CHAPTER 1

INTRODUCTION AND FUNDAMENTAL CONCEPTS

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1.1 Fluid

Substances that flow when shear stress is applied are referred to be fluids. Gases and liquids are both fluids.

1.2 Newtonian fluid

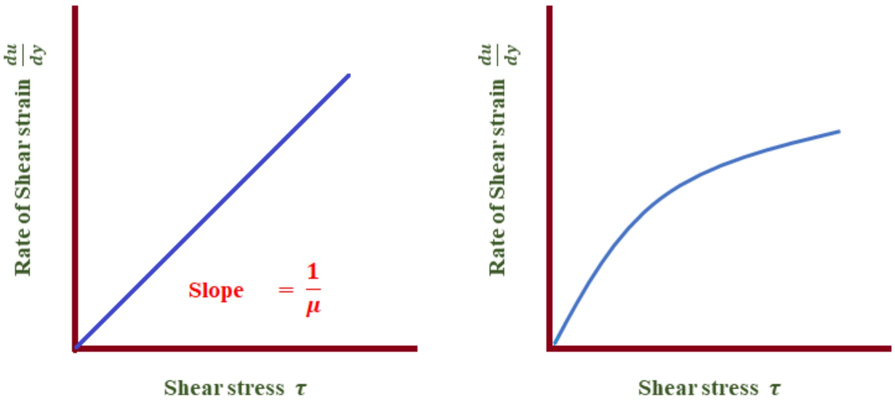
Newtonian fluids include substances like water, benzene, alcohol, & hexane, and many more that correspond to Newton's law of viscosity. Figure 1.1a illustrates the linear relationship between the stress tensor and the rate of strain in a Newtonian fluid. Gasoline, glycerin, air, water, and other Newtonian fluids are just a few examples.

1.3 Non-Newtonian fluid

Non-Newtonian fluids, such as pastes, gels, polymer solutions, Carreau fluid, William -son fluid, Micropolar fluid, etc., are those that oppose Newton's law of viscosity. Non-Newtonian Fluid is a fluid in which shear stress and rate of shear strain are not linearly related as shown in figure 1.1b. Blood, grease, honey, shampoo, custard, toothpaste, paint are few examples of non-Newtonian fluids.

1.4 Applications of Non-Newtonian fluid

The study of non-Newtonian fluids is very important since it has many engineering and industrial applications. Such fluids are utilised specifically in the following



(a) Newtonian fluid (b) Non-Newtonian fluid

Figure 1.1: Types of fluid

fields: prescription medications, physiology, material processing, fibre technology, chemical and nuclear industries, oil reservoir engineering, and meals. Shampoos, apple sauce, ketchup, blood at low shear rates, polymer solutions, paints, food items, milk, coating of wires, grease, crystal development, and many more fluids are examples of this type.

1.5 Steady and unsteady flow

A flow in which the various quantities like velocity, pressure and density at any point do not change with time is said to be a steady flow. For steady flow, if u is the velocity at a point then $\frac{\partial u}{\partial t} = 0$. A flow in which the parameter depends on time is called unsteady flow.

1.6 Compressible and incompressible flow

It is usual to classify flows into two categories. Gases can be compressed, and changes in temperature and pressure quickly affect their density. On the other hand, liquids can technically be referred to as incompressible fluids because they are so difficult to compress.

1.7 Laminar flow

Every liquid particle must follow a precise path for a flow to be classified as laminar, viscous, or stream line flow. One particle's route does not cross any other particle's path.

1.8 Magnetohydrodynamics

The study of electrically conducting fluid flow in the presence of magnetic field is known as magnetohydrodynamics (MHD).

1.9 Applications of Magnetohydrodynamics

This includes liquid metals like gallium, mercury, and sodium in addition to molten iron. Petroleum, chemical, and metallurgical processing industries provide as the best examples of the significance of MHD fluid flow over a deforming body. Additional real-world applications include surface cooling in technology, wind-up roll processes, and polymer film.

