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NUCLEAR INTERACTIONS IN THE SHELL MODEL

S. P. PANDYA and S. K. SHAH
Physical Research Laboratory, Ahmedabad 9, India †

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Abstract: A central two-body interaction that fits the energy levels of the $(d_{\frac{5}{2}})(d_{\frac{5}{2}})$ and $(d_{\frac{5}{2}})(d_{\frac{3}{2}})^{-1}$ configurations observed in K^{38} and Cl^{36} is deduced. It is shown that this does not explain the very small splittings of the $s_{\frac{1}{2}}$-particle doublets in P^{30}, P^{32} etc.

1. Introduction

During the last decade the nuclear shell model has enjoyed many remarkable successes. In recent years the work of Brueckner and others has also provided it with a strong qualitative justification for many of its basic assumptions. However, even the phenomenology of the shell model can hardly be regarded as being in a satisfactory state yet. For example, the independent particle wavefunctions are in all calculations approximated by the harmonic oscillator wavefunctions for obvious reasons of simplicity, and we have no rigorous estimate of how far the quantitative results would change if some other type of independent particle wavefunctions were to be used for the calculations. Again our knowledge of the effective perturbation interaction introduced to split up the degeneracy of many states of the same configuration is hardly satisfactory. Different authors have chosen different interactions for calculations for various groups of nuclei. The interactions chosen are almost always velocity-independent, central, and smooth (no hard-cores or singularities) two-body forces, with several arbitrary parameters. The only justification for a given choice of parameters is generally that it fits the problem in hand adequately.

For p-shell nuclei, Rosenfeld or Inglis interaction appears to give quite good results. Bilaniuk and French [1]) have shown however that these interactions cannot explain the observed very small s-particle doublet splittings in light nuclei. Moreover an analysis of the experimental data on deuteron stripping reactions on Calcium isotopes led French and Raz [2]) to pronounce the Rosenfeld interaction as inadequate for explaining these data. On the other hand recently Komoda [3]) has found the Rosenfeld interaction to be quite satisfactory for explaining the pairing energies and the magnetic moments of Ca isotopes.

Thieberger [4]) has determined the parameters of the central two-body

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interaction H_{12} from study of the ground state binding energies of a large number of nuclei, $A = 12$–60. While the justification for the use of pure spherical jj coupling wavefunctions for all such nuclei is itself in some doubt, it is also clear that the procedure adopted gives an average interaction, wiping out the configuration-dependence of H_{12} if any exists, and the results perhaps show only the insensitivity of the binding energy calculations to the choice of the wavefunctions and H_{12}. Arima [5]) has made similar calculations for the $2s_{\frac{1}{2}}$ and $1d_{\frac{3}{2}}$ shells, also with satisfactory results, and has derived the parameters of the two-body interaction consisting of central, tensor and spin-orbit forces. While Thieberger obtains a Serber-type two-body interaction, Arima does not.

Although very little is known of the three-body or many-body effective interactions in nuclei, it seems that they may conveniently be ignored for the purpose of the shell model calculations [6]). However, regarding the importance of the role of velocity-dependence of the interactions or of non-central spin-dependent forces, our knowledge is far from satisfactory. It is not unreasonable to expect that such residual effective nuclear interactions in the shell model may be configuration dependent. Of this aspect too there exists no systematic investigation.

In this paper we make an attempt to derive somewhat differently information on the nature of the effective two-body nuclear interaction (assumed central and velocity-independent) operative in the region of nuclei with $A = 30$–40. Our emphasis is on relative separations of the whole group of energy levels of the given configurations, since the interaction is primarily introduced to explain these. The energy levels of the odd nuclei in the $d_{\frac{3}{2}}$ shell are first analysed in section 2 with the help of some of the hole-particle theorems established earlier by one of the authors [7]). This provides us with a simple central interaction which correctly accounts for the splitting of the energy levels of the $(d_{\frac{3}{2}})^2$ configuration. It is well known that the states of a neutron-proton configuration $(s_{\frac{1}{2}})(l_j)$ form a close doublet, such as observed in nuclei Al^{28}, P^{30}, P^{32}, etc. Since the splitting of these doublet states depends only upon the strength of the spin-dependent part of the neutron-proton interaction, an analysis of such splittings would provide us with information on the spin-dependence of the effective nuclear forces. This is next done in section 3, and in section 4 the results thus obtained are compared with the results obtained in section 2.

2. Odd Nuclei in the $d_{\frac{3}{2}}$ Shell

In this section we shall analyse the low-lying energy levels of the odd nuclei Cl^{34}, Cl^{36} and K^{38}, which may be described in the jj coupling scheme in terms of the configurations $(d_{\frac{3}{2}})$ $(d_{\frac{3}{2}})$, $(d_{\frac{3}{2}})(d_{\frac{3}{2}})^{-1}$ and $(d_{\frac{3}{2}})^{-1}$ $(d_{\frac{3}{2}})^{-1}$ respectively. We shall assume also harmonic oscillator type radial dependence of the particle wavefunctions. A brief remark was made regarding the level schemes of Cl^{34}

and Cl^{36} by one of the authors earlier [7]). Now we shall discuss these nuclei in somewhat more detail in view of the additional available experimental data.

The known levels of Cl^{34} are $J = 0^+$ (ground state), 3^+ (0.145 MeV), and levels of unknown spin-parity at 1.1 MeV, 1.9 MeV, 2.7 MeV and above [8]). The earlier analysis [7]) predicted the identification of the 1.1 and 1.9 MeV levels with the 1^+ and 2^+ levels of the $(d_{\frac{3}{2}})(d_{\frac{3}{2}})$ configuration, unless there remains an undiscovered level in Cl^{34} below 2.5 MeV. If the *jj* coupling scheme were exactly valid in these nuclei, we would expect the level schemes of Cl^{34} and K^{38} to be identical. However, the energy levels known in K^{38} are 3^+ (ground state), 0^+ (0.123 MeV) and a level of unknown spin-parity (but very probably $T = 0$) at 0.44 MeV [9]). It is generally agreed that the *jj* coupling scheme has a better validity near the end of the subshell than at the beginning, the excellent validity of the *jj* coupling has been demonstrated for the low levels of Cl^{38} and K^{40}, and whereas the level spectrum of S^{33} (or Cl^{33}) shows many low levels (three levels below 2.5 MeV), no level below 2.5 MeV is seen in K^{39}. These considerations suggest that the energy levels of the $(d_{\frac{3}{2}})(d_{\frac{3}{2}})$ configuration are seen to a better approximation in K^{38} rather than in Cl^{34}. Although $J = 2^+$ level is not seen here, the 0^+—2^+ separation can be taken as 2.15 MeV in view of the observed 2^+ state in A^{38} at this excitation energy. Finally, the observed $T = 0$ level at 0.44 MeV in K^{38} may be identified as remaining $J = 1^+$ level of the $(d_{\frac{3}{2}})(d_{\frac{3}{2}})$ configuration.

