CHAPTER II

EFFECTIVE INTERACTIONS IN NUCLEI

1. Introduction

A really interesting problem in shell model is to determine the nature of the effective two-body interaction in nuclei. As we mentioned in chapter I, it is possible to evaluate nuclear properties using a free nucleon-nucleon interaction such as Gammel-Thaler potential and the Brueckner-Bethe-Goldstone formalism; however, such calculations are rather complicated. Thus there exists a case for a serious attempt to obtain a relatively simple effective nuclear interaction for use in shell model calculations to predict nuclear properties with a fair degree of accuracy. This may be done by a detailed systematic study of the matrix elements of the effective interaction in states of two nucleons in various configurations. In this chapter we outline a method which we believe should be very useful for such a study and apply it to some simple calculations.

It is well known that the remarkable successes of the spherical shell model in predicting many of the nuclear properties and particularly the energy level schemes, have been obtained for nuclei near the closed shells, i.e. for nuclei with one, two or three nucleons (or holes) outside a closed shell. For simplicity, and to ensure that the results are dependable, we consider only nuclei with two nucleons outside a closed shell.

We further restrict ourselves to consideration of only T = 1 of two nucleons. The work described here can be extended also to T = 0 states in a similar way. However, experimental information on T = 0 states is not in an equally satisfactory state. The energy levels of three-nucleon systems (closed shell + 3) can easily be written down in terms of the matrix elements of the effective interaction in two nucleon states, with the use of fractional parentage coefficients. We discuss an example in the next chapter. A simultaneous analysis of related two- and three-nucleon systems would provide information on the role of three-body forces or change of effective interaction, if any, with change in the number of extra-core nucleons.

A standard technique for evaluating the matrix elements of a given two-body potential† is to expand it in Legender polynomials[24]

$$V(\vec{r}_2 - \vec{r}_1) = \sum_k V_k(r_1, r_2) P_k(\cos\theta_{12}) \qquad (1.1)$$

and to express the matrix element in terms of Slater integrals,

$$F_k^{(abcd)} = \int V_k(r_1, r_2) R_a(r_1) R_b(r_2) R_c(r_1) R_d(r_2)\, d\vec{r}_1\, d\vec{r}_2 . \qquad (1.2)$$

The effect of the potential is then entirely given in terms of the Slater integrals F_k. The spin-angle integrations

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† We consider only local potentials, $V(\vec{r}_2 - \vec{r}_1)$.

are easily carried out in a general way using the tensor-algebraic techniques developed by Racah. Thus the energy levels of nuclei in p-shell can be expressed in terms of only two radial integrals F_0 and F_2, and in s-d shell by seven radial integrals[13]. Given an arbitrary local potential interaction, one may evaluate all the Slater integrals required in the calculation. However, for non-local potential this method becomes very cumbersome. There is however an alternate method which is much more flexible, more elegant and can reveal much better the detailed nature of the nuclear interaction. We shall show that it is also much easier to apply in actual calculations.

It may be remarked that in the analysis of the nucleon-nucleon scattering data, it is found convenient to analyse the data first in terms of the scattering phase shifts in states of different spin, isotopic-spin and relative orbital angular momentum l. These phase shifts can then be discussed in terms of different potential models. Such an approach may well be useful in nuclear spectroscopy. In this case, one can analyse the available data on nuclear energy level spectra to obtain the matrix elements of the interaction in states of two nucleons with different spin, isotopic-spin and relative orbital angular momentum. One may then analyse these matrix elements of the potential in different states of two nucleons, and the possible configuration dependence

of the potential, locality or velocity dependence (i.e. dependence on ℓ) of the interaction etc. In the next section we describe in detail the procedure to be followed for such an analysis.

2. Method of Analysis

We shall consider only simple light nuclei such as Li^6, O^{18} and Ti^{50} (Fe^{54}). The single particle potential due to the core consisting of closed shells is taken to be

$$V = V_c + a(\vec{\ell}\cdot\vec{s}) = \tfrac{1}{2}m\omega^2 r^2 + a(\vec{\ell}\cdot\vec{s}) = k\omega\nu r^2 + a(\vec{\ell}\cdot\vec{s}) \qquad (2.1)$$

and the value of the parameter $\underline{a}$ is suitably fixed. The single particle energy levels appropriate for the nucleus under consideration are taken from experimental data on closed shell plus one nucleon system. The single particle wavefunctions are chosen to be harmonic oscillator wavefunctions[24].

$$\Psi_{n\ell jm}(r,\theta,\varphi) = \frac{R_{n\ell}(r)}{r}\sum_{m_\ell m_s}\langle \ell s m_\ell m_s | jm\rangle\, Y_\ell^{m_\ell}(\theta,\varphi)\,\chi_s^{m_s} \qquad (2.2)$$

where $\langle \ell s m_\ell m_s | jm\rangle$ is the usual Clebsh-Gordan coefficient and $\chi_s^{m_s}$ is the spin wavefunction. The radial part $R_{n\ell}(r)$ is given as

$$R_{nl}(r) = \left[\frac{2^{l-n+2}(2l+2n+1)!!(2\nu)^{l+3/2}}{\sqrt{\pi}\, n!\,[(2l+1)!!]^2} \right]^{\frac{1}{2}} e^{-\nu r^2} r^{l+1} L^{l+1/2}_{n+l+1/2}(2\nu r^2) \qquad (2.3)$$

where

$$L^{l+\frac{1}{2}}_{n+l+\frac{1}{2}}(2\nu r^2) = \sum_{k=0}^{n} (-1)^k 2^k \binom{n}{k} \frac{(2l+1)!!}{(2l+2k+1)!!} (2\nu r^2)^k \qquad (2.4)$$

= Associated Laguerre polynomial.

The empirical parameter $r_l^2 = 1/\nu$ appearing in the wavefunction is so adjusted that the r.m.s. radius calculated with these wavefunctions,

$$\langle r^2 \rangle = \tfrac{1}{2}(n+\tfrac{3}{2}) r_l^2 \qquad (2.5)$$

is the same as that given by electron scattering experiments, or in absence of this data, by the simple expression $\sqrt{\frac{3}{5}} \times 1{\cdot}2$ fm.

The lowest few levels of a nucleus may be described in terms of an appropriate configuration space, in which, according to the usual practice, justified by the results of the Brueckner theory, only the lowest few energetically "almost degenerate" configurations are included. We can write the two-nucleon wavefunction

as†

$$|JM\rangle = |(n_1 l_1 s_1) j_1 (n_2 l_2 s_2) j_2 : JM\rangle$$

$$= \sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | JM\rangle \Psi_{n_1 l_1 j_1 m_1}(r_1, \theta_1, \varphi_1) \Psi_{n_2 l_2 j_2 m_2}(r_2, \theta_2, \varphi_2) \qquad (2.6)$$

and the energy as

$$E_J = \epsilon(n_1 l_1 j_1) + \epsilon(n_2 l_2 j_2). \qquad (2.7)$$

The degeneracies in the above spectrum are removed by the introduction of the effective two-body interaction V_{12}. The Hamiltonian matrix in the assumed configuration space is constructed for each value of J,

$$\langle (n_1 l_1 s_1) j_1 (n_2 l_2 s_2) j_2 : JM | V_{12} | (n_1' l_1' s_1') j_1' (n_2' l_2' s_2') j_2' : JM \rangle \qquad (2.8)$$

and is diagonalised explicitely to obtain the eigenvalues and eigenfunctions of the lowest few states of interest.

In the above equation (2.8) the two-nucleon wavefunctions are written in j-j coupling formalism. The first step is then to convert them to L-S foupling, by a suitable

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† We do not write down the isotopic-spin part explicitely since T = 1 is assumed in all cases. Again, although explicit antisymmetrisation of the above wavefunction is not displayed, it is taken account of in all calculations. See final equation (2.12).

transformation,

$$\left|(n_1 l_1 s_1) j_1 (n_2 l_2 s_2) j_2 : JM\right\rangle$$

$$= \sum_{LS} A \begin{pmatrix} l_1 & s_1 & j_1 \\ l_2 & s_2 & j_2 \\ L & S & J \end{pmatrix} \left|(n_1 l_1 n_2 l_2) L (s_1 s_2) S : JM\right\rangle \quad (2.9)$$

where A - represents LS-jj transformation coefficient[25].

In the next step the space part of the wavefunction is expanded in terms of centre of mass and relative orbital angular momentum states of the two particles. This transformation has recently been widely used in nuclear spectroscopy calculations[26,27,28,29].

$$\left|(n_1 l_1 n_2 l_2) L\right\rangle = \sum_{N\Lambda n l} B^{n_1 l_1 n_2 l_2}_{N\Lambda n l (L)} \left|(N\Lambda n l) L\right\rangle \quad (2.10)$$

where the coefficients $B^{n_1 l_1 n_2 l_2}_{N\Lambda n l (L)}$ are tabulated by Brody and Moshinsky[30].

Finally, we recouple the angular momenta, so that the two-nucleon spin state and the relative orbital angular momentum state are coupled together to give total angular momentum $\mathcal{K}$.

$$\begin{aligned} &\left|(n_1 l_1 n_2 l_2) L (s_1 s_2) S : JM\right\rangle \\ &= \sum_{N\Lambda n l} B^{n_1 l_1 n_2 l_2}_{N\Lambda n l (L)} \left|(N\Lambda n l) L (s_1 s_2) S : JM\right\rangle \\ &= \sum_{\substack{N\Lambda n l \\ \mathcal{K}}} B^{n_1 l_1 n_2 l_2}_{N\Lambda n l (L)} U(\Lambda l J S; L \mathcal{K}) \left|N\Lambda (n l, S) \mathcal{K} : JM\right\rangle . \end{aligned} \quad (2.11)$$

Here U (abcd; ef) is the normalised Racah coefficient.

Now combining above equations, it is possible to write the matrix element of the Hamiltonian in equation (2.8), in terms of the matrix elements of V_{12} in two nucleon states of total spin S, and relative orbital angular momentum ℓ.

$$\langle (n\ell, S)x \,|\, V_{12} \,|\, (n'\ell', S')x \rangle .$$

Thus

$$\langle (n_1\ell_1 s_1) j_1 (n_2\ell_2 s_2) j_2 : JM \,|\, V_{12} \,|\, (n_1'\ell_1' s_1') j_1' (n_2'\ell_2' s_2') j_2' : JM \rangle$$

$$= \sum_{\substack{L S L' S' \\ N\Lambda n\ell n'\ell' \\ x}} a a' \left[1 + (-1)^{S+\ell}\right] \left[1 + (-1)^{S'+\ell'}\right] \times$$

$$A \begin{pmatrix} \ell_1 & s_1 & j_1 \\ \ell_2 & s_2 & j_2 \\ L & S & J \end{pmatrix} A \begin{pmatrix} \ell_1' & s_1' & j_1' \\ \ell_2' & s_2' & j_2' \\ L' & S' & J \end{pmatrix} B^{n_1\ell_1 n_2\ell_2}_{N\Lambda n\ell (L)} B^{n_1'\ell_1' n_2'\ell_2'}_{N\Lambda n'\ell' (L')} \times \qquad (2.12)$$

$$U(\Lambda \ell J S; L x)\, U(\Lambda \ell' J S'; L' x) \times$$

$$\langle (n\ell, S)x \,|\, V_{12} \,|\, (n'\ell', S')x \rangle$$

where a and a' $= \frac{1}{2}$ for equivalent particles

$= \frac{1}{\sqrt{2}}$ for inequivalent particles.