1.10 Magnetohydrodynamics Flow

Magnetohydrodynamics deals with the dynamics of fluids having nonnegligible electrical conductivity which interact with a magnetic field. As a result, motion of an electrically conducting fluid in the presence of a magnetic field, electric current is induced in the fluid. An electrically conducting fluid moving in presence of a magnetic field (transverse) which generate a force called the Lorentz force. This force has a tendency to modify the initial motion of the conducting fluid. Moreover, the induced currents generate their own magnetic field, which is added to the primitive magnetic field. Thus there is an interlocking between the motion of the conductor and the electromagnetic field. The study of MHD has significantly used both in nature and in man-made devices such as cooling of nuclear reactors, metal-working processes, MHD generators, MHD-based micro-coolers, MHD-based stirrer and MHD-based micro-pumps and many more. One of the application of MHD can be seen in Figure 1.2.

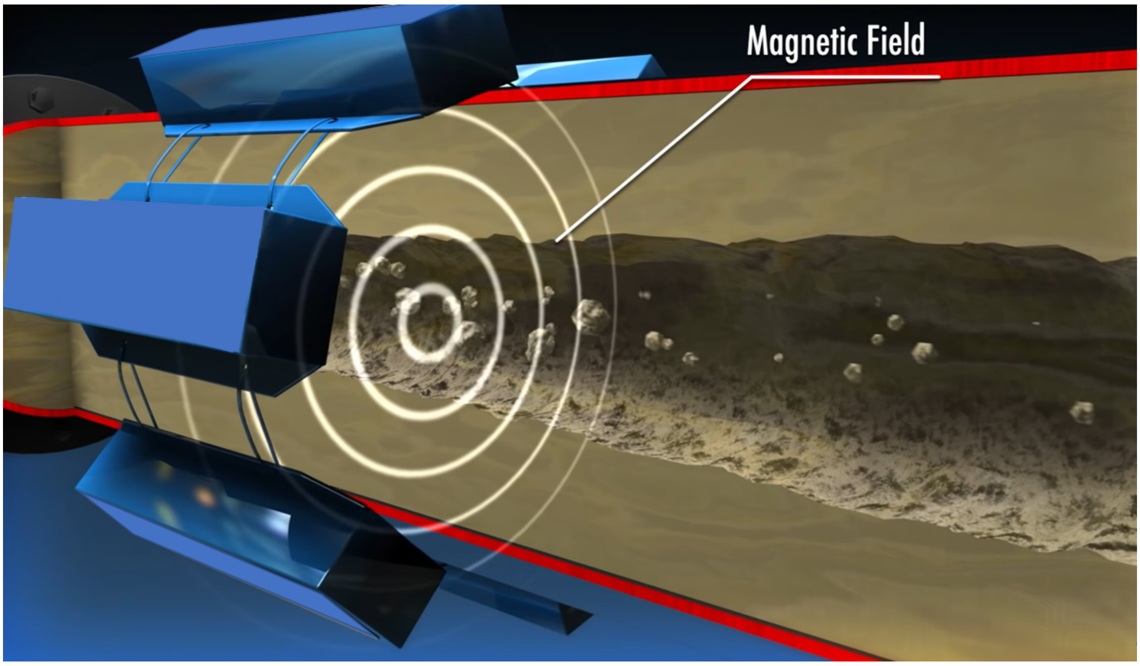


Figure 1.2: Application of MHD

1.11 Entropy

A measurement of molecular disorder or unpredictability is entropy. Finding ways to regulate the usage of efficient energy is currently one of engineers' and academics' top priorities. The main goal in the field of thermal engineering is to maximise device efficiency while minimising heat loss, friction, and dissipation during mechanical processes. Many energy-related issues, such as those involving thermal energy, cooling of contemporary electronic systems, geothermal energy systems, and solar power collectors, have drawn substantial interest to the research of entropy generation minimization. The benchmark of the destruction of accessible work of a considered structure as observed by Bejan [4, 5] is entropy generation analysis.