We now apply eq. (1) of ref. [7]) to evaluate the energy levels of the configuration $(d_{\frac{3}{2}})(d_{\frac{3}{2}})^{-1}$ expected to be seen in Cl^{36}. The result gives $J = 2^+$ (ground state), 3^+ (0.93 MeV), 1^+ (1.47 MeV) and 0^+ (4.23 MeV). The experimental data on Cl^{36} have been discussed by Segel [10]) and Endt and Braams [8]). Allowing for the configuration mixing effects (i.e. departure from exact *jj* coupling) which may displace the pure configuration states by ≈ 0.1 MeV, it is easy to identify the calculated states $J = 2^+$, 3^+ and 1^+ with the observed ground state, 0.79 MeV, and 1.60 MeV states. The remaining $J = 0$ state is quite high and is difficult to identify. The results of the earlier analysis and the assignments of Endt and Braams are in agreement with these results. Further explicit measurements of spins and parities in these nuclei will enable us to refine the analysis and also estimate the amount of departure from strict *jj* coupling scheme.

The result of the above arguments is that we have been able to interpret the experimental data to obtain the relative energy splittings of the levels of the neutron-proton configurations $(d_{\frac{3}{2}})(d_{\frac{3}{2}})$ and $(d_{\frac{3}{2}})(d_{\frac{3}{2}})^{-1}$ with errors of at most $\approx 10\ \%$.

We write the two-body neutron-proton interaction as

$$H_{12} = [A_0 + A_1 M + A_2 B + A_3 MB] J(r_{12}), \tag{1}$$

where M, B are the space- and spin-exchange operators, and A_k are the constants. The function $J(r_{12})$ is for convenience chosen to be of the Gaussian type,

$$J(r_{12}) = \exp[-(|\mathbf{r}_1 - \mathbf{r}_2|/r_0)^2]. \tag{2}$$

For the configuration of equivalent nucleons $(d_{\frac{3}{2}})(d_{\frac{3}{2}})$ we have in the states $J = 0, 2, MB = -1$ and in the states $J = 1, 3, MB = +1$. This enables us to write the interactions for these states in a simpler form:

$$H_{12} = (a_{\pm}+b_{\pm}\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)J(r_{12}) \quad \text{for} \quad MB = \mp 1, \tag{3}$$

$$a_{\pm} = A_0 \mp A_3 + \tfrac{1}{2}(A_2 \mp A_1), \qquad b_{\pm} = \tfrac{1}{2}(A_2 \mp A_1). \tag{4}$$

Matrix elements of H_{12} can be easily evaluated for various states, and we write the energy of a state with spin J as

$$E_J = a_{\pm}E_J^{(a)} + b_{\pm}E_J^{(b)}. \tag{5}$$

Table 1 lists the numerical values of $E_J^{(a)}$ and $E_J^{(b)}$ for $\lambda = r_0/r_d = 0.8$, and 1.0 where r_d is the parameter in the radial wavefunction $\exp[-(r/r_d)^2]$.

TABLE 1

The coefficients $E_J^{(a)}$ and $E_J^{(b)}$ for various values of J and λ

	J	0	1	2	3
$\lambda = 0.8$	$E_J^{(a)}$	0.1248	0.0946	0 0645	0 0946
	$E_J^{(b)}$	−0.1372	−0 0477	−0 0274	0.0597
$\lambda = 1.0$	$E_J^{(a)}$	0 1877	0.1542	0.1207	0 1542
	$E_J^{(b)}$	−0.1653	−0.0846	−0 0330	0.0834

It is well known [7, 11]) that there exists a simple relationship between the matrix elements of the Wigner and $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$ interactions for the states of neutron-proton configurations $(j_1)(j_2)$ and $(j_1)^{-1}(j_2)$, viz.,

$$\begin{aligned} E_J^{(a)}[(j_1)(j_2)] &= -E_J^{(a)}[(j_1)^{-1}(j_2)], \\ E_J^{(b)}[(j_1)(j_2)] &= \quad E_J^{(b)}[(j_1)^{-1}(j_2)]. \end{aligned} \tag{6}$$

Therefore, by comparing the relative separations of the states $J = 0$, 2 and $J = 1$, 3 of the configurations $(d_{\frac{3}{2}})(d_{\frac{3}{2}})$ and $(d_{\frac{3}{2}})(d_{\frac{3}{2}})^{-1}$ deduced earlier, we can at once obtain the values of $a_{\pm}$ and $b_{\pm}$. For example, we obtain from table 1 and eq. (6) for the separations $E_2 - E_0$ in K^{38} and Cl^{36}, and $\lambda = 1.0$,

$$-0.0670a_+ + 0.1323b_+ = 2.15 \text{ MeV},$$

and

$$0.0670\, a_+ + 0.1323\, b_+ = -4.23 \text{ MeV}.$$

This gives

$$\begin{aligned} a_+ &= -44.3 \text{ MeV}, \\ b_+ &= -\ 7.9 \text{ MeV}. \end{aligned} \tag{7}$$

Similarly we obtain

$$b_- = -2.6 \text{ MeV}. \tag{8}$$

Unfortunately, one cannot obtain a_- in this fashion, but one can obtain it by evaluating the separation of say $J = 2,3$ and using the above values of a_+ and $b_\pm$. We find

$$a_- = -44.9 \text{ MeV}. \tag{9}$$

Table 2 lists the values of A_0, A_1, A_2 and A_3 thus obtained for $\lambda = 0.8$ and 1.0. The results do not appear to be very sensitive to the range parameter λ.

TABLE 2

The parameters of the two-body central interaction (1) determined from analysis of the $d_{\frac{3}{2}}$-shell are listed in the first two columns, and those determined from analysis of the $(s_{\frac{1}{2}})(l_j)$ doublets are listed in the third and fourth columns for $\lambda = 0.8$ and 1.0

	$\lambda = 0.8$	$\lambda = 1.0$	$\lambda = 0.8$	$\lambda = 1.0$
A_0	−46.9	−39.3		
A_1	+5.4	+5.3		
A_2	−13.6	−10.5	−2.3	−1.5
A_3	−3.6	−3.1	−10.8	−6.8

We have shown that by assuming only the validity of the jj coupling scheme and absence of many-body forces, we can assign with a reasonable certainty spins and parities to many low lying levels of K^{38} (and Cl^{34}) and Cl^{36}, and with the help of the level schemes thus constructed we have decuced a simple two-body interaction which accounts for these. Conversely, one may now say that with the assumption of the interaction (1) with the parameters listed in table 1, and the validity (even approximate) of the jj coupling scheme, one is able to predict the low lying levels of the odd nuclei in the $d_{\frac{3}{2}}$ shell, belonging to the configurations $(d_{\frac{3}{2}})(d_{\frac{3}{2}})$ or $(d_{\frac{3}{2}})(d_{\frac{3}{2}})^{-1}$.

