# Appendix A

# Unitary decomposition for a one plus two-body Hamiltonian for spinless fermions

Let us consider a system of $m$ fermions in $N$ sp states with a (1+2)-body Hamiltonian $H = h(1) + V(2)$ where $h(1) = \sum_i \epsilon_i \hat{n}_i$ and $V(2)$ is defined by the two-body matrix elements $V_{ijkl} = \langle kl \mid V(2) \mid ij \rangle$. With respect to the $U(N)$ group, the two-body interaction $V(2)$ can be separated into scalar ($\nu = 0$), effective one-body ($\nu = 1$) and irreducible two-body ($\nu = 2$) parts. Then, we have [Ch-71, Ko-01a],

$$
\begin{array}{l}
V^{\nu=0} = \dfrac{\hat{n}(\hat{n}-1)}{2} \overline{V} \; ; \;\; \overline{V} = \dbinom{N}{2}^{-1} \displaystyle\sum_{i<j} V_{ijij} \, , \\[3ex]
V^{\nu=1} = \dfrac{\hat{n}-1}{N-2} \displaystyle\sum_{i,j} \zeta_{i,j} a_i^\dagger a_j \; ; \; \zeta_{i,j} = \left[ \sum_k V_{kikj} \right] - \left[ (N)^{-1} \sum_{r,s} V_{rsrs} \right] \delta_{i,j} \, , \\[3ex]
V^{\nu=2} = V - V^{\nu=0} - V^{\nu=1} \iff V^{\nu=2}_{ijkl} \; ; \\[2ex]
V^{\nu=2}_{ijij} = V_{ijij} - \overline{V} - (N-2)^{-1} \left( \zeta_{i,i} + \zeta_{j,j} \right) \, , \\[2ex]
V^{\nu=2}_{ijik} = V_{ijik} - (N-2)^{-1} \zeta_{j,k} \;\; \text{for } j \neq k \, , \\[2ex]
V^{\nu=2}_{ijkl} = V_{ijkl} \;\; \text{for all other cases} \, .
\end{array}
\tag{A1}
$$

Similar to Eq. (A1), the $h(1)$ operator will have $\nu = 0, 1$ parts,

$$
\begin{aligned}
h^{\nu=0} &= \overline{\epsilon}\,\hat{n}\,, \;\; \overline{\epsilon} = (N)^{-1} \sum_i \epsilon_i\,, \\
h^{\nu=1} &= \sum_i \epsilon_i^1 \hat{n}_i\;, \;\; \epsilon_i^1 = \epsilon_i - \overline{\epsilon}\,.
\end{aligned} \tag{A2}
$$

Then the propagation equations for the $m$-particle centroids and variances are,

$$
\begin{aligned}
E_c(m) &= \langle H \rangle^m = m\overline{\epsilon} + \binom{m}{2} \overline{V}\,, \\
\sigma^2(m) &= \left\langle H^2 \right\rangle^m - [E_c(m)]^2 \\
&= \frac{m(N-m)}{N(N-1)} \sum_{i,j} \left\{ \epsilon_i^1 \delta_{i,j} + \frac{m-1}{N-2} \zeta_{i,j} \right\}^2 \\
&+ \frac{m(m-1)(N-m)(N-m-1)}{N(N-1)(N-2)(N-3)} \left\langle\left\langle \left(V^{\nu=2}\right)^2 \right\rangle\right\rangle^2 .
\end{aligned} \tag{A3}
$$