1.11.1 Local rate of entropy generation

$$S'''_{gen} = \frac{\kappa}{T_0^2}\left(\nabla T\right)^2 + \frac{\mu}{T_0}\phi^*, \tag{1.11.1}$$

where, T_0 and ϕ^* are the absolute wall temperature and viscous dissipation, respectively. The first term of right-hand side of the equation (1.11.1) defines irreversibility due to heat transfer, whereas, the second term describes the entropy generation related with viscous dissipation.

1.12 Williamson fluid

The Williamson fluid model is one type of non-Newtonian fluid that thins under shear. Williamson examined the pseudoplastic materials flow, experimentally validated the findings, and provided a model equation to describe the pseudoplastic fluid flow. Consider the minimum and maximum viscosities (μ_0 and μ_∞) in the Williamson fluid model.

1.12.1 Constitutive equation of Williamson fluid

Following Williamson fluid model [110] is defined as

$$S = -pI + \tau \tag{1.12.1}$$

$$\tau = \left[\mu_\infty + \frac{(\mu_0 - \mu_\infty)}{1 - \Gamma\dot{\gamma}}\right] A_1, \tag{1.12.2}$$

where

$$\dot{\gamma} = \sqrt{\frac{\pi}{2}}; \quad \text{where} \quad \pi = \text{trace}(A_1^2) \tag{1.12.3}$$

for $\mu_\infty = 0$ and $\Gamma\dot{\gamma} < 1$, thus the equation 1.12.2 converted as

$$\tau = \left[\frac{\mu_0}{1 - \Gamma\dot{\gamma}}\right] A_1, \tag{1.12.4}$$

using binomial expansion on equation 1.12.4, we get

$$\tau = \mu_0 \left[1 + \Gamma\dot{\gamma}\right] A_1, \tag{1.12.5}$$

1.13 Carreau fluid

This kind of fluid is a generalised Newtonian fluid, meaning that its viscosity depends on the rate of shear. The Carreau fluid behaves like a viscous Newtonian fluid at low shear rates. Carreau fluid also behaves like a power-law fluid at intermediate shear rates. Additionally, the Carreau fluid behaves like Newtonian fluid once again along viscosity at high shear rates that are governed by power index n and infinite viscosity shear rates.

1.13.1 Carreau fluid model

For Carreau fluid model the constitutive equation is

$$S = -pI + \tau \tag{1.13.1}$$

$$\tau = \left[\mu_\infty + (\mu - \mu_\infty)\left\{1 + (\Gamma\dot{\gamma})^2\right\}^{\frac{p_i-1}{2}}\right] A_1, \tag{1.13.2}$$

where

$$\dot{\gamma} = \sqrt{\frac{\pi}{2}}, \tag{1.13.3}$$

Here $\pi = trace\,(A_1^2)$. We regard the equation as being Eq. (1.13.2) for the case $\mu_\infty = 0$, therefore,

$$\tau = \left[\mu\left\{1 + (\Gamma\dot{\gamma})^2\right\}^{\frac{p_i-1}{2}}\right] A_1, \tag{1.13.4}$$

Using binomial expansion, Eq. (1.13.4) becomes

$$\tau = \mu\left(1 + \Gamma^2\left(\frac{p_i - 1}{2}\right)\dot{\gamma}^2\right) A_1, \tag{1.13.5}$$

The fluid behavior can be characterized by the power-law index p_i. For $p_i < 1$, $p_i > 1$ and $p_i = 1$, fluid is called shear thinning, shear thickening and Newtonian respectively.

1.14 Micropolar fluid

Fluids having microstructure are known as micropolar fluids. They belong to a group of fluids known as polar fluids because they have non-symmetric stress tensors. Physically, micropolar fluids may imply fluids that disregard the deformation of the fluid's particles and suspend hard, randomly oriented particles in a viscous medium. While Newtonian Navier-Stokes equations cannot adequately describe the properties of fluid with suspended particles, the study of micropolar fluid has recently drawn the attention of many academics.