3. The $s_{\frac{1}{2}}$-Particle Doublets

To study the nuclear interactions which produce the observed very small splittings of the $s_{\frac{1}{2}}$-particle doublets in nuclei near $A = 30$, we again assume jj coupling scheme and harmonic oscillator radial wavefunctions. We shall also assume $r_s = r_d$ for the extension parameters of the radial wavefunctions. The wavefunction of a state of spin J obtained from coupling of an $s_{\frac{1}{2}}$ proton to an l_j neutron shall be written in the obvious notation $\langle s_{\frac{1}{2}} \times l_j : JM|$, where the $\times$ denotes the vector coupling. The technique of evaluating the matrix elements of a two-body interaction operator (1) for such states is well known, but we shall briefly sketch a derivation here, as it will be of use in our subsequent calculations involving non-central spin-dependent forces. The above wavefunction can

be transformed to the L–S coupling form as

$$\langle s\tfrac{1}{2}\times lj : JM| = \sum_{S=0,1} [(2j+1)(2S+1)]^{\frac{1}{2}} W(\tfrac{1}{2}jSl; J\tfrac{1}{2})\langle (0l)l\times(\tfrac{1}{2}\tfrac{1}{2})S : JM|, \quad (10)$$

where $W(abcd; ef)$ is the Racah coefficient.

The two-body interaction (1) is now written as

$$H_{12} = [a+bM+\alpha\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\beta\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 M]J(r_{12}), \quad (11)$$

with

$$\begin{aligned} a &= A_0+\tfrac{1}{2}A_2, \qquad \alpha = \tfrac{1}{2}A_2, \\ b &= A_1+\tfrac{1}{2}A_3, \qquad \beta = \tfrac{1}{2}A_3. \end{aligned} \quad (12)$$

As is well known the matrix elements of the first two terms of (11) give J-independent (but l-dependent) contributions to the energy of the state, viz.,

$$E_J^{(0)} = aF^{(0)}(0l0l)+bF^{(l)}(0ll0)/(2l+1), \quad (13)$$

where $F^{(0)}$ and $F^{(l)}$ are the radial direct and exchange integrals. The matrix elements of the spin-dependent terms can also be written down at once as

$$E_J^{(1)} = [\alpha F^{(0)}(0l0l)+\beta F^{(l)}(0ll0)/(2l+1)]\tfrac{3}{2}(-1)^{l+J+1}(2j+1) \times W(\tfrac{1}{2}j\tfrac{1}{2}j; l1)W(\tfrac{1}{2}j\tfrac{1}{2}j; J1). \quad (14)$$

Hence the knowledge of the splitting of the doublet immediately gives us the value of $[\alpha F^{(0)}+\beta F^{(l)}/(2l+1)]$.

We shall now apply these results to the doublets $(s_{\frac{1}{2}})(s_{\frac{1}{2}})$, $(s_{\frac{1}{2}})(d_{\frac{5}{2}})$ (see footnote †) and $(s_{\frac{1}{2}})(d_{\frac{3}{2}})$ which one may expect to see as ground states in P^{30}, Al^{28} and P^{32} respectively, and also as excited states in some of these or other nuclei. A result that follows directly from eq. (14) is

$$\Delta(s_{\frac{1}{2}}\times d_{\frac{5}{2}}) = \tfrac{3}{2}\Delta(s_{\frac{1}{2}}\times d_{\frac{3}{2}}), \quad (15)$$

where Δ denotes the doublet splittings E_2-E_3 and E_2-E_1 for the two cases. The experimental results show however, $\Delta(s_{\frac{1}{2}}\times d_{\frac{5}{2}}) = 0.031$ MeV and $\Delta(s_{\frac{1}{2}}\times d_{\frac{3}{2}}) = 0.077$ MeV from Al^{28} and P^{32} ground state doublets. Our conclusions should therefore be tempered by the knowledge that the jj coupling assumption may not be valid even for the ground states of these nuclei. We shall later remark generally on these other discrepancies we shall observe ††.

In P^{30} the ground and the first excited state at 0.685 MeV are easily interpreted as the $J = 1, 0$ states of the $(s_{\frac{1}{2}})(s_{\frac{1}{2}})$ doublet. These data, combined with

† Note that for jj coupling $\Delta(d_{\frac{5}{2}}\times s_{\frac{1}{2}}) \equiv \Delta(d_{\frac{5}{2}}^{-1}\times s_{\frac{1}{2}})$.

†† Recently excited states in Si^{28} have been observed [12]) corresponding to the ground state doublet of Al^{28} In this case the observed splitting is $\Delta = 0.065$ MeV which would improve the agreement substantially. One may easily argue that the spherical jj coupling shell model has a better validity in Si^{28} rather than in Al^{28}.

the doublet splitting in P^{32} and eq. (14), give

$$\left.\begin{aligned} 0.165(\alpha+\beta) &= -0.685 \\ 0.060\alpha+0.009\beta &= -0.077 \end{aligned}\right\} \lambda = 1.0,$$
$$\left.\begin{aligned} 0.105(\alpha+\beta) &= -0.685 \\ 0.035\alpha+0.007\beta &= -0.077 \end{aligned}\right\} \lambda = 0.8. \qquad (16)$$

The values of A_2 and A_3 derived from these equations are given in table 2.

Unfortunately not enough is known of the spins and parities of the excited states of P^{30} and P^{32} to enable us to identify more such $s_{\frac{1}{2}}$-particle doublets. When additional information of several doublets in the same nucleus is available, it would be possible to evaluate more precisely not only the spin-dependent but also the spin-independent part of the two-body interaction.