The factors in the first line of the right side of the above equation take care of explicit antisymmetrisation of the wavefunctions.

It is possible to simplify the above expression in some approximation. For example, if one considers only central interactions V_{12}^{c}, we obtain $\ell = \ell'$, $S = S'$ and the matrix element is independent of the value of α. In that case the sum over α can be carried out directly and one obtains,

$$\left\langle (n_1 \ell_1 s_1) j_1 (n_2 \ell_2 s_2) j_2 : JM \,\middle|\, V_{12}^{c} \,\middle|\, (n_1' \ell_1' s_1') j_1' (n_2' \ell_2' s_2') j_2' : JM \right\rangle$$

$$= \sum_{\substack{L L' S \\ N \Lambda n \ell n'}} a a' \left[1 + (-1)^{\ell + S} \right]^2 A \begin{pmatrix} \ell_1 & s_1 & j_1 \\ \ell_2 & s_2 & j_2 \\ L & S & J \end{pmatrix} A \begin{pmatrix} \ell_1' & s_1' & j_1' \\ \ell_2' & s_2' & j_2' \\ L' & S' & J \end{pmatrix} \times$$

$$B^{n_1 \ell_1 n_2 \ell_2}_{N \Lambda n \ell (L)} B^{n_1' \ell_1' n_2' \ell_2'}_{N \Lambda n' \ell (L')} \delta_{L L'} \left\langle n \ell, S \,\middle|\, V_{12}^{c} \,\middle|\, n' \ell, S \right\rangle . \qquad (2.13)$$

Now if we study only two-nucleon states with isotopic-spin T = 1, the matrix elements of the interaction need be considered only in singlet even (S = 0, ℓ = even) and triplet odd (S = 1, ℓ = odd) states. Further, if we make an additional approximation of considering only singlet even interactions (Serber-type force), we obtain

$$\left\langle (n_1 \ell_1 s_1) j_1 (n_2 \ell_2 s_2) j_2 : JM \,\middle|\, V_{12}^{cs} \,\middle|\, (n_1' \ell_1' s_1') j_1' (n_2' \ell_2' s_2') j_2' : JM \right\rangle$$

$$= \sum_{\substack{N \Lambda \\ n n' \ell}} 4 a a' A \begin{pmatrix} \ell_1 & s_1 & j_1 \\ \ell_2 & s_2 & j_2 \\ L & 0 & J \end{pmatrix} A \begin{pmatrix} \ell_1' & s_1' & j_1' \\ \ell_2' & s_2' & j_2' \\ L & 0 & J \end{pmatrix} \times$$

$$B^{n_1 \ell_1 n_2 \ell_2}_{N \Lambda n \ell (L=J)} B^{n_1' \ell_1' n_2' \ell_2'}_{N \Lambda n' \ell (L=J)} \left\langle n \ell, \,\middle|\, V_{12}^{cs} \,\middle|\, n' \ell \right\rangle . \qquad (2.14)$$

If we further consider only interactions in singlet s-states ($\ell = 0$), the above expression becomes even more simple.

We note that tables of A- and B-coefficients are available[25,30], and the radial matrix elements can be written down once and for all as a function of $\lambda = r_0/r_\ell$ for a given potential shape, so that the evaluation of any matrix element is quite simple. The expression for the radial matrix elements $\langle n\ell | V_{12} | n\ell \rangle$ in terms of I_ℓ are given in table 1. Table 2 gives the values of these matrix elements that we shall need in subsequent calculations. For this evaluation we assume the potential to have Gaussian shape $\exp[-(r/r_0)^2]$, and the matrix elements are listed as functions of λ, in units of the depth of the potential, V_0. We note that $I_\ell = \left(\lambda^2/1+\lambda^2\right)^{\ell+3/2}$. Figure 1 also shows a plot of the matrix elements $\langle n\ell | V_{12} | n\ell \rangle$ in units of I_0, as a function of λ. Here it may be remarked that when we take the limiting case, $\lambda \longrightarrow 0$, I_{os} tends to some finite value, so that $I_{os} \neq 0$, but some finite constant and all other $I_{n\ell}(\ell \neq 0)$ are zero. It can be seen from table 1 that in such a limiting case $I_{1s} = 1.5\, I_{os}$, $I_{2s} = 1.875\, I_{os}$, $I_{3s} = 2.187\, I_{os}$ etc. Similarly, for $\lambda \longrightarrow \infty$, we get in the limit, $I_{n\ell} = I_{os}$ for all $n\ell$. A further advantage of this formalism is that we can consider the matrix elements of V_{12} in states of different ℓ and S as arbitrary parameters, and provided sufficient experimental data is available, we can determine them empirically,

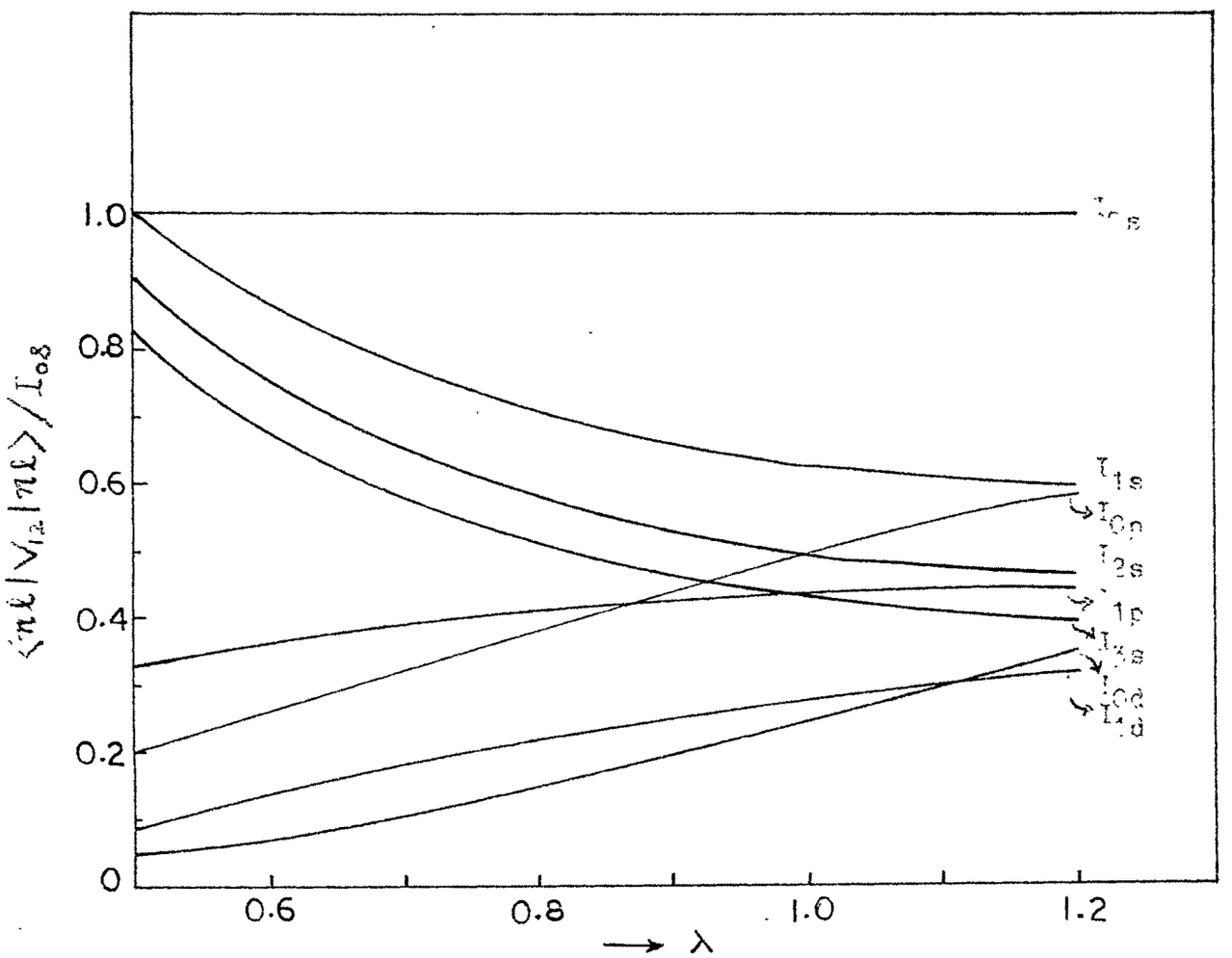


Fig. 1. The quantities $\langle n\ell|V_{12}|n\ell\rangle / I_{08}$ are plotted against the scaling parameter $\lambda = r_0/r_i$.

Table 1

The expressions for the matrix elements $\langle n\ell | V_{12} | n\ell \rangle$ in terms of $I_\ell = (\lambda^2/1+\lambda^2)^{\ell+3/2}$, where $\lambda = r_0/r_\ell$

$\langle n\ell \mid V_{12} \mid n\ell \rangle$	The expressions in terms of I_ℓ.
$\langle os \mid V_{12} \mid os \rangle$	I_0
$\langle op \mid V_{12} \mid op \rangle$	I_1
$\langle od \mid V_{12} \mid od \rangle$	I_2
$\langle of \mid V_{12} \mid of \rangle$	I_3
$\langle og \mid V_{12} \mid og \rangle$	I_4
$\langle oh \mid V_{12} \mid oh \rangle$	I_5
$\langle oi \mid V_{12} \mid oi \rangle$	I_6
$\langle 1s \mid V_{12} \mid 1s \rangle$	$1.5I_0-3I_1+2.5I_2$
$\langle 2s \mid V_{12} \mid 2s \rangle$	$1.875I_0-7.5I_1+16.25I_2-17.5I_3+7.87I_4$
$\langle 3s \mid V_{12} \mid 3s \rangle$	$2.1875I_0-13.1250I_1+45.9375I_2-96.25I_3$ $+122.0625I_4-86.6250I_5+26.8125I_6$
$\langle 1p \mid V_{12} \mid 1p \rangle$	$2.5I_1-5I_2+3.5I_3$
$\langle 2p \mid V_{12} \mid 2p \rangle$	$4.375I_1-17.5I_2+33.25I_3-31.50I_4+12.375I_5$
$\langle 1d \mid V_{12} \mid 1d \rangle$	$3.50I_2-7I_3+4.5I_4$
$\langle 2d \mid V_{12} \mid 2d \rangle$	$7.875I_2-31.50I_3+56.25I_4-49.50I_5+17.875I_6$
$\langle 1f \mid V_{12} \mid 1f \rangle$	$4.5I_3-9.0I_4+5.5I_5$
$\langle 1g \mid V_{12} \mid 1g \rangle$	$5.5I_4-11.0I_5+6.5I_6$

Table 2

The matrix elements $I_{nl} = \langle nl | V_{12} | nl \rangle$ for the Gaussian potential $exp(-r^2/r_0^2)$ for different values of $\lambda = k_0/k_l$.