1.14.1 Constitutive equations of micropolar fluid

The constitutive equations for micropolar fluids with stress tensor τ_{ij} and couple stress tensor C_{ij} are given as Lukaszewicz [35]

$$\tau_{ij} = (-P + \lambda v_{k,k})\,\delta_{ij} + \mu\,(v_{i,j} + v_{j,i}) + \mu_r\,(v_{j,i} - v_{i,j}) - 2\mu_r \epsilon_{mij} G_m, \tag{1.14.1}$$

and

$$C_{ij} = c_0 G_{k,k}\delta_{ij} + c_d\left(G_{i,j} + G_{j,i}\right) + c_a\left(G_{j,i} - G_{i,j}\right), \qquad (1.14.2)$$

where λ and μ are the usual viscosity coefficients, μ_r is the dynamic microrotation viscosity, and c_0, c_a, c_b are constants, called coefficients of angular viscosities.

The symmetric part of the stress tensor τ_{ij} in equation (1.14.1) is given by

$$\tau_{ij}^{(S)} = \left(-P + \lambda v_{k,k}\right)\delta_{ij} + \mu\left(v_{i,j} + v_{j,i}\right), \qquad (1.14.3)$$

Using the stress tensor given by (1.14.1) or (1.14.3) and couple stress (1.14.2) as well as an extra equation known as the angular momentum equation, which is given by, local conservation rules of mass, linear and angular momentum, and energy for polar fluids were found.

$$\rho j^* \frac{DG}{Dt} = \vec{\nabla}\cdot\vec{C}_{ij} + \epsilon_{ijk}\tau_{jk}, \qquad (1.14.4)$$

where j^* is the microinertia coefficient.

1.15 Heat Transfer

Heat transfer is transfer of energy from higher temperature region to lower temperature region, which is because of temperature difference. The basic driving force for heat transfer is temperature variance. This transfer of heat continues till both the regions attains same temperature. Heat transfer issues are involved in different industrial technologies like power engineering, thermal transport etc.

Essentially, there are three types of Heat transfer

1.15.1 Conduction

It refers to the transfer of heat due to a temperature gradient and by the inter molecular interactions in a stationary medium. In this model of heat transfer, heat flows from a region of higher temperature to a region of lower termperature by kinetic motion or by direct impact of the molecules irrespective of whether the body is at rest or in motion. It takes place in solids, liquids and gases.

1.15.2 Radiation

Transfer of energy through electromagnetic waves is called radiation. It is the only form of heat transfer that can occur in the absence of an intervening medium.

Radiation which is emitted by a volume in fluid is due to the thermal agitation of its composing molecules. Transfer of heat by radiation becomes important when the temperature differences are high. It may be noted that radiation also depends on the nature of the fluid.

1.15.3 Convection

It is the transfer of energy between a surface and a moving fluid which are at different temperatures. It depends on the bulk movement of the fluid and therefore occurs in liquids and gases.
Out of these three modes, in fluids, the heat transfer through convection is the most predominant one. This can be categorized into three ways

Natural (Free) convection

Natural convection is the process of transferring heat energy through fluid circulation brought on by buoyancy changes brought on by temperature variations. Natural convection occurs when fluid near a heat source absorbs heat, loses density, and rises. The fluid then moves to be replaced by the cooler fluid around it. Convection current is created when the process of heating this cooler fluid continues. Natural convection is driven by buoyancy, which results from variations in fluid density. Examples of natural convection include sea wind creation, the rising plume of hot air from a fire.

Forced convection

Transfer of heat which occurs due to movement of fluid from forces other than buoyancy, i.e., due to forces, such as a fan or a pump or a moving boundary is known as forced convection. It is encountered when designing or analyzing heat exchangers, pipe flow, air conditioning apparatus etc.

Mixed convection

Mixed convection refers to heat transport in fluids that involves both buoyancy and external forces. It is used in a variety of industrial and technical operations, including the cooling of nuclear reactors during emergency shutdown, the exposure of solar central receivers to wind, the cooling of electronic equipment by fans, and the placement of heat exchangers in low-velocity environments. Also, it is widely accepted that the dimensionless ratio of the Grashof number to the Reynolds number serves as the controlling variable for laminar boundary layer free-forced convective flow.