4. Discussion

It is clear that there exists a very real discrepancy between the parameters of the spin-dependent part of the central two-body interaction determined from the $(s_{\frac{1}{2}})(l_j)$ doublet splittings and from the splitting of the levels of the $(d_{\frac{3}{2}})(d_{\frac{3}{2}})$ and $(d_{\frac{3}{2}})(d_{\frac{3}{2}})^{-1}$ configurations. That the discrepancy is outside the limits of errors can be seen from the fact that if the spin-dependent interaction obtained from the d-shell analysis is used to evaluate the s-doublet splittings, we obtain $\Delta(s_{\frac{1}{2}}\times s_{\frac{1}{2}}) = 0.93$ and 1.13 MeV, and $\Delta(s_{\frac{1}{2}}\times d_{\frac{3}{2}}) = 0.25$ and 0.32 MeV for $\lambda = 0.8$ and 1.0 respectively. The discrepancy for the $(s_{\frac{1}{2}}\times s_{\frac{1}{2}})$ doublet is not so serious as that for the $(s_{\frac{1}{2}}\times d_{\frac{3}{2}})$ doublet. This perhaps suggests the presence of non-central spin-dependent terms in the two-body interaction. If such terms would displace the energy levels by 0.1–0.2 MeV in the right direction, the discrepancy would be resolved. One may also note that while such small contritions to the energy of the levels by the non-central interactions would be important for $(s_{\frac{1}{2}})(l_j)$ doublets, they would be relatively insignificant for other configurations, and hence their neglect in the $d_{\frac{3}{2}}$-shell may be justified.

Of course, the above discrepancies (including that between the observed and expected splitting of the ground state doublet in Al^{28}) may also be attributed to failure of jj coupling or of the spherical shell model etc. However, our attempt has been to see how far one may succeed in understanding the observed level schemes in a large group of nuclei in terms of simple assumptions for angular momentum coupling and two-body interactions.

In conclusion, we feel that the results presented here show that it is not possible to define a simple unique central two-body interaction which will with the assumption of jj coupling scheme explain the low lying energy levels ascribable to different configurations. Such a simple interaction must either be regarded as configuration-dependent or may have to be modified by introducing additional non-central or velocity-dependent terms.

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NEUTRON–PROTON INTERACTIONS IN Li^6

S. K. SHAH and S. P. PANDYA

he simplicity of the structure and the energy levels of Li^6 has prompted ıany attempts to determine the nature of the nuclear forces operating beveen the two nucleons in the open p-shell. In view of the compact structure f the small nuclear core (s^4), and since the p-nucleons are relatively on the ırface, one may expect the interaction between them to resemble to a large xtent the forces observed between free nucleons. (In other words, this is not very-many-body problem.) The properties of these latter interactions in arious spin and isotopic-spin states of the two nucleons are now fairly ell known. These interactions consist of central, tensor and two-body ɔin-orbit terms, are strong and short-ranged, have repulsive hard cores, nd show strong spin and i-spin dependence. We note that the free nucleon–ucleon interactions are derived from phase-shift analysis, so that the inɔrmation is expressed in terms of the nature of the interactions in states of ɛlative angular momenta of the two nucleons. We now attempt to derive ·om the known energy levels of Li^6, the nature of the effective neutron–roton interaction and its matrix elements in various states of relative ngular momenta which will enable a straightforward comparison to be ıade with the free two-nucleon interaction.

For simplicity we describe the energy levels in terms of L–S coupling. The -states are not as yet identified. We consider only triplet and singlet S- and)-states. The $^1S_0$ and $^1D_2$ states are identified as the 3·56 MeV and 5·35 leV states. The value of the matrix-element of the interaction in the ground-tate is taken as −8·1 MeV following Sopers, and Pinkston and Brennan. low the wave-functions of these states are easily written in terms of centre-f-mass and relative coordinates, and the energies are

$$\begin{aligned} 2E_0 &= \langle 0s \mid V \mid 0s\rangle + \langle 1s_{\frac{1}{2}} \mid V \mid 1s\rangle \\ 2E_2 &= \langle 0s \mid V \mid 0s\rangle + \langle 0d \mid V \mid 0d\rangle \end{aligned} \tag{1}$$

here $\langle nl \mid V \mid nl\rangle$ denotes the matrix-element of the interaction in the tate of relative angular momentum nl. We choose $V = V_0 \exp(-\mu_0 r)/\mu_0 r$. lote that we choose the effective interaction to be of Yukawa type, and free f the repulsive hard-core. Then the parameters of the interaction in the inglet even states, V_0, μ_0 are easily determined,

$$V_0 = -26 \text{ MeV}; \qquad \mu_0 = 0{\cdot}6 f^{-1} \tag{2}$$

Similarly for the triplet states we identify $^3S_1$ and $^3D_{1,\,2,\,3}$ states as the round state, 5·5 MeV, 4·52 MeV, and the 2·18 MeV states. The interaction ow consists of central, tensor and two-body spin-orbit forces (no single article **l** . **s** interaction is invoked), of Yukawa type and we find

$$V_1 = -92 \text{ MeV}; \qquad \mu_1 = 0{\cdot}8 f^{-1} \tag{3}$$

$$\begin{aligned} \langle 0d \mid V^{\text{tensor}} \mid 0d\rangle &= 2{\cdot}42 \text{ MeV} \\ \langle 0d \mid V^{LS} \mid 0d\rangle &= -1{\cdot}38 \text{ MeV} \end{aligned} \tag{4}$$

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and from the values of equation (4) we obtain

μ_T	0·35	0·56	0·70	0·90	1·06	f^{-1}
V_T	19	69	146	406	746	MeV

μ_{LS}	0·35	0·56	0·70	0·90	1·06	f^{-1}
V_{LS}	−10·8	−39·5	−83·2	−232	−426	MeV

(5

In all these calculations the radial parameter in the wave-function of th p-nucleon, $\exp(-r^2/2r_p^2)$, is taken to be $r_p = 2{\cdot}0\,f$, from the results of electron scattering experiments.

We now make the following remarks:

1. The central singlet and triplet interactions are effectively the residua forces, operating between the two nucleons after the single-particl central potential is extracted out. We find that they are weaker, an have larger ranges than their counterparts in free nucleon–nucleo interactions.

2. The triplet even tensor and spin-orbit forces cannot be uniquel determined; however, it is clear that the two-body spin-orbit force i required (as remarked earlier, we do not include any spin-orbit force in the single-particle potential) to produce correct splitting of the ^{3}I states, and a deep, short-range attractive spin-orbit interaction is quit acceptable.

3. The tensor force is apparently repulsive; however, this should not b taken too seriously, since the contribution of the tensor interactio to the energies of the ^{3}D states is only $\simeq$0·1–0·2 MeV, ten time smaller than that of the spin-orbit interaction. We can accept a attractive tensor force of an equal magnitude, with small ($\simeq$10 per cent corresponding change in the strength of the spin-orbit force, withou disturbing the agreement of the calculated and the observed ^{3}D levels

4. The tensor force mixes the $^3S_1$ and the $^3D_1$ states, but the matri element is rather small ($\simeq$0·2 MeV) and does not effectively distur the assumption of the L–S coupling.

