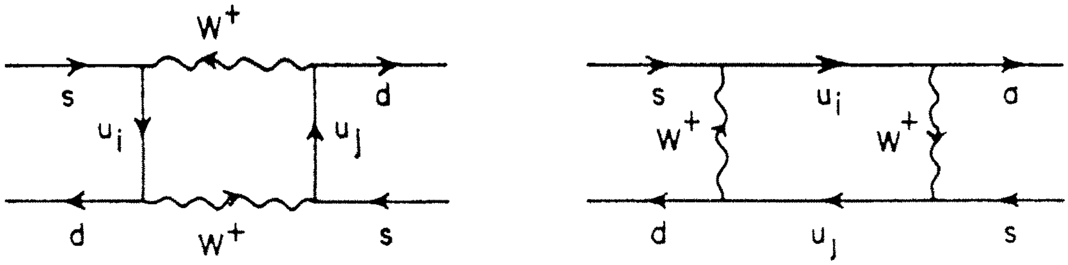

Figure 2.1: "Box"-diagiam generating $K_0 - \overline{K^0}$ mixing and ϵ_K in the SM

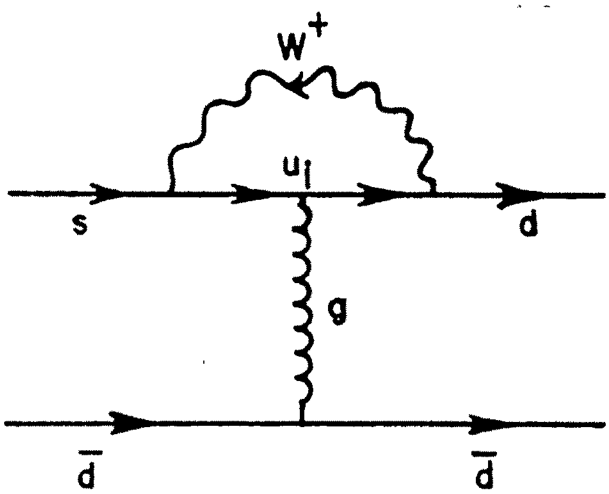

Figure 2.2: "Penguin"-diagram iesponsible for ϵ'_K in the SM

Consequently, the mass and width differences in the above approximations are given as

$$\Delta m_K \approx 2ReM_{12}, \ \ \Delta\gamma_K \approx 2Re\Gamma_{12}, \tag{2.112}$$

and

$$|\epsilon_0| = \frac{1}{\sqrt{2}}\frac{ImM_{12}}{\Delta M_K} \approx \frac{1}{2\sqrt{2}}\frac{ImM_{12}}{ReM_{12}}. \tag{2.113}$$

In the 3-generation SM, which, for a complex CKM matrix, is a milliweak theory, $K_0 - \overline{K^0}$ mixing and $K_L \to 2\pi$ come about because of the 1-loop Feynman diagrams in Figures (2.1) and (2.2) respectively, giving rise to

$$ImM_{12} = \frac{G_F^2}{12\pi^2} f_K^2 m_K m_W^2 B_K \left[\lambda_c^2 \eta_1 S(y_c) + \lambda_t^2 \eta_2 S(y_t) + \lambda_c \lambda_t \eta_3 S(y_c, y_t)\right], \tag{2.114}$$

and

$$\tan\theta_0 = \frac{s_{13}s_{23}}{s_{12}} \sin\delta \left[\frac{150MeV}{m_s(1\ GeV)}\right]^2 \bar{H}, \tag{2.115}$$

where

$$\begin{array}{rclrcl} \lambda_t & \equiv & K_{td}^* K_{ts} & y_t & \equiv & m_t^2/m_W^2 \\ f_K & = & 0.16\,GeV & m_W & = & 81.8\,GeV. \end{array} \tag{2.116}$$

Whereas f_K is the pion decay constant, the bag parameter B_K reflects our ignorance of the hadronic matrix elements. If vacuum saturation approximation were correct then one would have $B_K = 1$, but theoretical estimates only put the rather loose bound of $1/3 \leq B_K \leq 1$. The functions $S(x)$ and $S(x, y)$ arise from the loop integral and are given by

$$\begin{array}{rcl} S(x) & = & x\left[\frac{1}{4} + \frac{9}{4(1-x)} - \frac{3}{2(1-x)^2}\right] + \frac{3}{2}\left[\frac{x}{x-1}\right]^3 \ln x \\ S(x,y) & = & xy\left[\left\{\frac{1}{4} + \frac{3}{4(1-x)} - \frac{3}{4(1-x)^2}\right\}\frac{\ln x}{x-y} - \frac{3}{8}\frac{1}{1-x}\frac{1}{1-y}\right] + (x \leftrightarrow y) \end{array}$$

The quantities η_i represent QCD corrections [25]. While η_1 does not depend on m_t and is evaluated to be 0.85, η_2 is essentially independent of m_t for $40\ GeV \leq m_t^{\rm phys} \leq 130\ GeV$ and $\eta_2 = 0.61$. η_3 and $\bar{H}$ are slowly varying functions of m_t and are approximately 0.25 and 0.37 respectively [26] However we shall allow for their full variation in our calculations.