This dimensionless quantity in mixed convection situations denotes the proportion of buoyancy forces to inertial forces within the boundary layer.

1.16 Mass Transfer

Mass transfer is the tendency of movement of molecules from one location having high molecule concentration to another location having lower molecule concentration. Mass transfer is one of the most vital characteristics of chemical engineering. The force behind mass transfer is concentration gradient. The mass transfer depends on the diffusion of molecules one phase to another phase and also depends on properties like vapour pressure, concentration etc.

1.17 Convective flows

A cold body warms up when it comes into contact with a hot body, and the hot body cools down. The heated body's internal energy is reduced during this process, whereas the internal energy of the cool body is increased. As a result, energy moves from the heated body to the cool body. We observe that no mechanical work is performed during this energy transfer (neglect any change in volume of the body). This is as a result of the absence of any displacements. A heated body can transfer energy to a cold body without the use of mechanical means. Heat is the term used to describe energy that is transmitted from one body to another without the need of any mechanical effort. As a result, heat is a type of energy. Every time there is a temperature difference, energy is in motion. After being transferred, it becomes the receiving body's internal energy. It should be obvious that only when energy is being transmitted does the word "heat" have any real meaning.

1.18 Thermal Conductivity and temperature gradient

The term "heat current" refers to the flow of heat through any cross-section in time $\triangledown t$ if a quantity of heat $\triangledown Q$ is present. At steady state, it is discovered that the heat current is inversely proportional to the length x and is proportional to the cross-sectional area A and the temperature differential $(T_1 - T_2)$ between the ends.

Thus

$$\frac{\triangledown Q}{\triangledown t} = \kappa \frac{A(T_1 - T_2)}{x} \tag{1.18.1}$$

where κ, often known as the material's "thermal conductivity," is a constant for the slab's composition.

The equation can only be used to describe a thin layer of material that is perpendicular to the heat flow if the area of cross-section is not uniform or if the steady state conditions are not attached. If A is the cross-sectional area at a given location, dx is a thin layer that runs perpendicular to the direction of heat flow, and dT is the temperature differential across the layer, the heat current through this cross-section is

$$\frac{\bigtriangledown Q}{\bigtriangledown t} = -\kappa A \frac{dT}{dx} \tag{1.18.2}$$

The term "temperature gradient" refers to the value $\frac{dT}{dx}$. The minus sign denotes a negative $\frac{dT}{dx}$ along the direction of heat flow. The motion of the fluid is affected by so many factors. The boundaries of the fluid affect the flow. The boundaries of the fluid flow can be stationary boundaries, fluctuating boundaries, moving boundaries, oscillatory boundaries and so on.

1.19 Governing equations

The laws of conservation of mass, momentum, energy, and mass flux are observed in fluid flows.

1.19.1 Conservation of mass (Equation of continuity)

According to this equation, the excess of mass that flows in over the amount that flows out must match the increase in fluid mass at the surface during any given period of time. This results in the equation of continuity, which can be stated as follows when represented using vector notations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{V}\right) = 0 \tag{1.19.1}$$

If a fluid's density does not change with pressure, it is said to be incompressible. The continuity equation has the following form when this happens

$$\nabla \cdot \vec{V} = 0 \tag{1.19.2}$$

1.19.2 Conservation of Momentum (Equation of Motion)

The second law of motion, which states that the sum of all forces acting on a fluid mass confined in an arbitrary volume fixed in space is equal to the time rate of change in linear momentum, yields the Navier-Stokes equations of motion, which govern the flow behaviour. When represented in vector notation, this law which generates the equation of motion can be written as follows:

$$\rho\frac{D\vec{V}}{Dt} = \rho f_i - \nabla p + \mu\nabla^2\vec{V} \tag{1.19.3}$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + \left(\vec{V}\cdot\nabla\right)$ is called the material derivative, ∇^2 is the Laplacian operator, and f_i is the external body forces acting on the enclosed volume.