We conclude from the above analysis that the observed energies of th singlet and triplet S and D states of Li^6 can be easily explained by assumin tensor and two-body spin-orbit forces very similar to those observed in fre nucleon–nucleon interactions, and central forces which are weak, attractive longer-ranged, and without hard cores, as would be expected of the effectiv residual interactions.

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V. L. NARASIMHAM, S. K. SHAH and S. P. PANDYA

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SEPARABLE NON-LOCAL INTERACTIONS IN NUCLEI

V. L. NARASIMHAM †, S. K. SHAH and S. P. PANDYA
Physical Research Laboratory, Ahmedabad, India

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Abstract: The lowest six even parity states of Li^6 are calculated in the *L-S* coupling scheme with the use of separable non-local potentials proposed by Mitra *et al.* It is shown that the non-central terms of the potential give very small cc ributions to the energies of these states. The reaction matrix evaluated with the central terms of the interaction does give matrix elements in these states that are in good agreement with experiment.

1. Introduction

A major open problem in the theory of nuclei is the nature of the nuclear forces, and in particular the non-locality of the nuclear interaction. It appears that at present it is possible to fit the low-energy nucleon-nucleon interaction data by using either local interactions which are singular and have hard repulsive cores, or non-local interactions which do not have these unpleasant properties, but are to a large extent arbitrary. For example, Mitra and coworkers [1]), have used a non-local separable potential to obtain a reasonable description of the nucleon-nucleon bound state and scattering data. It then remains to be seen if these two-body interactions (local or non-local) will provide a suitable description of the nuclear many-body problem, and if we may be enabled to decide in favour of one or the other of these interactions as the better one. With this in mind, Mitra and Pandya [2]) calculated the $T = 0$ energy levels of the shell model configuration $(d_{\frac{5}{2}})^2$ using the non-local separable potential with parameters fitted by Mitra and coworkers. In this calculation the reaction matrix was approximated by the potential matrix, and it was later shown [3]) that by taking into account the complete reaction matrix the results change very considerably, and actually tend to decrease significantly the large disagreement between the theory and the experimental results obtained from the use of the potential matrix only. Again, the conclusions drawn from the results of the calculations of the $(d_{\frac{5}{2}})^2$ configuration are rendered somewhat uncertain, since for a *j-j* coupling state, the calculation of energy involves the parameters of both the singlet and triplet interactions. In this case the singlet (P-) interaction parameters were not as precisely known as the triplet parameters, and it turns out that the reaction matrix affects the singlet P contributions to the energies of the states considered in a very drastic manner. It would be desirable to make a calculation in which the parameters of the two-body interaction in different (triplet or singlet) states of spin and i-spin can be independently checked against observed data.

† Present address: Department of Physics, University of California, San Diego, La Jolla.

The nucleus Li^6, because of its simple structure, has been used by many authors to study the nature of the nuclear forces. The six low-lying even-parity states of Li^6 have been described in terms of the shell model configuration $(p)^2$ and it has been shown that the *L-S* coupling scheme is a very good approximation for these states. Pinkston and Brennan [4]) have analysed this level spectrum assuming a single-particle spin-orbit potential $\zeta \mathbf{l}\cdot\mathbf{s}$ plus a simple central two-body interaction V_c, and have obtained the values of ζ as well as the matrix elements of V_c in various states, ^3S, ^3D, ^1S, ^1D, etc., of the $(p)^2$ nucleons. Since *L-S* coupling is a good approximation, and the mixing of the singlet and the triplet states is small, it should be possible to check independently the parameters of the nuclear forces in the triplet and singlet states by calculating the energies of these states and comparing them with the observed values. In this note we use the non-local, separable potential of Mitra and coworkers [1]) to evaluate the energy levels of Li^6 and obtain a much more effective check of this interaction than was possible in the previous calculations of Mitra and Pandya. Only S- and D-states are calculated, since experimentally the ^3P and ^1P states are not yet identified. Thus only singlet and triplet even parity forces enter our calculations. The calculations of the energy levels with the potential as well as the reaction matrix are described in section 2, and the results are discussed in section 3.

2. Calculations

Since we consider only the S- and D-states of Li^6, we need only the even parity terms of the singlet and triplet potentials. These interactions are given in refs. [1]), and the evaluation of the corresponding *G*-matrix is described in refs. [1, 3]). We collect all the relevant results in the appendix.

The wavefunctions in *L-S* coupling for the states considered are written as

$$|(p)^2L(s)^2S : JM\rangle = [\Phi_{Lm_L}(\mathbf{p}_1, \mathbf{p}_2) \times \chi_{Sm_S}]_{JM}, \tag{1}$$

where the notation $[\ldots \times \ldots]$ denotes the vector coupling of the angular momenta L and S to the resultant spin J; while χ_{Sm_S} with $S = 0$ or 1 is the spin part of the wavefunction, and the space part of the wavefunction is given by

$$\Phi_{Lm_L}(\mathbf{p}_1, \mathbf{p}_2) = [\phi_{1m_1}(\mathbf{p}_1) \times \phi_{1m_2}(\mathbf{p}_2)]_{Lm_L}, \tag{2}$$

ϕ_{1m} being the single particle harmonic oscillator wave function in the 1p-subshell:

$$\phi_{1m}(\mathbf{p}) = \left[\frac{8}{3\sqrt{\pi}}(2\mu)^{\frac{1}{2}}\right]^{\frac{1}{2}} \exp(-\mu p^2) p Y_1^m(\hat{p}). \tag{3}$$

The parameter $\sqrt{2\mu}$ is a measure of the radial extension of the wavefunction. (In our units $\hbar = 1$.)

Now for evaluation of the matrix element it is convenient to write the above wavefunction in terms of centre-of-mass and relative momenta

$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2, \qquad 2\mathbf{p} = \mathbf{p}_2 - \mathbf{p}_1 \,.$$

Then we can write immediately

$$\Phi_{Lm_L}(\mathbf{p}_1, \mathbf{p}_2) = \frac{1}{\sqrt{2}}[\Psi_{n\lambda}(\mathbf{P})\psi_{0s}(\mathbf{p}) - \Psi_{0s}(\mathbf{P})\psi_{n\lambda}(\mathbf{p})], \tag{4}$$

where the quantum numbers $n\lambda$ take the values 1s if $L = 0$ and 0d if $L = 2$. It is now possible to write

$$|(\mathrm{p})^2L(\mathrm{s})^2S : JM\rangle$$

$$= \frac{1}{\sqrt{2}}\left[\Psi_{n\lambda}(\mathbf{P}) \times [\psi_{0s}(\mathbf{p}) \times \chi_{SM_S}]_{SM_S} - \Psi_{0s}(\mathbf{P}) \times [\psi_{n\lambda}(\mathbf{p}) \times \chi_{SM_S}]_{JM}\right]_{JM}. \quad (5)$$

