Appendix C

Here we shall show that if we start with a scalar band averaged density ρ_0 , only the zeroth multipole $(\rho_p \rho_n)_0$ of the product $\rho_p \rho_n$ contributes in the derivative term of the HF equations. We saw in appendix B that the definition of spherical density ρ_0 given in IV(8) lead **us** to the evaluation of the quantity

$$J = \frac{1}{2j_\alpha+1} \sum_{m_\alpha} \langle n_\alpha l_\alpha j_\alpha m_\alpha | \rho_p \rho_n | n_\beta l_\alpha j_\alpha m_\alpha \rangle$$

C(1)

Consider L^{th} multipole of the product $\rho_p \rho_n$

Then ,

$$J = \frac{1}{2j_\alpha+1} \sum_{m_\alpha} \langle n_\alpha l_\alpha j_\alpha m_\alpha | (\rho_p \rho_n)^L_M | n_\beta l_\alpha j_\alpha m_\alpha \rangle$$

$$= \frac{1}{2j_\alpha+1} \sum_{m_\alpha} C\begin{pmatrix} j_\alpha & L & j_\alpha \\ m_\alpha & M & m_\alpha \end{pmatrix}$$

$$\cdot \langle n_\alpha l_\alpha j_\alpha || (\rho_p \rho_n)^L || n_\beta l_\alpha j_\alpha \rangle \qquad C(2)$$

Multiplying by $C\begin{pmatrix} \partial_\alpha & 0 & \partial_\alpha \\ m_\alpha & 0 & m_\alpha \end{pmatrix} = 1$ and using the proper symmetry relations for the Clebsch-Gordon coefficients, one gets

$$J = \sum_{m_\alpha} \frac{1}{\sqrt{2L+1}} \, C\begin{pmatrix} \partial_\alpha & \partial_\alpha & L \\ m_\alpha & -m_\alpha & 0 \end{pmatrix} C\begin{pmatrix} \partial_\alpha & \partial_\alpha & 0 \\ m_\alpha & -m_\alpha & 0 \end{pmatrix}$$

$$\cdot \langle n_\alpha l_\alpha \partial_\alpha \| (\rho_p \rho_n)^L \| n_\beta l_\alpha \partial_\alpha \rangle$$

$$= \frac{1}{\sqrt{2L+1}} \, \delta_{L,0} \, \langle n_\alpha l_\alpha \partial_\alpha \| (\rho_p \rho_n)^L \| n_\beta l_\alpha \partial_\alpha \rangle \tag{C(3)}$$

Thus only the zeroth multipole $(\rho_p \rho_n)_0$ of the product $\rho_p \rho_n$ contributes if we start with a spherical density ρ_0.