1.19.3 Conservation of Energy(Equation of Energy)

According to this law, the rate of increase of fluid energy in the volume V is equal to the opposite of the energy's outward flux, plus any energy produced by the body's work, surface forces, thermal conduction, and any other heat sources that may be present.

This law, which results in the equation of energy can be represented as follows for an incompressible fluid when written in vector notation:

$$\rho C_p\frac{DT}{Dt} = \kappa\nabla^2 T + \frac{\partial Q}{\partial t} + \phi \tag{1.19.4}$$

where ϕ is the heat generated due to friction forces and is usually known as dissipation function.

1.19.4 Conservation of Mass Flux (Equation of Concentration)

When it comes to the conservation of mass, a fluid with a density of ρ that is clearly composed may well have been a blend of two or more fluids. Every component in the mixture is treated according to the mass conservation principle. Mass transfer is the name of this process. In the absence of constituent generation, the law's statement is as follows:

$$\frac{DC}{Dt} = -\nabla\cdot J \tag{1.19.5}$$

In the same way that the local temperature gradient drives the conduction heat flux, the concentration gradient ∇C drives the diffusion flux vector J. This idea was introduced by the German Physiologist Fick. It is responsible for the analytical developments of the fluid of mass transfer in the same way that Fourier's idea on heat conduction in the thermal boundary layer.

Fick's law of mass diffusion is

$$J = -D_M \nabla C \tag{1.19.6}$$

A transport property called mass diffusivity, or D_M (units $m^2 s^{-1}$), has a numerical value that is often dependent on the mixture's pressure, temperature, and composition. Substituting $J = -D_M \nabla C$ in the mass conservation statements yields

$$\frac{DC}{Dt} = D_M \nabla^2 C \tag{1.19.7}$$

1.20 Joule heating

Thermal energy is created when a current flows through an electrical conductor. This effect is called the Joule heating. This thermal energy increases the conductor material temperature. The heating effect was first studied by famous scientist James Prescott Joule.

1.21 Viscous dissipation

Viscosity of the fluid converts some kinetic energy into thermal energy during the motion of fluid particles. It is caused due to viscosity and as process is irreversible, this is called viscous dissipation.

1.22 Dufour effect

Energy flux because of change in concentration of mass is called Dufour effect. In many flow problems potential of chemical differs, and this drive the flow of heat, this process is called Dufour process. This process is reciprocal phenomenon of Soret effect. The name is Dufour is named after Swiss physicist L. Dufour.

1.23 Soret effect

When heat and mass transfer in a moving fluid occurs together, there is strong connection between potentials and fluxes. It is observed that temperature difference creates mass fluxes which represents the thermal-diffusion effect or Soret effect. This name Soret is named after the scientist Charles Soret. Thermal diffusion disrupts the mixture arrangement's equality, due to which concentration is improved and temperature is reduced.

1.24 Chemical reaction

A chemical reaction is an interaction between one or more substances that results in a chemical change and the production of one or more products that are distinct from the reactants. The molecular diffusion of a species in such flows may occur uniformly throughout a given phase and is known as homogenous reaction or it may take place in a restricted region or within the boundary of a phase and is known as heterogeneous reaction. Homogeneous reactions include those that occur between gases, liquids, or solids. A reaction between a gas and a liquid, a gas and a solid or a liquid and a solid is heterogeneous.

1.25 Forces acting on the fluid

A fluid element is under the influence of surface forces or body forces. The forces acting at a distance on a fluid particle are known as body forces. Similar to this, surface forces are the forces brought about by a particle coming into direct touch with other liquid particles or solid walls or plates. In addition to other forces like viscosity, pressure, gravity, and inertia forces, some of the body forces taken into account in MHD problems include the electric force and the magnetic force.