Since the form of the potential is

$$\langle \mathbf{p}_1\mathbf{p}_2|V|\mathbf{p}_1'\mathbf{p}_2'\rangle = \delta(\mathbf{P}-\mathbf{P}')\langle \mathbf{p}|V|\mathbf{p}'\rangle, \quad (6)$$

we can write the matrix element as

$$\langle LSJM|V|LSJM\rangle$$

$$= \tfrac{1}{2}\left[\langle \psi_{0s}(\mathbf{p}) \times \chi_S|V|\psi_{0s}(\mathbf{p}') \times \chi_S\rangle + \langle [\psi_{n\lambda}(\mathbf{p}) \times \chi_S]_{JM}|V|[\psi_{n\lambda}(\mathbf{p}') \times \chi_S]_{JM}\rangle\right]. \quad (7)$$

The evaluation of the space and spin integrals is now straightforward, as already described by Mitra and Pandya [2]) or Moshinsky [5]). Similar expressions will hold also for the G matrix.

We now note that the evaluation of the energy of the ^{1}S and ^{1}D states involves only the parameters of the interaction in $T = 1$ and $S = 0$ state. Further, the evaluation of the energy of the ^{1}S state needs only the parameters of the interaction in the relative s- state, that is only λ_0 and β_0. Thus by comparing the calculated matrix elements with those deduced from empirical analysis of the observed energy levels, we should be able to check the validity of the various terms in the non-local interaction in great detail. The matrix elements involved in evaluating these states are easily shown to be

$$\begin{aligned}
\langle \psi_{0s}(\mathbf{p})|V|\psi_{0s}(\mathbf{p}')\rangle &= \left(-\frac{\lambda_0}{M}\right) 2^5(2\mu)^{\frac{1}{2}}(2\pi)^{\frac{1}{2}}X_0^2, \\
\langle \psi_{0d}(\mathbf{p})|V|\psi_{0d}(\mathbf{p}')\rangle &= \left(-\frac{\lambda_2}{M}\right) \frac{2^9(2\mu)^{\frac{1}{2}}(2\pi)^{\frac{1}{2}}}{3} Y_2^2, \\
\langle \psi_{1s}(\mathbf{p})|V|\psi_{1s}(\mathbf{p}')\rangle &= \left(-\frac{\lambda_0}{M}\right) \frac{2^7(2\mu)^{\frac{1}{2}}(2\pi)^{\frac{1}{2}}}{3} \left(X_0 - \frac{8\mu}{3} X_2\right)^2.
\end{aligned} \quad (8)$$

the angular and the spin-integrations are trivial, and the result depends only on the radial integrals

$$\begin{aligned}
X_{2n} &= \int_0^\infty p^{(2n+2)}(\beta_0^2 + p^2)^{-1}\exp(-2\mu p^2)\mathrm{d}p, \\
Y_{2n} &= \int_0^\infty p^{(2n+4)}(\beta_2^2 + p^2)^{-2}\exp(-2\mu p^2)\mathrm{d}p.
\end{aligned} \quad (9)$$

The method for evaluation of these integrals is described in appendix 2 of ref. [2]).

The corresponding results for the G-matrix are obtained by substituting the radial integrals X_0X_0' for X_0^2, Y_2Y_2' for Y_2^2 and $(X_0 - \frac{8}{3}\mu X_2)(X_0' - \frac{8}{3}\mu X_2')$ for $(X_0 - \frac{8}{3}\mu X_2)^2$ in equations (8), where the definition of integrals X_{2n}' and Y_{2n}' are the same as in the

equations (9) except that the integrand is now multiplied by the factor $\bar{A}_0(p)$ and $\bar{A}_2(p)$ respectively, as defined in appendix, equations (A.11) and (A.12), e.g.

$$X'_{2n} = \int_0^\infty \bar{A}_0(p) p^{(2n+2)} (\beta_0^2 + p^2)^{-1} \exp(-2\mu p^2) \mathrm{d}p.$$

The results for the triplet states are similar. We again note that for ^{3}S state, the non-central forces do not contribute to the energy, when calculations are made with the V-matrix; however when the G-matrix is taken into account, there will be central-tensor interference terms contributing to the energy. It will be seen that these latter terms are small, and thus calculation of the energy of the ^{3}S state gives an effective check on the parameters of the central potential which in this case has been so chosen as to act in the relative s- states only. The calculation of the ^{3}D state energy then checks the validity of the parameters of the non-central forces. The matrix elements in these cases for the V- and G-matrix are listed below:

$$\langle \psi_{0s}(\mathbf{p}) \times \chi_1 | V_c | \psi_{0s}(\mathbf{p}') \times \chi_1 \rangle = \left(-\frac{\lambda}{M} \right) 2^5 (2\mu)^{\frac{3}{2}} (2\pi)^{\frac{1}{2}} X_0^2,$$

$$\langle \psi_{1s}(\mathbf{p}) \times \chi_1 | V_c | \psi_{1s}(\mathbf{p}') \times \chi_1 \rangle = \left(-\frac{\lambda}{M} \right) \frac{2^7 (2\mu)^{\frac{3}{2}} (2\pi)^{\frac{1}{2}}}{3} (X_0 - \tfrac{8}{3}\mu X'_2)^2,$$

$$\langle [\psi_{0d}(\mathbf{p}) \times \chi_1]_J | V_t | [\psi_{0d}(\mathbf{p}') \times \chi_1]_J \rangle = \left(-\frac{\lambda}{M} \right) \frac{2^9 (2\mu)^{\frac{3}{2}} (2\pi)^{\frac{1}{2}}}{15} t^2 Y_2^2 (2J+1) W(2112; J1),$$

$$\langle [\psi_{0d}(\mathbf{p}) \times \chi_1]_J | V_{L\cdot S} | [\psi_{0d}(\mathbf{p}') \times \chi_1]_J \rangle = \left(-\frac{\lambda_2}{M} \right) (-)^J \sqrt{\tfrac{1}{5}} 2^9 (2\mu)^{\frac{3}{2}} (2\pi)^{\frac{1}{2}} Y_2^2 W(2121; J1).$$

Here, X_{2n}, Y_{2n} are defined by equations similar to the equations (9) with the proper range parameters β, γ replacing β_0, β_2 respectively.