I_{nl} \ λ	0.5	0.8	1.0	1.2
I_{os}	0.0895	0.2437	0.3536	0.4535
I_{1s}	0.0896	0.1730	0.2210	0.2725
I_{2s}	0.0809	0.1377	0.1739	0.2127
I_{3s}	0.0736	0.1229	0.1518	0.1803
I_{op}	0.0179	0.0951	0.1768	0.2676
I_{1p}	0.0292	0.1030	0.1547	0.2052
I_{2p}	0.0342	0.0966	0.1361	0.1743
I_{od}	0.0036	0.0371	0.0884	0.1580
I_{1d}	0.0083	0.0540	0.0994	0.1481
I_{2d}	0.0128	0.0632	0.1008	0.1367
I_{of}	0.0007	0.0145	0.0442	0.0932
I_{1f}	0.0020	0.0261	0.0605	0.1032
I_{og}	0.0001	0.0057	0.0221	0.0550
I_{1g}	0.0005	0.0130	0.0363	0.0698
I_{oh}	0.0000	0.0022	0.0110	0.0325
I_{oi}	0.0000	0.0009	0.0055	0.0192

without having to assume anything about the nature of the potential. Unfortunately, in general, not enough experimental data is available, and one has to make some assumptions on the nature of the potential. In the next section we shall treat in some detail the extraction of the matrix elements from available data on various nuclei.

3. Pairing Energies

We would now describe some results on the nature of the energy levels to be expected on the basis of a very simple assumption viz., consider singlet s-state interactions only.

We consider two equivalent nucleons $(n\ell j)^2$. The energy of a state of spin J of this configuration is then, for singlet s-state (S = 0, $\ell = 0$) interactions only,

$$E\left[(n\ell j)^2 : J\right] = \sum_{Nn} A^2 \begin{pmatrix} \ell & \tfrac{1}{2} & j \\ \ell & \tfrac{1}{2} & j \\ J & 0 & J \end{pmatrix} B^2_{NJn0}(J) \langle ns | V_{12} | ns \rangle . \quad (3.1)$$

Now if we consider the states with J = 0, then we can study the behaviour of pairing energies $E\left[(j)^2_0\right]$ for s-interactions only in different configurations. For such a case the expression for the energy of the state is particularly simple,

$$E\left[(j)^2_0\right] = \frac{2j+1}{2(2\ell+1)} \sum_{Nn} B^2_{N0n0}(0) \langle ns | V_{12} | ns \rangle . \quad (3.2)$$

Thus for a given ℓ, the dependence on j is only given by the factor $(2j+1)$. The values of $B^2/2\ell+1$ required in the above formula are given in table 3. The final results would depend upon the behaviour of the matrix elements with n, and this in turn would depend upon the range of the potential or the parameter $\lambda = r_0/r_\ell$. Let $I_0 = \langle os|V_{12}|os\rangle$, $I_1 = \langle op|V_{12}|op\rangle$ etc. Then we put $X_1 = I_1/I_0$, $X_2 = I_2/I_0$ etc. and write the expressions for $2E_0/(2j+1)I_0$ knowing the values of $B^2/2\ell+1$ from table 3, as

$$\begin{aligned}
\frac{2E_0}{(2j+1)I_0} &= 1 \quad \text{for } (os)^2 \\
&= 0.166(1+X_1) \quad \text{for } (op)^2 \\
&= 0.033(1+X_2) + 0.111X_1 \quad \text{for } (od)^2 \qquad (3.3) \\
&= 0.007(1+X_3) + 0.050(X_1+X_2) \quad \text{for } (of)^2 \\
&= 0.0015(1+X_4) + 0.019(X_1+X_3) \\
&\quad + 0.040X_2 \quad \text{for } (og)^2
\end{aligned}$$

We also give a plot of $2E_0/(2j+1)I_0$ for different oscillator orbits ℓ, and different choice of behaviour of I_n with n in figure 2. The results are given in table 4. The general trend is quite clear, viz., E_0 increases with j for the given ℓ, and $E_0/2j+1$ decreases with ℓ.

The energy levels (with T = 1) of some simple configuration $(p_{3/2})^2$, $(d_{3/2})^2$, $(d_{5/2})^2$, $(f_{5/2})^2$ and $(f_{7/2})^2$ are evaluated for singlet s-state interactions only using equation (2.14). Figure 3 shows the separation of levels J = 0 and J = 2 as a function of λ, and figure 4 shows

Table 3

The table gives the values of $B^2/2\ell+1$ for different configuration showing the ℓ dependence of E_o in equation (3.2).

Configuration	N	n	B	B^2	$\frac{B^2}{2\ell+1}$
$(os)^2$	0	0	1.0	1.0	1.0
$(op)^2$	1	0	0.707	0.5	0.166
	0	1	-0.707	0.5	0.166
$(od)^2$	0	2	0.408	0.166	0.033
	2	0	0.408	0.166	0.033
	1	1	-0.745	0.555	0.111
$(of)^2$	0	3	0.224	0.050	0.007
	3	0	-0.224	0.050	0.007
	1	2	-0.592	0.350	0.050
	2	1	0.592	0.350	0.050
$(og)^2$	0	4	0.119	0.014	0.0015
	4	0	-0.119	0.014	0.0015
	1	3	0.414	0.171	0.019
	3	1	-0.414	0.171	0.019
	2	2	0.600	0.360	0.040

Table 4

The values of $2E_o/(2j+1)I_o$ for different configurations corresponding to different nature of interactions.

No.	Parameters	$(os)^2$	$(op)^2$	$(od)^2$	$(of)^2$	$(og)^2$
a	Long range force X_n=1 for all n	1	0.332	0.177	0.114	0.121
b	Hypothetical X_n=0 for all n	1	0.166	0.033	0.007	0.0015
c	δ-fn. X_1 = 1.50 X_2 = 1.875 X_3 = 2.00 X_4 = 2.10	1	0.415	0.261	0.190	0.146
d	λ = 0.8 X_1 = 0.71 X_2 = 0.57 X_3 = 0.50 X_4 = 0.40	1	0.284	0.131	0.075	0.048
e	λ = 1.0 X_1 = 0.63 X_2 = 0.49 X_3 = 0.43 X_4 = 0.40	1	0.271	0.119	0.066	0.042

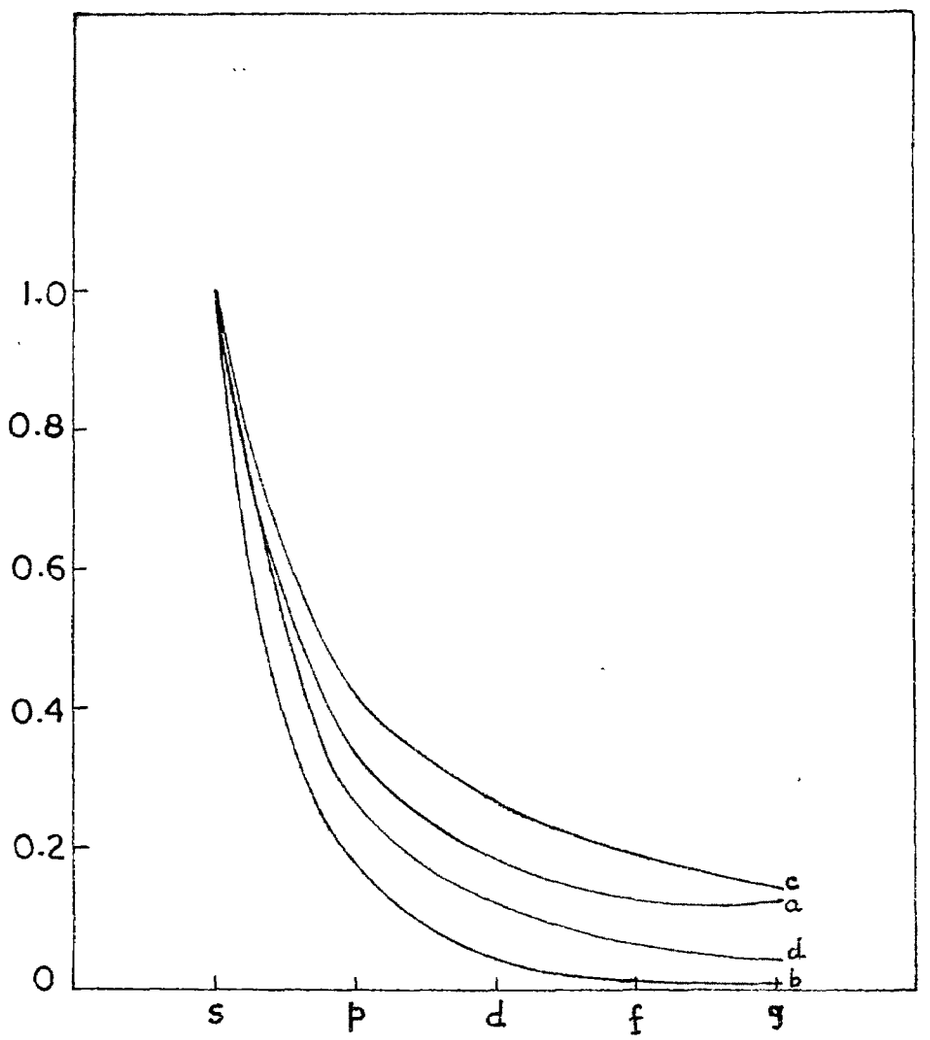
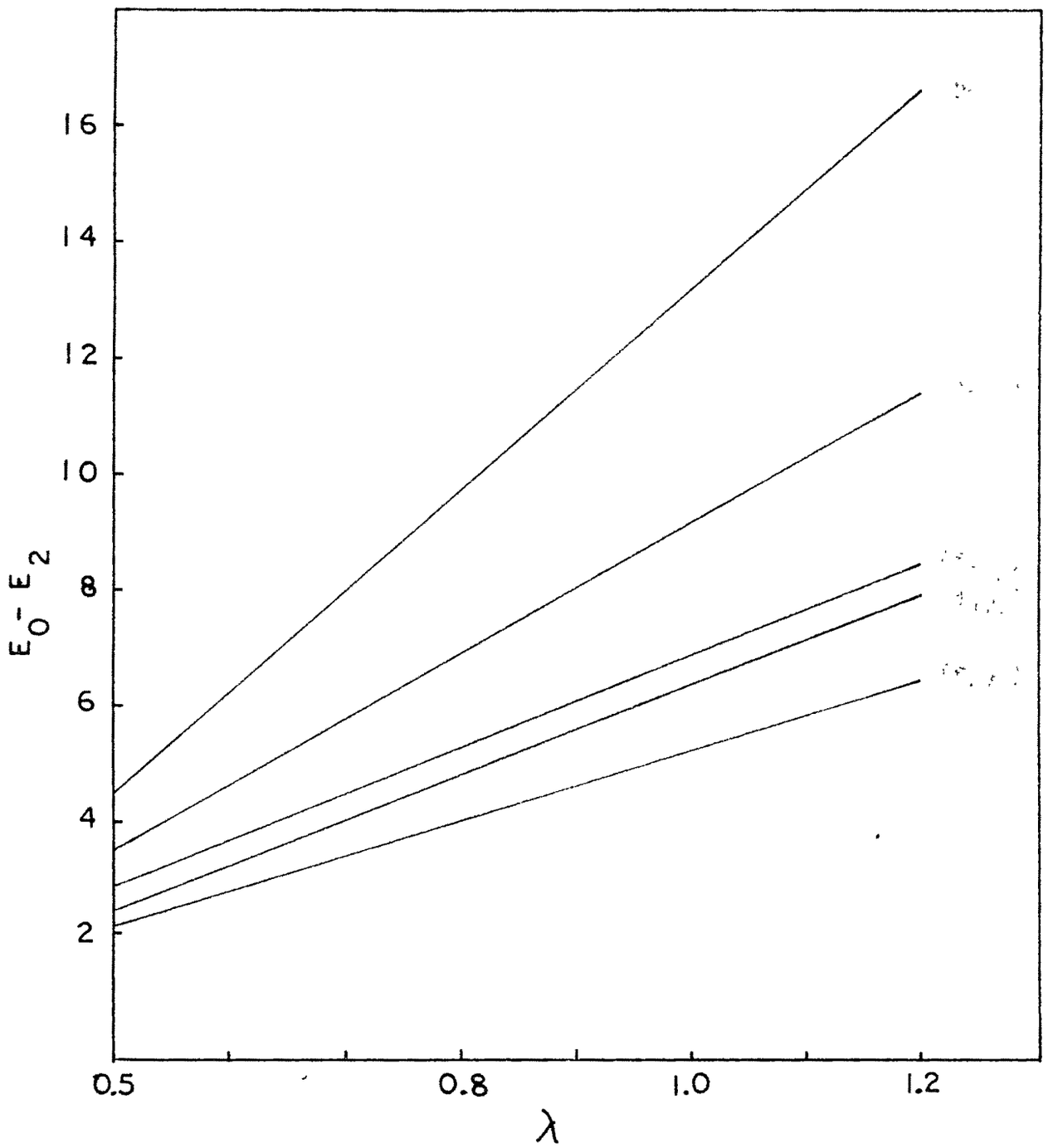