1.25.1 Buoyancy

If a body is floating in a fluid and at rest, it will be in equilibrium in a vertical plane, hence the sum of the forces acting on it must be equal to the sum of the forces acting on it downward. Whether the body is submerged in a liquid or a gas, this is true. Gravity will exert a downward force on the body, and the fluid in which the body is floating will exert an upward force as a result. The buoyancy is the term for the resulting upward pressure.

1.25.2 Boussinesq approximation

The Boussinesq approximation [27, 58] is of great significance in problems concerning free convection and mixed convection flows. It is well known established fact that free convection as well as mixed convection flows occur due to changes in the fluid density resulting from temperature gradients. In case of thermal boundary layers, the Boussinesq approximation emphasizes that:

$$\rho_\infty - \rho = \rho\beta_T (T - T_\infty) \tag{1.25.1}$$

Here, β_T is the coefficient of volume expansion for heat transfer and ρ_∞ is the free stream density. This is a thermodynamic property which provides a measure of the amount by which the density varies in response to a change in fluid temperature at a constant pressure. Similarly, for concentration boundary layers, the Boussinesq approximation state that:

$$\rho_\infty - \rho = \rho\beta_C (C - C_\infty) \tag{1.25.2}$$

where, β_C is the coefficient of volume expansion for mass transfer. When both thermal and Solutal expressions (heat and mass transfer) are taken into account in free convection boundary layers, the Boussinesq approximation is modified to include the thermal and Solutal buoyancy forces as:

$$\rho_\infty = \rho [1 + \beta_T (T - T_\infty) + \beta_C (C - C_\infty)]\,. \tag{1.25.3}$$

1.26 Dimensionless Parameters

We can better understand the physical significance of a given phenomena by using dimensionless parameters. Simple equations can be turned dimensionless by using certain dependent or independent characteristic values. The following provides clarification on a few of the dimensionless parameters used in the thesis.

1.26.1 Reynolds Number

Reynolds number is the ratio of inertial forces to viscous forces. The importance of Reynolds number in the dynamics of viscous fluids demonstrated by the British Scientist Osborne Reynolds in 1883. It is denoted by Re and mathematically given by

$$Re = \frac{(\rho U^2/L)}{(\mu U/L^2)} = \frac{UL}{\nu} \tag{1.26.1}$$

where U is and characteristic velocity and L is characteristic length respectively.

1.26.2 Eckert number

The non-dimensional parameter Ec is defined as

$$Ec = \frac{U^2}{C_p \left(T_w - T_\infty\right)} \tag{1.26.2}$$

In compressible fluids, the Eckert number determines the relative increase in temperature of the fluid due to adiabatic compression. It can also retain in incompressible fluid, if the frictional heat is to be considered.

1.26.3 Prandtl Number

The ratio of the kinematic viscosity to the thermal diffusivity i.e.

$$Pr = \frac{(\mu/\rho)}{(\kappa/\rho C_p)} = \frac{\mu C_p}{\kappa} \tag{1.26.3}$$

1.26.4 Magnetic parameter

The ratio of electromagnetic force to viscous force is used to define it. It assesses the relative significance of drag forces caused by viscous forces and magnetic induction in flow.

$$M = \frac{\sigma B^2 L^2}{\nu \rho} \tag{1.26.4}$$

1.26.5 Thermal Grashof Number

The ratio of the thermal buoyancy to viscous force acting on a fluid. It often arises in the study of situations involving free convection

$$Gr_T = \frac{g \beta_T L^3 \left(T_w - T_\infty\right)}{\nu^2} \tag{1.26.5}$$

1.26.6 Solutal Grashof Number

The ratio of the mass buoyancy force to viscous force acting on a fluid. It often arises in the study of situations involving free convection

$$Gr_C = \frac{g \beta_C L^3 \left(C_w - C_\infty\right)}{\nu^2}. \tag{1.26.6}$$

1.26.7 Thermal Biot number

Higher values of the Thermal Biot number increase the thermal heat transfer coefficient included therein. When the Thermal Biot number is zero, there is no heat transfer at the surface, and as it tends to infinity, the case of the desired surface temperature is recovered.