To calculate the corresponding results for the reaction matrix, we have neglected the spin-orbit terms in the interaction entirely, and have used only the central terms of the expression for the reaction matrix, given in equation (A.6) of the appendix. The resulting expression for the matrix elements is obtained from equation (8) by replacing X_0^2 by $X_0 X'_0$ and $(X_0 - \frac{8}{3}\mu X)^2$ by $(X_0 - \frac{8}{3}\mu X_2)(X'_0 - \frac{8}{3}\mu X'_2)$, where X'_{2n} are similar to X_{2n} but with the factor $A(p)$ multiplying the corresponding integrands. $A(p)$ is defined by equation (A.7), for example:

$$X'_{2n} = \int_0^\infty A(p) p^{(2n+2)} (\beta^2 + p^2)^{-1} \exp(-2\mu p^2) \mathrm{d}p.$$

We have not calculated the energies with the non-central terms of the G-matrix. It will be shown later that these terms will be quite small and insignificant.

To evaluate the numerical results we have chosen $\sqrt{2\mu} = 2.24$ fm, as given by the results of electron scattering on Li^6, analysed by Elton [6]) in terms of the modified

harmonic oscillator model. We take $M^*/M = 0.6$ (ref. [3])). The final results are given in tables 1 and 2.

TABLE 1
Matrix elements using central terms of the V- and G-matrices in MeV

States	V-matrix	G-matrix	Pinkston and Brennan
^3S	−3.98	−7.56	−8.00
^3D	−2.02	−4.93	−4.40
^1S	−2.99	−4.45	−3.85
^1D	−1.90	−1.96	−2.12

TABLE 2
Matrix element using non-central terms of the V-matrix only for ^3D_J states in MeV

States	Tensor	$L.S$
$^3D_1$	−0.007	+0.0311
$^3D_2$	+0.0353	−0.0104
$^3D_3$	−0.099	+0.0207

3. Discussion

Most of the earlier shell model calculations for the energy levels of Li^6 have used a central harmonic oscillator potential which generates in the zeroth approximation the independent particle wave functions given in eq. (3), a weak spin-orbit potential acting on each independent particle $\zeta \mathbf{l} \cdot \mathbf{s}$ and also a weak local two-body interaction H_{12} which is generally chosen for simplicity to be central, but spin- and i-spin-dependent. Perhaps the most satisfactory analysis in this respect is that of Pinkston and Brennan [5]) who have determined the value of ζ as well as the matrix elements of the interaction H_{12} in the various states of the two nucleons which we have considered, so as to obtain the best fit for the observed energy levels. We can therefore directly compare the matrix elements of the non-local two-body interaction that we have computed with the matrix elements obtained by Pinkston and Brennan. Table 1 also lists the values of Pinkston and Brennan.

Firstly we remark that the matrix elements of the tensor as well as the spin-orbit potential in the ^3D_J states (the only states in which they act) are quite small. Calculations of Mitra and Pandya [2]) on the energy levels of the $(d_{\frac{3}{2}})^2$ configuration using the potential matrix as well as those of Narasimham [3]) using the reaction matrix show that in both these cases the tensor and the spin-orbit forces give rather small contributions. This is why we have not evaluated the contributions of these non-central forces using the corresponding reaction matrix terms. It is now perhaps safe to conclude that the non-central forces which play a very important role in the interaction of the free nucleons, and in binding energy of nuclear matter are relatively unimportant for evaluation of the energies of the nuclear states in straightforward shell model type

calculations. It is enough to take into account their indirect contribution through the single particle spin-orbit potential $\zeta \mathbf{l} \cdot \mathbf{s}$. While this conclusion appears to be quite justified in the light of our calculations for the realistic non-local interactions that we have used, it is important to check if this is true also for realistic local potentials such as those of Gammel-Thaler or Signell-Marshak. Banerjee and Dutta Roy [7]) have indeed evaluated the energies of the states of Ca^{42} using the reaction matrix for the Gammel-Thaler potential, but the relative contributions of the central and non-central terms are not apparent to us from their paper.

We now compare our results for central non-local forces with those of Pinkston and Brennan. It is clear from table 1 that while the results obtained with the approximation of using the potential matrix only are in serious disagreement, the use of the reaction matrix gives quite satisfactory results, in good agreement with those of Pinkston and Brennan. One may therefore conclude that the non-local interaction in the triplet and singlet even parity states determined by Mitra *et al.*, will give reasonably satisfactory (correct to $\approx 10\,\%$) results for the energy of the nuclear states. It should be noted that some of the discrepancy for the ^{3}D state can be attributed to the restriction of choosing the central potential to act in the $l = 0$ state only.

4. Conclusion

We can now summarise the results of this calculation. The non-central terms of the non-local interaction gives negligible contribution to the energies of the states considered, and this appears to be a generally valid property. The central terms in the interaction give reasonably correct values for the matrix elements, provided the reaction matrix (and not just the potential) is taken into account, and these matrix elements of the central non-local two-body forces, together with the generally accepted spin-orbit single particle potential $\zeta \mathbf{l} \cdot \mathbf{s}$ ($\zeta = -1.5$ MeV as shown by several authors) give a reasonable description of the S- and D- states of the Li^6 nucleus. We therefore feel that the triplet and singlet even parity potentials of Mitra *et al.* give a satisfactory description of the nuclear properties.

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Appendix

We give here the potential defined in ref. [1]) employed in these calculations for triplet even and singlet even states.

Triplet even state $T = 0$. In this case we have

$$\langle \mathbf{p}|V|\mathbf{p}'\rangle = \left(-\frac{\lambda}{M}\right) g(\mathbf{p})g(\mathbf{p}') - \frac{5\lambda_2}{M} v_2(p)v_2(p')(\mathbf{L}\cdot\mathbf{S})P_2(\hat{p}\cdot\hat{p}'), \tag{A.1}$$

where

$$g(\mathbf{p}) = C(p) + \frac{1}{\sqrt{8}} T(p)S(\hat{p}), \tag{A.2}$$

$$S(\hat{p}) = 3(\sigma_1 \cdot \hat{p})(\sigma_2 \cdot \hat{p}) - (\sigma_1 \cdot \sigma_2), \tag{A.3}$$

$$C(p) = 1/(\beta^2 + p^2),$$

$$T(p) = -tp^2/(\gamma^2 + p^2)^2, \tag{A.4}$$

$$v_2(p) = p^2/(\delta^2 + p^2)^2,$$

$$\lambda = 24.1\alpha^3, \quad \beta = 5.8\alpha, \quad t = 0.75, \tag{A.5}$$

$$\lambda_2 = -20.1\alpha^3, \quad \gamma = \delta = 6.0\alpha,$$

$$\alpha^2/M = 2.226 \text{ MeV} = \text{(binding energy of deuteron)}$$

The G matrix is given by the relation

$$\langle \mathbf{p}|G|\mathbf{p}'\rangle = -\frac{\lambda}{M} A(p) g(\mathbf{p}) g(\mathbf{p}'), \tag{A.6}$$

where

$$A(p) = \left[1 - \frac{M^*}{M} \lambda J(p')\right]^{-1}, \qquad J(p') = \int_0^\infty \frac{C^2(p'') + T^2(p'')}{(p''^2 - p'^2)} \, \mathrm{d}^3\mathbf{p}''. \tag{A.7}$$