2.6.2 The Neutral Beauty meson system

Although $\mathcal{CP}$ violation has been observed so far only in neutral Kaon decays, one would expect to have non-zero effects in other processes onvolving heavy neutral mesons like $B^0 - \overline{B^0}$ and $D^0 - \overline{D^0}$ if one believes the KM mechanism for $\mathcal{CP}$ violation is correct. The phenomenology of the $B^0 - \overline{B^0}$ systems is quite similar to that of $K^0 - \overline{K^0}$. The physical situation, however, is very different since B^0 involves the bound states of a heavy quark a light quark and there are many intermediate states and the multi-particle final states dominate the decay as the case in deep inelastic scattering.

A new property which makes the B system very interesting is the recent observation of $B_d - \overline{B_d}$ mixing by the ARGUS collaboration. Their result fully justifies the expectation that the studies regarding neutral beauty mesons can reveal new phenomena and motivates the serious consideration of $\mathcal{CP}$ assymetries in this system. To study this particle-antiparticle mixing consider the time-integrated mixing parameters proposed by Pais and Treiman [6.4 p]

$$\begin{array}{rcl} r_d & \equiv & \dfrac{\int_0^\infty |<\overline{B_d}|B_d>|^2 dt}{\int_0^\infty |<B_d|B_d>|^2 dt} = |e^{i\xi_{B_d}}|^2 \dfrac{(\Delta m_B)^2 + (\Delta\Gamma_B)^2/4}{2\Gamma_B^2 + (\Delta m_B)^2 + (\Delta\Gamma_B)^2/4} \\ \overline{r_d} & \equiv & \dfrac{\int_0^\infty |<B_d|\overline{B_d}>|^2 dt}{\int_0^\infty |<\overline{B_d}|\overline{B_d}>|^2 dt} = |e^{i\xi_{\overline{B_d}}}|^2 \dfrac{(\Delta m_{\overline{B_d}})^2 + (\Delta\Gamma_{\overline{B_d}})^2/4}{2\Gamma_{\overline{B_d}}^2 + (\Delta m_{\overline{B_d}})^2 + (\Delta\Gamma_{\overline{B_d}})^2/4}. \end{array} \tag{2.117}$$

If $\mathcal{CP}$ is violated we expect r to differ from $\overline{r}$ by a quantity proportional to

$$|e^{i\xi_{\overline{B_d}}}|^2 - |e^{i\xi_{B_d}}|^2 \approx 8 Re\epsilon_B, \quad |\epsilon_B| \ll 1, \tag{2.118}$$

otherwise $r = \overline{r}$. The above considerations are relevant for reactions where only one B^0 or $\overline{B_0}$ meson is produced. However, often in the actual experimental situation a pair of B^0 and $\overline{B_0}$ is produced instead of a single B^0 or $\overline{B_0}$ As the beam evolves in time, both of them oscillate in their B^0 and $\overline{B_0}$ content and one cannot directly measure either r_d or $\overline{r_d}$.

Okun, Zakharov and Pontecorvo proposed the observations of following two parameters in dilepton decay mode, characterising particle-antiparticle mixing

$$R_d = \frac{N^{++} + N^{--}}{N^{+-} + N^{-+}} \tag{2.119}$$

and the $\mathcal{CP}$ violating leptonic charge assymetry

$$A_d = \frac{N^{++} - N^{--}}{N^{++} + N^{+-} + N^{-+} + N^{--}} \tag{2.120}$$

where N's denote the number of dilepton pairs with the associated charges. For example, in the process

$$e^+e^- \longrightarrow \Upsilon(4S) \longrightarrow B_d^0\overline{B_d} \tag{2.121}$$

these relations reduce to

$$R_d = \frac{1}{2}(r_d + \overline{r_d}), \quad A_d = \frac{r_d - \overline{r_d}}{2 + r_d + \overline{r_d}} \tag{2.122}$$

Neglecting the possibility of large $\mathcal{CP}$ violation and introducing the approximation $\Delta\Gamma/\Delta m \approx 0$ we have

$$r_d = \frac{(\Delta m_d/\Gamma_d)^2}{2 + (\Delta m_d/\Gamma_d)^2} \tag{2.123}$$

If one assumes that for 3 generations SM with a relatively heavy top quark, the dominant contribution to r_d comes from the corresponding box-diagram with the top flowing in it then

$$x_d = (\Delta m_d/\Gamma_d) = \tau_b \frac{G_F^2}{6\pi^2}\eta M_B \left(B_B f_B^2\right) M_W^2 y_t f_2(y_t)|V_{tb}V_{td}^*|^2 \tag{2.124}$$

where τ_B is the B_d^0 lifetime, f_B the decay constant, B_B the bag factor and η a QCD correction factor

The ARGUS result permits the range

$$\begin{aligned} \Delta m_B &= (4.2 \pm 0.9) \times 10^{-13}\text{GeV} \\ \text{or } x_d &= 0.73 \pm 0\ 18 \\ \text{and } m_B &= 5.28GeV, \quad B_B f_b^2 = (0.15 \pm 0.05GeV)^2 \\ \eta &= 0.85 \quad \tau_B = (1.16 \pm 0.16) \times 10^{-12}s \end{aligned} \tag{2.125}$$