Fig. 1. $2E_a/(l_a+1)Z_a$ for [illegible] as a function of the azimuthal quantum number l.



$E_0 - E_2$ (in arbitrary units) as a function of λ for [illegible] configurations.

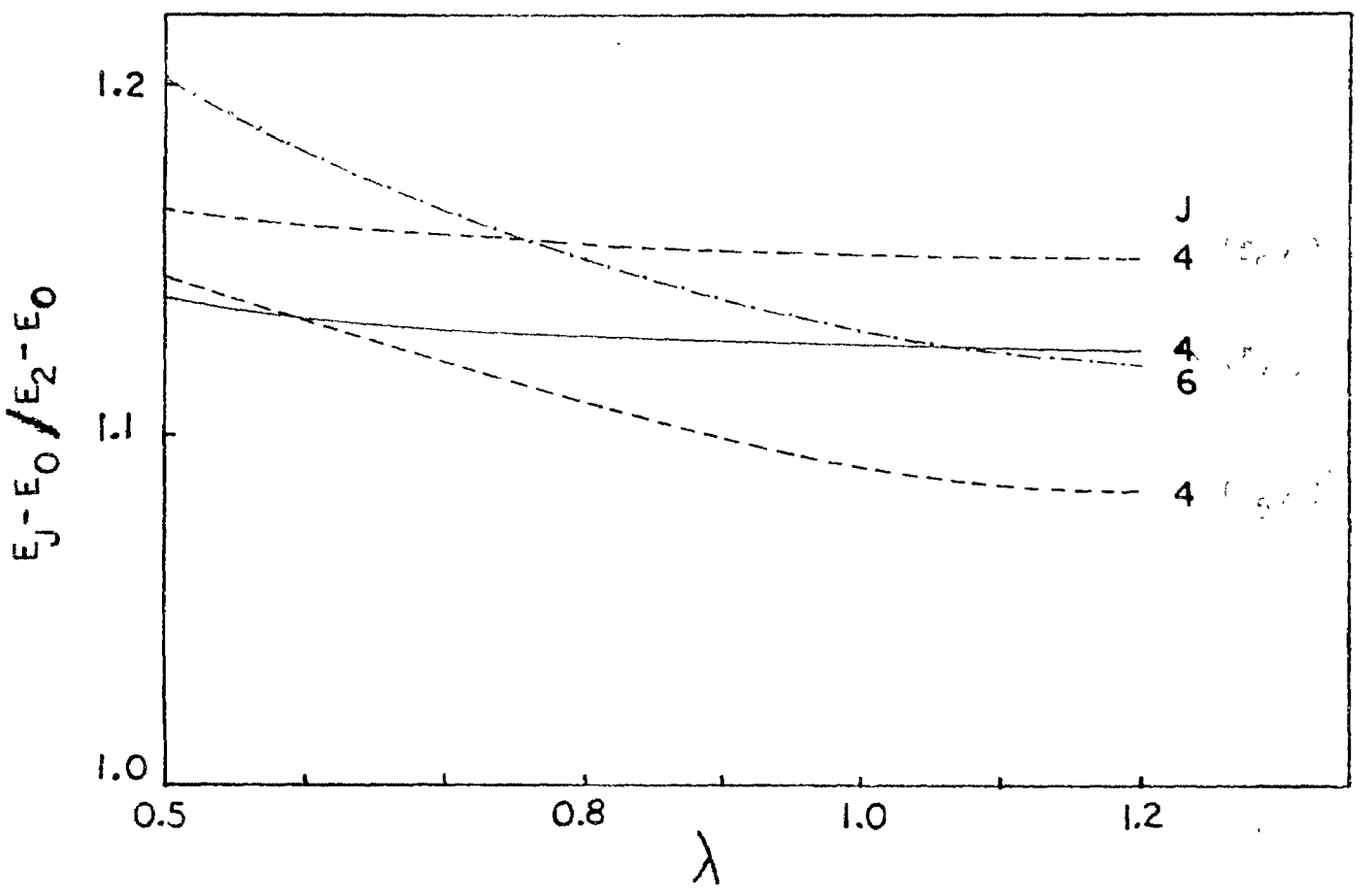


Figure. [illegible] of the [illegible] positive [illegible] as a function of λ [illegible] configurations.

the splitting of the seniority 2 levels $J \neq 0$ relative to the $J = 0$, 2 separation varying with λ.

4. Analysis of closed shell+2 nucleon configurations

In this section we use the formalism outlined in section 2 to study the nature of the effective interaction which would correctly predict the energy levels of some simple configurations in nuclei Li^6, O^{18} and Ti^{50}. Our purpose is to see if a single set of interaction parameters can explain energy levels in all these nuclei, or if there is any evidence for configuration-dependence of the interaction. This analysis can be further extended to cover heavier nuclei such as Zr^{90} or two-nucleon configuration in the vicinity of A = 208. Since the present investigation reported here is tentative and designed primarily to see if the methodology of section 2 can be successfully applied, our programme for the present has not been ambitious. It is however hoped to undertake a thorough systematic analysis of a large number of two nucleon configurations in near future.

Although in principle we can consider all the matrix elements $\langle n\ell, S : \alpha | V_{12} | n'\ell', S : \alpha \rangle$ in a Hamiltonian matrix as independent variables to be determined empirically from an analysis of the experimental data, we shall find that in general there is not enough data to do this. We therefore make some simple assumptions.

Firstly, we consider only central two-body interactions. Secondly, we neglect the contributions to the energy of the matrix elements in states with $\ell \geq 3$. This seems reasonable in view of the combined effects of the Pauli principle (which gives rise to a finite healing distance) and the centrifugal barrier. One can also see from figure 1 that at a reasonable value of range parameter λ, the matrix elements of higher states are quite small. We note that if the non-central forces are included in the analysis, the coupling of the $\ell = 1$ and $\ell = 3$ states may play an important role.

We may then consider the strength and range of the potential in each of the states $\ell = 0$, 2 ($S = 0$) and $\ell = 1$ ($S = 1$) as independent parameters. (Note that for a definite range of the interaction the matrix elements $\langle n\ell | V_{12} | n\ell \rangle$ for different n but same ℓ are all related). Even this gives too many arbitrary parameters for the data available at present, as we shall see. Therefore we shall consider two types of potentials: (a) we assume the even state interaction to be operative only in $\ell = 0$ state, and no interaction in $\ell = 2$ state. This makes the interaction rather strongly non-local. (b) More conventionally, we shall consider the same potential to be operative in $\ell = 0$ and $\ell = 2$ states. We shall allow the odd and even state interaction to have different range and strengths. In most of the usual shell model calculations the same range is assumed for interactions in all states.