Singlet even states $T = 1$, $l = 0, 2$. In this case we have

$$\langle \mathbf{p}|V_l|\mathbf{p}'\rangle = (2l+1) \left(-\frac{\lambda_l}{M}\right) v_l(p) v_l(p') P_l(\hat{p} \cdot \hat{p}'), \tag{A.8}$$

where

$$\lambda_0 = 18.6\alpha^3, \quad \beta_0 = 5.8\alpha, \quad v_0(p) = 1/(p^2 + \beta_0^2), \tag{A.9}$$

$$\lambda_2 = 10.5\alpha^3, \quad \beta_2 = 5.8\alpha, \quad v_2(p) = p^2/(p^2 + \beta_2^2)^2.$$

The G matrix is defined as

$$\langle \mathbf{p}|G_l|\mathbf{p}'\rangle = \bar{A}_l(p) \langle \mathbf{p}|V_l|\mathbf{p}'\rangle, \tag{A.10}$$

where

$$\bar{A}_l(p) = \left[1 - \frac{M^*}{M} \lambda_l J_l(p')\right]^{-1}, \tag{A.11}$$

$$J_l(p') = \int_0^\infty \frac{v_l^2(p'')}{(p''^2 - p'^2)} \, \mathrm{d}^3\mathbf{p}''. \tag{A.12}$$

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ENERGY LEVELS OF O^{19}

S. K. SHAH
Physical Research Laboratory, Ahmedabad, India *

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In a previous paper [1] we have discussed the nature of the effective nuclear interaction in $T = 1$ states of nuclei O^{18}, Ti^{50} etc. The parameters of the interaction in singlet even and triplet odd states of the two nucleons outside the closed shells were determined under two assumptions (a) the even state interactions are non-local and are effective in $l = 0$ (l refers to relative orbital angular momentum) s-states only, and (b) the even state interactions are the same in all states $l = 0$, 2 etc. For O^{18} the parameters of the even interactions (assumed to be Gaussian shape $V_0 \exp(-\gamma^2/\gamma_0^2)$) were determined to be (a) $V_0 = -25$ MeV, $\lambda = \gamma_0/\gamma_l = 1.0$ and (b) $V_0 = -40$ MeV, $\lambda = 0.65$, and several sets of corresponding odd state interactions which give a good fit to the O^{18} spectrum were given. In this note we apply these different sets of parameters to calculate the energy level spectrum of O^{19}, to see if these additional data can help to distinguish between the different sets.

$T = 3/2$ states of O^{19} arising from the configurations $(d_{5/2})^2$, $(d_{5/2})^2 (s_{1/2})$ and $(d_{5/2}) (s_{1/2})^2$ are considered. The results are shown in fig. 1. We find that in case (a) the odd state interactions appear to have only a small effect on the energies of the different levels, and all the different sets of odd interaction given in ref. [1] give essentially the same results. The level scheme for one interaction is shown in fig. 1.

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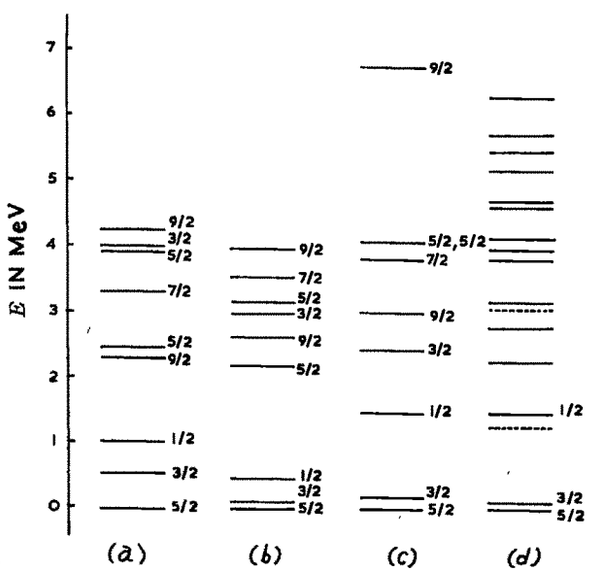

Fig. 1. The energy levels of O^{19}: (a) calculated with $V_0 = -25$ MeV, $\lambda = 1.0$ (1s-state interactions only) and $V_1 = -5.3$ MeV, $\lambda = 1.0$ (3p-state interactions); (b) calculated with $V_0 = -40$ MeV, $\lambda = 0.65$ (^{1}s- and ^{1}d-state interactions) and $V_1 = -25.3$ MeV, $\lambda = 0.8$ (^{3}p-state interactions); (c) calculated by Talmi and Unna; (d) experimental level spectrum. The state $J = 5/2$ is normalised as ground state.

On the other hand for case (b) the odd state interactions change considerably the energies of the states and in particular the energy of the lowest $J = 3/2$ is quite different for different odd-state forces which gave equally good fits for O^{18}. We give in fig. 1 the results for the interaction which gives the best fit for the excitation energy of the lowest $J = 3/2$ state. The results recently reported by Talmi and Unna [2] are also shown.

It is now clear that a measurement of the spins and parities of the levels between 2 and 3.5 MeV would be very important for distinguishing between the various schemes. In particular, it would be possible to distinguish between the schemes (a) and (b) if the spin of the 3.16 MeV level is determined and the existence of the level at 3.05 MeV is confirmed.

We would like to point out that if the low levels of O^{18}, O^{19} are discussed in terms of $d_{5/2}$ and $s_{1/2}$ orbitals only, the spectroscopy involves only 8 matrix elements of the effective nuclear interaction, and in principle these can already be determined from the available information of the lowest 5 levels of O^{18} and the lowest 3 levels of O^{19}. In practice considerable computation is necessary *. A determination of these matrix elements would not only enable us to derive considerable information on the nature of the effective interaction, but would also predict the excited states of O^{19}, and a comparison with experiments would immediately reveal if the nuclear interaction is exactly the same or not in O^{18} and O^{19}. This programme is now being carried out.

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* Talma and Unna [2] assume two of the matrix elements $\langle (d_{5/2})^2 0 | V | (s_{1/2})^2 0 \rangle$ and $\langle (d_{5/2})^2 2 | V | (d_{5/2} s_{1/2}) 2 \rangle$ to be zero, for simplicity in computation.

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