Appendix C

EGUE(2)-s ensemble

For m fermions occupying Ω number of sp orbitals each with spin $\mathbf{s} = \frac{1}{2}$ so that the number of sp states $N = 2\Omega$, we consider Hamiltonians that preserve total m-particle spin S. Then the m-particle states can be classified according to $U(N) \supset U(\Omega) \otimes SU(2)$ algebra with $SU(2)$ generating spin S. The $U(\Omega)$ irrep that corresponds to spin S is $f_m = \{2^p, 1^q\}$ where $m = 2p + q$ and $S = q/2$ and therefore the m-particle states are denoted by $\left|f_m\, v_m M\right\rangle$; v_m are the additional quantum numbers that belong to f_m and M is the S_z quantum number. With this, a general two-body Hamiltonian operator preserving spin S can be written as,

$$\widehat{H} = \sum_{f_2, v_2^i, v_2^f, m_2} V_{f_2 v_2^i v_2^f}(2)\, A^\dagger(f_2\, v_2^f\, m_2)\, A(f_2\, v_2^i m_2)\,. \tag{C1}$$

Here, $A^\dagger(f_2\, v_2 m_2)$ and $A(f_2\, v_2 m_2)$ denote creation and annihilation operators for the normalized two-particle states and $V_{f_2 v_2^i v_2^f}(2) = \left\langle f_2\, v_2^f\, s m_2 \mid \widehat{H} \mid f_2\, v_2^i\, s m_2 \right\rangle$ independent of the m_2's. Note that the two-particle spin $s = 0$ and 1 and the corresponding $U(\Omega)$ irreps f_2 are $\{2\}$ (symmetric) and $\{1^2\}$ (antisymmetric), respectively. The EGUE(2)-**s** ensemble for a given (m, S) is generated by the action of $\widehat{H}$ on m-particle basis space with a GUE representation for the H matrix in two-particle spaces. Then, the two-particle matrix elements $V_{f_2 v_2^i v_2^f}(2)$ are independent Gaussian variables with zero center and variance given by,

$$\overline{V_{f_2 v_2^1 v_2^2}(2)\, V_{f_2' v_2^3 v_2^4}(2)} = \lambda_{f_2}^2\, \delta_{f_2 f_2'}\, \delta_{v_2^1 v_2^4}\, \delta_{v_2^2 v_2^3}\,. \tag{C2}$$

Thus $V(2)$ is a direct sum of GUE matrices for $s=0$ and $s=1$. Just as for EGUE(2), tensorial decomposition of $\widehat{H}$ with respect to $U(\Omega)\otimes SU(2)$ algebra gives analytical results for the spin ensemble. As $\widehat{H}$ preserves S, it is a scalar in spin $SU(2)$ space. However with respect to $SU(\Omega)$, the tensorial characters for $f_2=\{2\}$ are $\boldsymbol{F}_\nu=\{0\}$, $\{21^{\Omega-2}\}$ and $\{42^{\Omega-2}\}$. Similarly for $f_2=\{1^2\}$ they are $\{0\}$, $\{21^{\Omega-2}\}$ and $\{2^21^{\Omega-4}\}$. Here the unitary tensors B's are

$$
\begin{aligned}
&B(f_2\boldsymbol{F}_\nu\omega_\nu)\\
&=\sum_{v_2^i,v_2^f,m_2}\left\langle f_2v_2^f\,\overline{f_2}\,\overline{v_2^i}\mid \boldsymbol{F}_\nu\omega_\nu\right\rangle\left\langle sm_2\,\overline{s}\,\overline{m_2}\mid 00\right\rangle A^\dagger(f_2v_2^fm_2)\,A(f_2v_2^im_2)\,.
\end{aligned}
\tag{C3}
$$

In Eq. (C3), $\langle f_2---\rangle$ are $SU(\Omega)$ Wigner coefficients and $\langle s--\rangle$ are $SU(2)$ Wigner coefficients. Then we have $\widehat{H}(2)=\sum_{f_2,\boldsymbol{F}_\nu,\omega_\nu}W(f_2\boldsymbol{F}_\nu\omega_\nu)B(f_2\boldsymbol{F}_\nu\omega_\nu)$. The expansion coefficients W's are also independent Gaussian random variables, just as V's, with zero center and variance given by

$$
\overline{W(f_2\boldsymbol{F}_\nu\omega_\nu)W(f_2'\boldsymbol{F}_\nu'\omega_\nu')}=\delta_{f_2f_2'}\delta_{\boldsymbol{F}_\nu\boldsymbol{F}_\nu'}\delta_{\omega_\nu\omega_\nu'}(\lambda_{f_2})^2(2s+1)\,.
$$

The m-particle H matrix will be a direct sum matrix with the diagonal blocks represented by f_m. Then $H(m)=\sum_{f_m}H_{f_m}(m)\oplus$ and the EGUE(2)-**s** is generated for each $H_{f_m}(m)$.

Using Wigner-Eckart theorem, the matrix elements of B's in f_m space can be decomposed as,

$$
\begin{aligned}
&\left\langle f_mv_m^fM\mid B(f_2\boldsymbol{F}_\nu\omega_\nu)\mid f_mv_m^iM\right\rangle\\
&=\sum_\rho\left\langle f_m\mid\mid B(f_2\boldsymbol{F}_\nu)\mid\mid f_m\right\rangle_\rho\left\langle f_mv_m^i\boldsymbol{F}_\nu\omega_\nu\mid f_mv_m^f\right\rangle_\rho\,,
\end{aligned}
\tag{C4}
$$

where the summation is over the multiplicity index ρ and this arises as $f_m\otimes\boldsymbol{F}_\nu$ gives in general more than once the irrep f_m. Applying Eq. (C4) and the expansion of $\widehat{H}$ in terms of B's, exact analytical formulas are derived for the ensemble averaged spectral variances, cross-correlations in energy centroids and also for the cross-correlations in spectral variances. In addition, the ensemble averaged excess parameter for the fixed-(m,S) density of states is given in terms of $SU(\Omega)$ Racah coefficients [Ko-07]. For finite m and $\Omega\to\infty$, some important results are: (i) the ensemble averaged variances, to the leading order, just as for the spinless fermion systems [Be-01a], are same for both EGOE(2)-**s** and EGUE(2)-**s** and this is inferred from the exact analytical formulas available for both the ensembles [comparing Eq. (2.3.11) with Eq. (19) of [Ko-07]]; (ii) similarly it is seen that the cross-correlations in energy centroids for EGOE(2)-**s** are twice that of EGUE(2)-**s** to the leading order [Ko-06a, Be-01a] [as an aside, let us point out that Eqs. (2.3.4) and (2.3.11) give for EGOE(2)-**s**, the exact formula for the normalized cross-correlations in the energy centroids]; (iii) combining (ii) with the exact analytical results for spinless fermion EGUE(2)-**s** (see Sec. 1.2.3 for details), it is conjectured that the covariances in spectral variances for EGOE(2)-**s** are twice that of EGUE(2)-**s** to the leading order [note that for EGUE(2)-**s** an analytical result is available but not for EGOE(2)-**s**]; and (iv) combining the analytical results for the excess parameter for EGOE(2) and EGOE(2)-**s** (see Appendix H and Sec. 2.9 for details), it is expected that the density of eigenvalues will be Gaussian for EGUE(2)-**s**.