The observed level schemes[31] of Li^6, O^{18} and Ti^{50} are shown in the figure 5. Spins and parities of the lowest six states of Li^6 are well known. These states are attributed to the $(p)^2$ configuration of the two extra-core nucleons, and have been analysed by a number of authors[9,10,32,33] to derive information on the nature of the nuclear forces. Our discussion shall be mainly in the framework of the analysis of Pinkston and Brennan[32]. The energy levels of the $(p)^2$ configuration in the intermediate coupling shell model scheme are described by the spin-orbit interaction parameter $\underline{a}$, and the matrix elements of V_{12} in the six states ^1S, ^1D, $^3P(T=1)$ and ^3S, ^3D, $^1P(T=0)$ of the two nucleons; here S, P, D refer to $L = 0, 1, 2$ in our notation. The Hamiltonian matrices can be atleast partially inverted to obtain some of the parameters directly in terms of the observed energies. The value of the parameter $\underline{a}$ is in this way determined to be $\underline{a} = -1.55$ MeV. It is also found that the major components of the lowest six states are singlet and triplet even states, and their energies are relatively insensitive to the matrix elements in the P-states. With reasonable assumptions, Pinkston and Brennan find

$$\begin{aligned} \langle {}^1S | V_{12} | {}^1S \rangle &= -3.85 \text{ MeV} \\ \langle {}^1D | V_{12} | {}^1D \rangle &= -2.12 \text{ MeV} \\ \langle {}^3P | V_{12} | {}^3P \rangle &\approx 0 \text{ MeV}. \end{aligned} \tag{4.1}$$

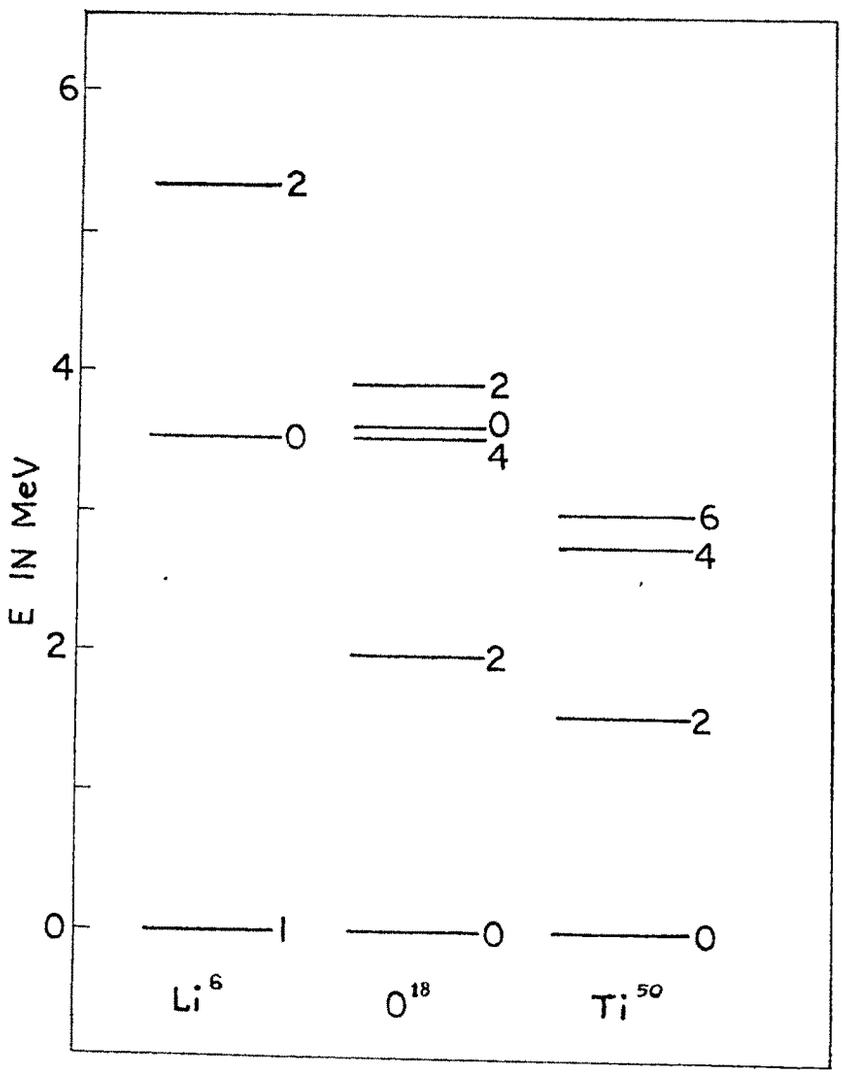


Fig.3. The energy levels observed in nuclei Li^6, O^{18} and Ti^{50}.

These values can be made more precise, when the P-states which are expected to occur at $\sim$ 8-10 MeV are identified. In terms of states of relative orbital angular momenta† matrix elements in ^1S and ^1D states can immediately be written down

$$\langle {}^1S | V_{12} | {}^1S \rangle = \frac{1}{2}\left[\langle 0s | V_{12} | 0s \rangle + \langle 1s | V_{12} | 1s \rangle\right]$$

$$\langle {}^1D | V_{12} | {}^1D \rangle = \frac{1}{2}\left[\langle 0s | V_{12} | 0s \rangle + \langle 0d | V_{12} | 0d \rangle\right] \qquad (4.2)$$

From the known values of these two matrix elements on the left hand side, and with the assumption of V_{12} to be a Gaussian shape potential it is possible to determine the parameters λ and the depth V_0 of this potential in singlet even states. If we assume the effective interaction to be operative only in s-states ($\ell = 0$) we obtain

$$\langle 0s | V_{12} | 0s \rangle = -4.24 \text{ MeV}$$

$$\langle 1s | V_{12} | 1s \rangle = -3.46 \text{ MeV} \qquad (4.3)$$

and

$$V_0 = -24 \text{ MeV}$$

$$\lambda = 0.68 .$$

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† The analysis of Pinkston and Brennan has already yielded matrix elements in L-S coupling representation, hence we do not need the A-coefficient in this case.

On the otherhand, if we assume that the same interaction operates in $\ell = 0$, 2 states, we find

$$\begin{aligned} \langle 0s | V_{12} | 0s \rangle &= -4.0 \text{ MeV} \\ \langle 0d | V_{12} | 0d \rangle &= -0.20 \text{ MeV} \\ \langle 1s | V_{12} | 1s \rangle &= -3.70 \text{ MeV} \end{aligned} \tag{4.4}$$

and

$$\begin{aligned} V_0 &= -34 \text{ MeV} \\ \lambda &= 0.57. \end{aligned}$$

The range of the potential can be obtained by calculating the value of r_ℓ. For harmonic oscillator wavefunction, the root mean square radius is given by

$$\langle r^2 \rangle = \frac{5}{4} r_p^2 = (2.7)^2 fm^2 \tag{4.5}$$

where the last step is obtained from the electron scattering data[34]. Thus $r_p = 2.5$ fm, and for $\lambda = 0.68$ and 0.57 the range r_0 has the values 1.7 fm and 1.4 fm respectively.

In the above analysis the p-state matrix element was assumed to be zero following Pinkston and Brennan. We wish to emphasise that the knowledge of the odd-state interactions in light nuclei is very very inadequate, particularly since the energies of the lowest few levels (against which our nuclear force parameters can be tested) are not affected to any large extent by these odd-state interactions. We have already listed in table 1 of chapter I many different exchange interactions used in

light nuclei. We note that the range of the interaction in all cases is assumed to be same for odd- and even-state interactions.

One of the aims of our analysis is to obtain a better knowledge of p-state interactions. It is clear empirically that at least for Li^6 the p-state matrix element is small. Taking the potential strength of Soper[9] or Meshkov and Ufford[10], and the same range for odd and even forces, we find

$$\langle 0p|V_{12}|0p\rangle \simeq -0.5 \text{ MeV}.$$

This is consistent with the zero value of Pinkston and Brennan, and inserting this value in the Hamiltonian matrices, we can easily show that the singlet even energies are not essentially affected.

Now let us consider the nuclei O^{18} and Ti^{50}. The lowest few levels will be attributed to the configurations of the last two nucleons in the unfilled subshells. In the spirit of the Brueckner theory we include for configuration mixing only approximately degenerate orbits. We therefore restrict ourselves to configurations which lie within 3-4 MeV of the ground state configuration. Thus for O^{18}, we only consider the states

$(d_{5/2})^2$ J = 0, 2, 4 $(d_{5/2}s_{1/2})$ J = 2,3 and

$(s_{1/2})^2$ J = 0,

and for Ti^{50}

$(f_{7/2})^2$ J = 0,2,4,6 $(f_{7/2}p_{3/2})$ J = 2, 4 and

$(p_{3/2})^2$ J = 0,2.

The energy separation of the single particle levels $d_{5/2}$ and $s_{1/2}$ is taken as 0.88 MeV and that of $f_{7/2}$ and $p_{3/2}$ to be 1.95 MeV. Hamiltonian matrices are now constructed using equation (2.13).

Let us first consider effective interactions to be operative in s-state only. Hamiltonian matrices are diagonalised for V_o = -25, -30, -35 and -40 MeV and ranging from 0.5 to 1.2. In table 5 we give the matrix elements in units of V_o for O^{18} and in table 6, those of Ti^{50}. One can take a fixed value of V_o and multiply the matrix elements given in tables 5 and 6 and add the appropriate single particle energy and carry out the explicit diagonalisation for each value of λ. We give the exact calculations for two values of V_o = -25 and -40 MeV as shown in figures 6, 7, 8 and 9.

We see that for O^{18}, the qualitative features of the level spectrum can be easily obtained. For each value of V_o and a corresponding value of λ (λ = 0.95 for V_o = -25 MeV, and λ = 0.65 for V_o = -40 MeV), we can obtain a first excited state 2 at about 2 MeV, and a group of states† with J = 4,0* and 2* at 2.8-3.5 MeV.

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† The notation here is that the unstarred and starred values refer to the lowest and the first excited states of given J.

Table 5

The matrix elements in units of V_o for O^{18} for different values of $\lambda = r_o/r_\ell$ (singlet s-state interactions only).

Configuration.	J \ λ	0.5	0.8	1.0	1.2
$\langle d^2_{5/2} \mid d^2_{5/2} \rangle$	0	0.0469	0.0958	0.1264	0.1575
	2	0.0118	0.0259	0.0348	0.0436
	4	0.0067	0.0183	0.0265	0.0340
$\langle s^2_{1/2} \mid s^2_{1/2} \rangle$	0	0.0380	0.0843	0.1160	0.1463
$\langle d_{5/2} s_{1/2} \mid d_{5/2} s_{1/2} \rangle$	2	0.0164	0.0440	0.0638	0.0816
$\langle d^2_{5/2} \mid d_{5/2} s_{1/2} \rangle^2$		0.0100	0.0252	0.0358	0.0455
$\langle d^2_{5/2} \mid s^2_{1/2} \rangle$	0	0.0160	0.0384	0.0548	0.0699

Table 6

The matrix elements in units of V_o for Ti^{50} for different values of λ (singlet s-state interactions only).

