Appendix E

Some properties of $SU(\Omega)$ Wigner coefficients

Some properties of the $SU(\Omega)$ Wigner coefficients used in Chapter 4 are given here and they are similar to those used for EGUE(2) and EGUE(2)-**s** in [Ko-05, Ko-07] and discussed in detail in [Bu-81]. Firstly (dropping the multiplicity index ρ everywhere for simplicity),

$$\left\langle f_a\, v_a\, f_b\, v_b \mid f_{ab}\, v_{ab} \right\rangle = (-1)^{\phi(f_a, f_b, f_{ab})} \left\langle f_b\, v_b\, f_a\, v_a \mid f_{ab}\, v_{ab} \right\rangle , \tag{E1}$$

where ϕ is a function of (f_a, f_b, f_{ab}) that defines the phase for $a \to b$ interchange in the Wigner coefficient. With $\left|\overline{f_a}\,\overline{v_a}\right\rangle$ denoting the time-reversal partner (complex conjugate) of $\left|f_a\, v_a\right\rangle$, we have

$$\left\langle f_a\, v_a\, f_b\, v_b \mid f_{ab}\, v_{ab} \right\rangle = \left\langle \overline{f_a}\,\overline{v_a}\,\overline{f_b}\,\overline{v_b} \mid \overline{f_{ab}}\,\overline{v_{ab}} \right\rangle . \tag{E2}$$

Similarly,

$$\left\langle f_a\, v_a\, f_b\, v_b \mid f_{ab}\, v_{ab} \right\rangle = (-1)^{\phi(f_a, f_b, f_{ab})} \sqrt{\frac{d_\Omega(f_{ab})}{d_\Omega(f_a)}} \left\langle f_{ab}\, v_{ab}\, \overline{f_b}\,\overline{v_b} \mid f_a\, v_a \right\rangle . \tag{E3}$$

In addition we also have,

$$\left\langle f_a\, v_a\, \overline{f_a}\,\overline{v_a} \mid \{0\}\, 0 \right\rangle = \frac{1}{\sqrt{d_\Omega(f_a)}} , \tag{E4}$$

$$\left[\left\langle f_a\, v_a \overline{f_a}\,\overline{v_b} \mid f_{ab}\, v_{ab}\right\rangle\right]^* = \left\langle f_a\, v_b \overline{f_a}\,\overline{v_a} \mid f_{ab}\, v_{ab}\right\rangle . \tag{E5}$$

Orthonormal properties of the Wigner coefficients are,

$$\sum_{v_a, v_b} \langle f_a\, v_a\, f_b\, v_b \mid f_{ab}\, v_{ab}\rangle \left[\langle f_a\, v_a\, f_b\, v_b \mid f_{cd}\, v_{cd}\rangle\right]^* = \delta_{f_{ab}, f_{cd}}\, \delta_{v_{ab}, v_{cd}} , \tag{E6a}$$

$$\sum_{f_{ab}, v_{ab}} \langle f_a\, v_a\, f_b\, v_b \mid f_{ab}\, v_{ab}\rangle \langle f_a\, v_c\, f_b\, v_d \mid f_{ab}\, v_{ab}\rangle = \delta_{v_a, v_c}\, \delta_{v_b, v_d} . \tag{E6b}$$

Finally,

$$\begin{aligned} &\sum_{v_{ab}} \langle f_a v_a f_b v_b \mid f_{ab} v_{ab}\rangle \langle f_{ab} v_{ab} f_c v_c \mid f\, v\rangle \\ &= \sum_{f_{bc}, v_{bc}} \langle f_b v_b f_c v_c \mid f_{bc} v_{bc}\rangle \langle f_a v_a f_{bc} v_{bc} \mid f\, v\rangle\, U(f_a f_b f f_c; f_{ab} f_{bc}) . \end{aligned} \tag{E7}$$