Configuration.	J \ λ	0.5	0.8	1.0	1.2
$\langle f^2_{7/2} \mid f^2_{7/2} \rangle$	0	0.0388	0.0726	0.0934	0.1151
	2	0.0098	0.0190	0.0246	0.0305
	4	0.0058	0.0121	0.0160	0.0200
	6	0.0040	0.0109	0.0158	0.0203
$\langle p^2_{3/2} \mid p^2_{3/2} \rangle$	0	0.0290	0.0650	0.0895	0.1123
	2	0.0074	0.0175	0.0245	0.0311
$\langle f_{7/2}p_{3/2} \mid f_{7/2}p_{3/2} \rangle$	2	0.0154	0.0352	0.0482	0.0608
	4	0.0060	0.0162	0.0236	0.0302
$\langle f^2_{7/2} \mid p^2_{3/2} \rangle$	0	0.0156	0.0333	0.0451	0.0563
	2	0.0035	0.0078	0.0109	0.0138
$\langle f^2_{7/2} \mid f_{7/2}p_{3/2} \rangle$	2	0.0082	0.0180	0.0242	0.0303
	4	0.0045	0.0099	0.0132	0.0165
$\langle p^2_{3/2} \mid f_{7/2}p_{3/2} \rangle$	2	0.0031	0.0107	0.0165	0.0214

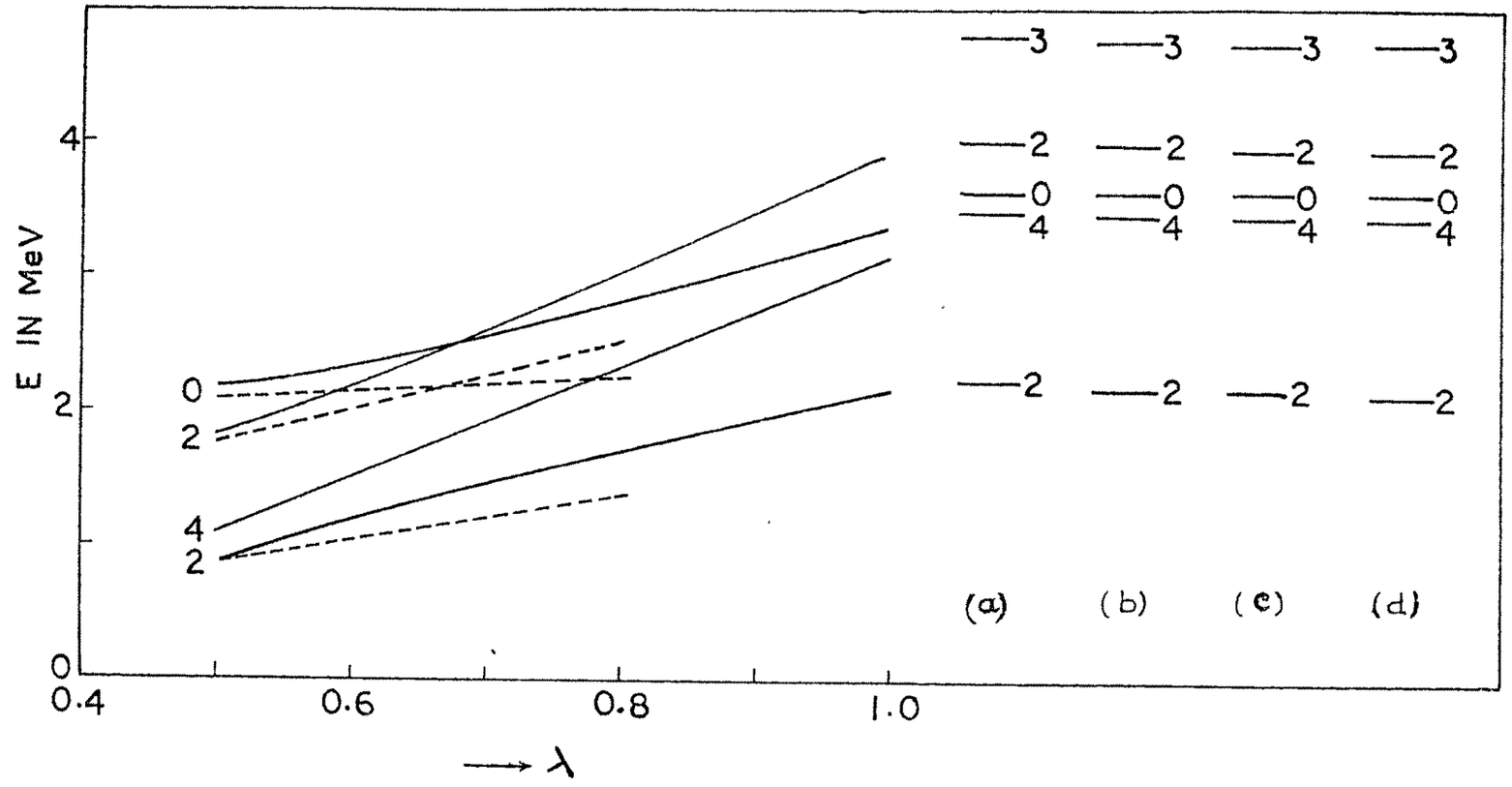


Fig.6. Theoretical level ordering in O^{18} as a function of range parameter λ for V_0 = -25 MeV. The full curve represent the levels with singlet s- state interactions only. The dotted curves represent the levels with singlet s- and d- state interactions. The right hand side of the figure gives the level schemes obtained with parameters V_0 = -25 MeV, λ = 1.0 (singlet s- state interactions only) and triplet p-state interactions with parameters (a) V_1 = -6.58 MeV, λ = 0.5 (b) V_1 = -8.29 MeV, λ = 0.8 (c) V_1 = -5.32 MeV, λ = 1.0 and (d) V_1 = -7.11 MeV, λ = 1.2. The state J = 0 is normalised to the value E = 0.

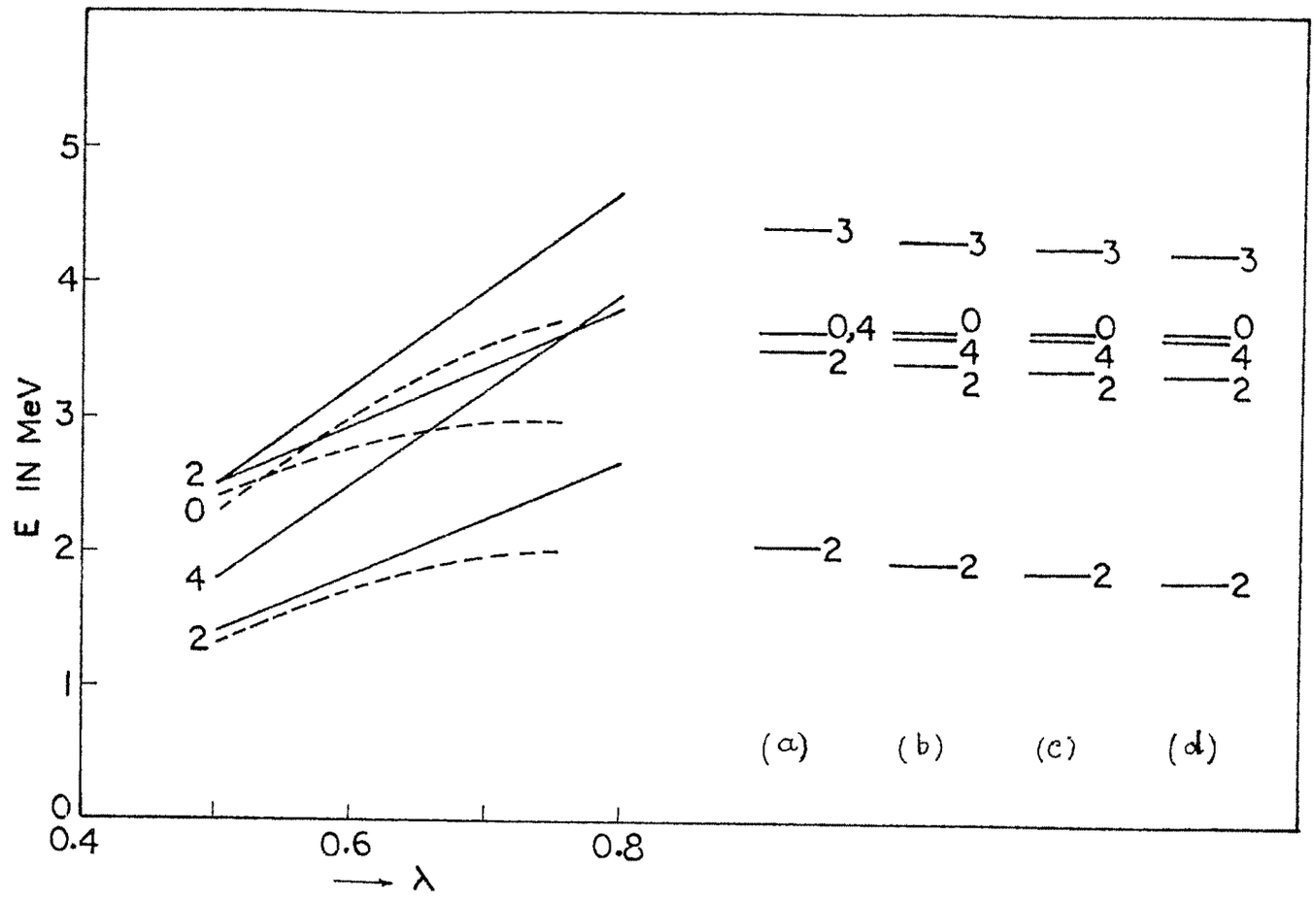
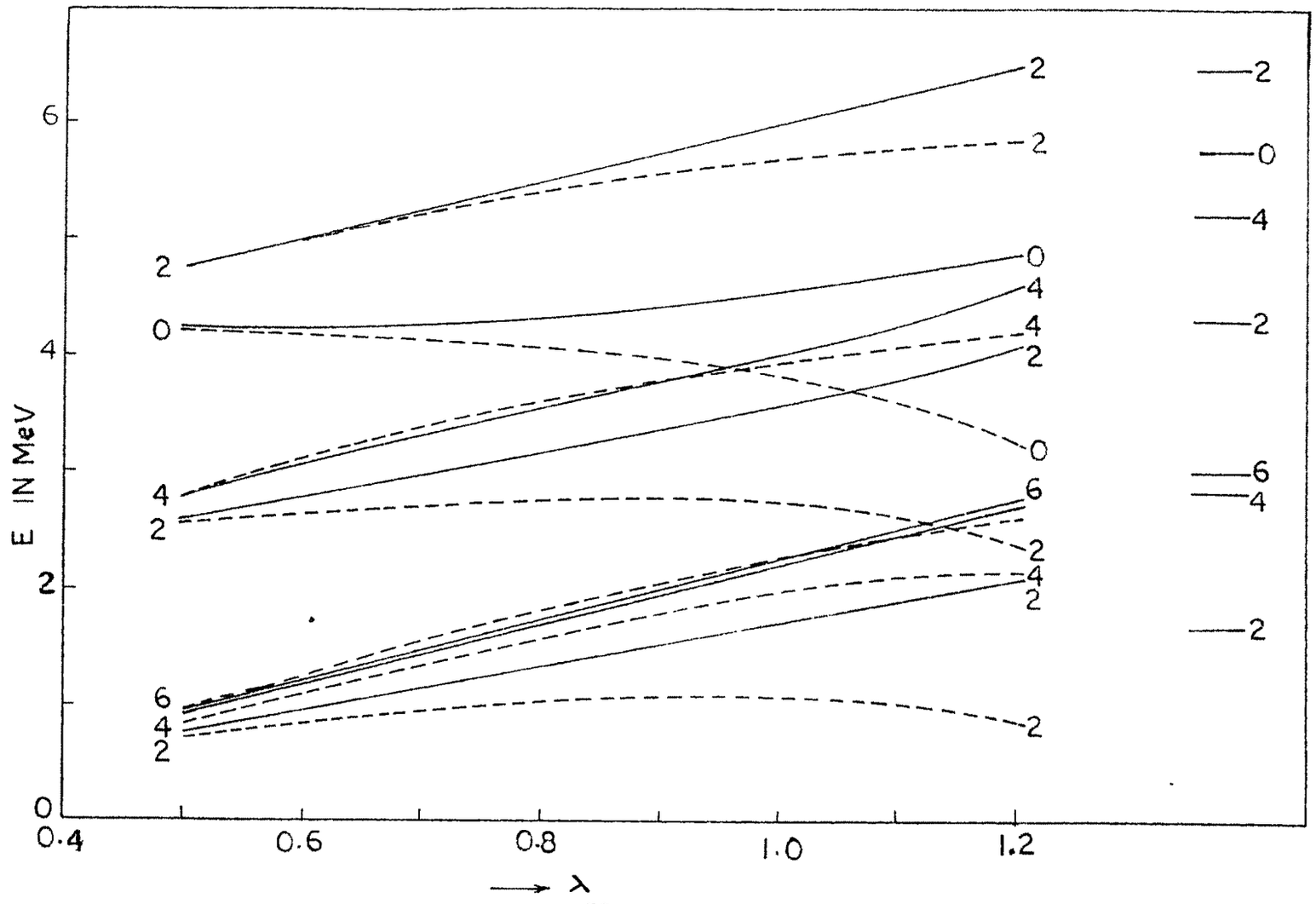


Fig. 1. Theoretical level ordering in O^{16} as a function of range parameter λ for $V_s = -40$ MeV. The full curves represent the levels with singlet s-state interactions only. The dotted curves represent the levels with singlet s- and d-state interactions. The right hand side of the figure gives the level schemes obtained with parameters $V_0 = -40$ MeV, $\lambda = 0.55$ (singlet s- and d- state interactions) and triplet s-state interaction with parameters (a) $V_t = -10.4$ MeV, $\lambda = 0.4$; (b) $V_t = -25.2$ MeV, $\lambda = 0.43$; (c) $V_t = -41.4$ MeV, $\lambda = 1.0$ and (d) $V_t = -10.6$ MeV, $\lambda = 1.0$. The triplet s-state [illegible] if required to fit


E IN MeV
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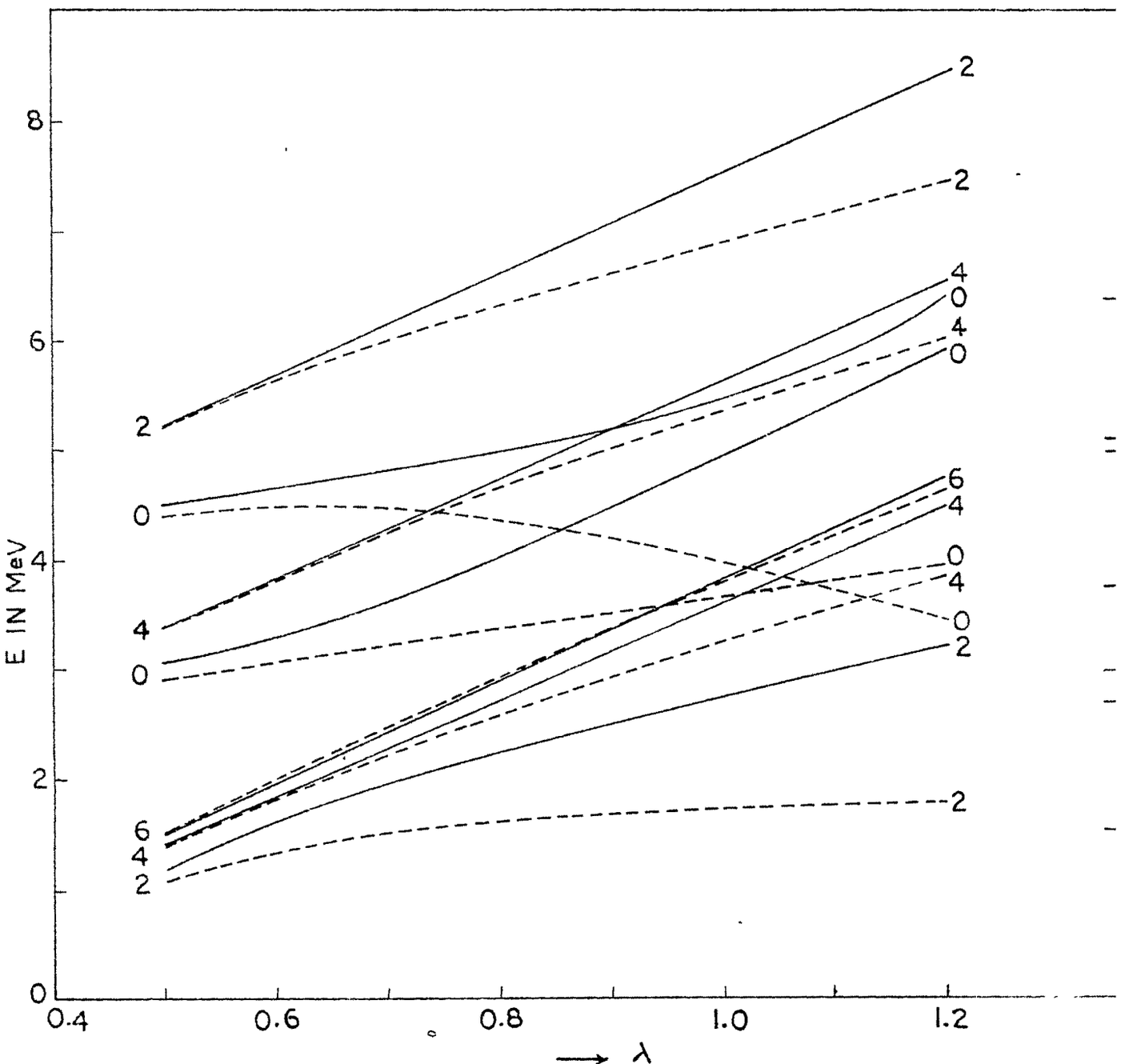

[illegible] of the [illegible] in Ti^{50} as a function of range parameter λ [illegible] with [illegible]-state interaction. [illegible] and [illegible]-state interactions. The [illegible] λ [illegible]

However this triad of states is somewhat low compared to the experimental results. On the other hand for Ti^{50}, the agreement between theory and experiment is quite poor. If the first excited state is correctly fitted (at $V_o = -25$ MeV, $\lambda = 0.85$ or $V_o = -40$ MeV, $\lambda = 0.57$) the states 4, 6 occur about 1 MeV too low (at 1.75 MeV). We next note that for $V_o = -25$ MeV, the values of λ obtained above $\lambda = 0.95$ (O^{18}) and 0.85 (Ti^{50}) give for the range of the potential $\beta_o = 1.7$ and 2.0 fm respectively, values which are in fair agreement with the range obtained for Li^6, see equation (4.3). Thus out of many possible sets of parameters which give almost similar results for singlet s-state interactions we choose for further investigation parameters corresponding to $V_o = -25$ MeV.

It is now of interest to investigate the effect of triplet p-state interactions. We give in tables 7 and 8, the matrix elements in triplet p-state in units of V_1, the strength of the potential, for O^{18} and Ti^{50} respectively. The results of the calculations show that the excitation energy of the states $J = 2$ and 2^* for O^{18} and the state $J = 2$ for Ti^{50} are not significantly affected by the p-state forces. However, for O^{18} the states $J = 4$ and 0^* are pushed up by attractive interactions and are depressed by repulsive interactions. This clearly argues in favour of attractive p-state forces, and show that by inclusion of suitable interactions one can obtain agreement with the experiments. In figure 6, we also show the energy levels

Table 7

The matrix elements in units of V_1 for O^{18} for different values of λ (triplet p-state interactions only)

Configuration.	J \ λ	0.5	0.8	1.0	1.2
$\langle d^2_{5/2} \vert d^2_{5/2} \rangle$	0	0.0094	0.0396	0.0663	0.0946
	2	0.0071	0.0367	0.0674	0.1012
	4	0.0009	0.0048	0.0088	0.0134
$\langle s^2_{1/2} \vert s^2_{1/2} \rangle$	0	-	-	-	-
$\langle s_{1/2} d_{5/2} \vert s_{1/2} d_{5/2} \rangle$	2	0.0054	0.0252	0.0446	0.0658
	3	0.0067	0.0315	0.0558	0.0833
$\langle d^2_{5/2} \vert s_{1/2} d_{5/2} \rangle$	2	-	-	-	-
$\langle d^2_{5/2} \vert s^2_{1/2} \rangle$	0	-	-	-	-

Table 8

The matrix elements, in units of V_1 for Ti^{50} for different values of λ (triplet p-state interactions only).

Configuration.	J \ λ	0.5	0.8	1.0	1.2
$\langle f^2_{7/2} \mid f^2_{7/2} \rangle$	0	0.0129	0.0411	0.0639	0.0871
	2	0.0123	0.0405	0.0632	0.0864
	4	0.0065	0.0264	0.0432	0.0606
	6	0.0057	0.0306	0.0568	0.0861
$\langle p^2_{3/2} \mid p^2_{3/2} \rangle$	0	0.0039	0.0143	0.0229	0.0319
	2	0.0078	0.0285	0.0458	0.0639
$\langle f_{7/2}p_{3/2} \mid f_{7/2}p_{3/2} \rangle$	2	0.0060	0.0250	0.0424	0.0610
	4	0.0034	0.0182	0.0330	0.0498
$\langle f^2_{7/2} \mid p^2_{3/2} \rangle$	0	-	-	-	-
	2	-	-	-	-
$\langle f^2_{7/2} \mid f_{7/2}p_{3/2} \rangle$	2	-	-	-	-
	4	-	-	-	-
$\langle p^2_{3/2} \mid p_{3/2}f_{7/2} \rangle$	2	-	-	-	-

calculated for O^{18} with p-state parameters adjusted to obtain the best fit. Again it is found that these parameters cannot be uniquely fixed with the given data, and several different values of V_1 and κ_1 are possible (such as $V_1 = -37$ MeV, $\kappa_1 = 0.9$ fm and $V_1 = -3.7$ MeV, $\kappa_1 = 2.2$ fm). Here, the s-state parameters are so chosen as to give the best fit for the J = 2 and 2* states which are not affected much by the p-state forces ($V_0 = -25$ MeV, $\kappa_0 = 1.8$ fm) and the p-state parameters are so chosen as to keep J = 0* state always at 3.62 MeV.

In case of Ti^{50} one finds that with suitable attractive p-state forces, one can raise the J = 4, 6 states and also increase their separation. In this case we also find that if the range of the triplet p-interaction is large (corresponding to $\lambda > 0.5$) the J = 6 state is depressed below the J = 4 state. This result leads us to effectively shorter range for p-state forces. In figure 8 we show one set of parameters ($V_1 = -155$ MeV, $\lambda = 0.5$) for p-states which give a good fit to the experimental data.

We can now summarise the results. It is possible to choose a singlet s-state interaction which has the same parameters in Li^6, O^{18} and Ti^{50} nuclei. Its parameters seem to be reasonable. On the other hand p-state interaction parameters appear to change systematically as we go from A = 6 to A = 50. For Ti^{50} the interaction appears to be quite strong with a rather short range ($V_1 = -155$ MeV, $\kappa_1 = 1.2$ fm).

For O^{18}, the parameters are not uniquely fixed, but if we take the same range viz., $\ell_1 = 1.2$ fm the strength V_1 definitely appears to be weaker, perhaps by a factor of 5. For comparison, the matrix element $\langle op | V_{12} | op \rangle$ in O^{18} would have a value of ≈ -0.7 MeV, whereas in Ti^{50} it has the value ≈ -2.8 MeV. It was concluded that in Li^6, although the p-states are not yet observed, $\langle op | V_{12} | op \rangle \lesssim 0.5$ MeV.

Next, we consider the results of the alternative assumption according to which the same interaction is operating in both the even states $\ell = 0$ and $\ell = 2$. In the same way as above, the Hamiltonian matrices were diagonalised for $V_o = -25$, -30, -35 and -40 MeV and λ ranging from 0.5 to 1.2. In table 9, we give the matrix elements in units of V_o for O^{18} and those for Ti^{50} in table 10. Here also we give the exact calculations for only two values of V_o viz., -25 and -40 MeV. The energy levels calculated on this basis for even state interactions only are shown by dotted lines in the figures 6 - 9. It will be noticed the changes in the excitation energies are most marked for the states with $J = 0^*$, 2 and 2^*, which are now depressed. As we have previously remarked, the p-state interaction do not affect the $J = 2$ state much, and hence we choose the even state parameters to obtain a good fit for this state. (Here also, it may be noted that in O^{18} the p-state parameters are so chosen as to give $J = 0^*$ state at 3.62 MeV). This leads us to choose $V_o = -40$ MeV,

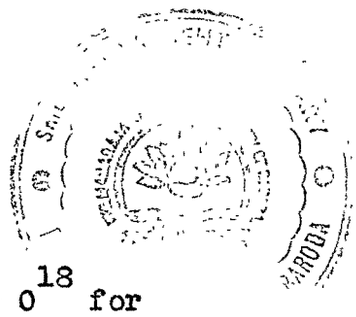

Table 9

The matrix elements, in units of V_o for O^{18} for different values of λ (singlet s- and d-state interactions)

Configuration.	J \ λ	0.5	0.3	1.0	1.2
$\langle d^2_{5/2} \mid d^2_{5/2} \rangle$	0	0.0471	0.0983	0.1323	0.1680
	2	0.0132	0.0394	0.0659	0.0980
	4	0.0069	0.0202	0.0309	0.0419
$\langle s^2_{1/2} \mid s^2_{1/2} \rangle$	0	0.0400	0.1049	0.1651	0.2341
$\langle d_{5/2} s_{1/2} \mid d_{5/2} s_{1/2} \rangle$	2	0.0186	0.0624	0.1026	0.1456
$\langle d^2_{5/2} \mid s^2_{1/2} \rangle$	0	0.0153	0.0313	0.0378	0.0395
$\langle d^2_{5/2} \mid s_{1/2} d_{5/2} \rangle$	2	0.0100	0.0225	0.0269	0.0270

Table 10

The matrix elements in units of V_o for Ti^{50} for different values of λ (singlet s- and d-state interactions)

Configuration.	J \ λ	0.5	0.3	1.0	1.2
$\langle f^2_{7/2} \mid f^2_{7/2} \rangle$	0	0.0395	0.0778	0.1041	0.1326
	2	0.0122	0.0373	0.0618	0.0907
	4	0.0064	0.0173	0.0280	0.0410
	6	0.0041	0.0119	0.0182	0.0245
$\langle p^2_{3/2} \mid p^2_{3/2} \rangle$	0	0.0309	0.0792	0.1187	0.1599
	2	0.0086	0.0256	0.0408	0.0576
$\langle f_{7/2}p_{3/2} \mid f_{7/2}p_{3/2} \rangle$	2	0.0182	0.0530	0.0820	0.1132
	4	0.0064	0.0192	0.0302	0.0414
$\langle f^2_{7/2} \mid p^2_{3/2} \rangle$	0	0.0145	0.0247	0.0274	0.0275
	2	0.0033	0.0062	0.0075	0.0082
$\langle f^2_{7/2} \mid f_{7/2}p_{3/2} \rangle$	2	0.0081	0.0147	0.0159	0.0147
	4	0.0045	0.0093	0.0114	0.0127
$\langle p^2_{3/2} \mid f_{7/2}p_{3/2} \rangle$	2	0.0038	0.0034	0.0195	0.0238

$\lambda = 0.70$ for Ti^{50} and $V_o = -40$ MeV, $\lambda = 0.65$ for O^{18}. (We remark that for $V_o = -25$ MeV and $\lambda > 0.8$ in O^{18}, the wavefunction of the $J = 2$ state gives a dominant component of $(s_{1/2}d_{5/2})$ configuration). The corresponding ranges in the two nuclei would be 1.5 fm and 1.2 fm. It may be remembered that for Li^6, we had very similar parameters, equation (4.4). The above choice of the parameters gives rather low values for the excitation energies of the higher states. This can be remedied to a large extent as in the previous case by inclusion of p-state forces. Figures 7 and 9 also give the results obtained for the level spectra with the inclusion of the p-state forces. Again a reasonably good agreement between theory and experiment is obtained. For O^{18} the $J = 2^*$ state is predicted a little lower than its observed position. For Ti^{50} the p-state interaction required has again a short range $\beta_1 = 1.2$ fm and depth -85 MeV, whereas for O^{18} one can fit the data with several different p-state interactions, one of which would be $V_1 = -45$ MeV, $\beta_1 = 1.2$ fm. Thus again the p-state force appears to grow stronger as we go to heavier nuclei.

It should be kept in mind that there is yet some uncertainty about the energy level scheme of Ti^{50}: the 6 level is yet to be identified, although we have assumed it to be at 3.0 MeV. The identification of this level as well as of the spins and parities of other excited states would certainly enable us to refine and improve the present analysis.

Now we add one remark on yet unobserved $J = 3^+$ state which is predicted from $(s_{1/2}d_{5/2})$ configuration in O^{18}. This state is solely due to triplet p-state interactions and the matrix elements in units of V_1 are given in table 7. Since it has not been possible to choose a unique set of parameters for triplet p-state interactions, we give the predicted position of this level for all sets of parameters which give equally good fit for the remaining known energy levels of O^{18}. The position of this level corresponding to parameters given in figure 6 is at $\sim$ 4.7 MeV and corresponding to those in figure 7 is at $\sim$ 4.3 MeV. The experimental evidence for the existence of this state will be able to throw some light on the nature of triplet odd-state interactions. It may be noted that $J = 3^-$ level has been detected at $\sim$ 4.5 MeV. The existence of this level should be a point of interest.

5. Conclusions

In the previous sections we have made an attempt to study systematically the parameters of the effective two-body nuclear interaction for a few configurations in terms of some simple models. It is hardly possible with the available data to decide in favour of a unique model for this effective "residual" interaction. We have considered only central forces, and neglected their contribution to the energy from states of relative orbital angular momenta $\ell \geq 3$. This seems to us a good assumption. We find that the available energy level

data on the configurations chosen for study here, can be explained in terms of either

(i) a non-local even interaction which operates only in $\ell = 0$ state with $V_0 = -25$ MeV and range $\beta_0 = 1.8 - 2.0$ fm and an odd-state interaction in $\ell = 1$ state which appears to change as we go to heavier nuclei or higher configurations, growing stronger and perhaps shorter ranged, or

(ii) an even state interaction with $V_0 = -40$ MeV, $\beta_0 = 1.2 - 1.4$ fm operating in both s- and d-states, and an interaction in p-state which again appears to increase in strength as one goes to higher shell model levels. We also note that the interaction in the p-state is definitely found to be attractive.

Much additional work remains to be done. The effect of including tensor forces in the odd-states has to be investigated. It is of interest to consider configurations with three identical nucleons outside a closed shell and compare the effective interactions in such configurations with our results above, e.g. in the pairs of nuclei O^{18}, O^{19} and Ti^{50}, V^{51}. Of these pairs, we make study of O^{18}, O^{19} pair in the chapter III. Similar calculations may also be carried out for heavier nuclei such as Ni^{60}, Zr^{90}, Zr^{92} and even Pb^{206} etc.

One may expect that in relative orbital angular momentum states with $\ell \geq 1$, the effect of hard cores or singularities in the realistic nucleon-nucleon interactions would not be of any critical importance for shell model wavefunctions, and in such states the reaction matrix and the potential matrix should give rather similar results. Thus to a reasonable approximation the contribution to the energy of a state from such states with $\ell \geq 1$ in equation (2.12), may be calculated by using a simple potential which fits well the low energy nucleon-nucleon data. Then the matrix elements in the s-state may alone be considered as empirical parameters to be determined so as to give a good fit to the observed level spectrum. These matrix elements would then be the reaction matrix elements in singlet s-states for finite nuclei, and may be compared with similar matrix elements calculated from realistic potentials (local or non-local) that have been proposed by many authors. It would not be too difficult to evaluate the reaction matrix for singlet s-states even for finite nuclei.

We wish to emphasise the approach and methodology of this chapter, which we hope will enable a more useful and flexible analysis to be made of the effective interaction when enough experimental data is available on both T = 1 and T = 0 states of two nucleon configurations.

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