$$\lambda_1 = \frac{h_{ft}}{\kappa}\sqrt{\frac{\nu}{a}}. \tag{1.26.7}$$

1.26.8 Solutal Biot number

The Solutal mass transfer coefficient, which increases with Solutal Biot number value, is contained in the Solutal Biot number. There is no mass transfer at the surface when the Solutal Biot number is zero, and the case of the prescribed surface concentration is recovered when the Solutal Biot number tends to infinity.

$$\lambda_2 = \frac{h_{fc}}{D_M}\sqrt{\frac{\nu}{a}}. \tag{1.26.8}$$

1.26.9 Soret Number

It is noticed that, mass fluxes can also be created by temperature gradients and this embodies thermal diffusion (Soret) effect. Soret number is represented by

$$Sr = \frac{D_T}{D_M}$$

1.26.10 Schmidt number

Schmidt number is a non-dimensional parameter defined as the ratio of momentum and mass diffusivity

$$Sc = \frac{\mu C_p}{D_M} \tag{1.26.9}$$

1.26.11 Suction parameter

Suction/Injection parameter is a dimensionless quantity, which is defined as

$$f_w = -\frac{v_w}{\sqrt{a\nu}} \tag{1.26.10}$$

hence, if the fluid is injected through the surface, v_w will be positive; if the fluid is extracted from the surface, v_w will be negative.

1.26.12 Heat generation/absorption coefficient

Heat generation/absorption coefficient β is defined as

$$\beta = \frac{Q^*}{a\rho C_p} \tag{1.26.11}$$

Here β represents heat generation, as $\beta > 0$ and heat absorption, as $\beta < 0$.

1.26.13 Radiation parameter

The inclusion of radiation terms is applied by using Rosseland approximation, radiative heat flux, q_r is given by

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{1.26.12}$$

So, Radiation parameter Rd is defined as

$$Rd = \frac{4\sigma^* T_\infty^3}{k^*\kappa} \tag{1.26.13}$$

1.26.14 Slip parameter

First order velocity slip parameter γ is defined as

$$\gamma = l\sqrt{\frac{a}{\nu}}. \tag{1.26.14}$$

1.26.15 Brinkman number

Brinkman number (Br) is a dimensionless number related to heat conduction from a wall to a flowing viscous fluid. Brinkman number is defined as

$$Br = PrEc \tag{1.26.15}$$

1.26.16 Bejan number

Bejen number Be is ratio of heat and mass transfer irreversibilities to the total entropy generation.

1.26.17 Temperature Ratio parameter

Temperature ratio parameter θ_w is ratio of the temperature on the surface to the free stream temperature. It is defined as the mathematically as

$$\theta_w = \frac{T_w}{T_\infty}. \tag{1.26.16}$$

1.26.18 Skin friction factor

It occurs between solid and fluid surface through which motion of fluid becomes slow. Skin friction coefficient can be defined as,

$$C_f = \frac{\tau_w}{\rho U_w^2}. \tag{1.26.17}$$

1.26.19 Nusselt number

Nusselt number represents the dimensionless temperature gradient at the surface. It is the ratio of convective heat transfer coefficient to conductive heat transfer coefficient.

$$Nu = \frac{hL}{\kappa} \tag{1.26.18}$$

1.26.20 Sherwood number

Sherwood number represents dimensionless concentration gradient at the surface. It is the ratio of convective mass transfer coefficient to conductive mass transfer coefficient.

$$Sh = \frac{h_m L}{D_M} \tag{1.26.19}$$

1.26.21 Chemical Reaction parameter

The non-dimensional chemical reaction parameter is defined as

$$K_c = \frac{k_c x}{U_w}. \tag{1.26.20}$$

1.26.22 Unsteadiness parameter

The unsteadiness parameter is expressed as the ratio of unsteadiness positive constant and initial stretching rate. It occurs in the research of unsteady fluid flow.

The unsteadiness parameter A is given by

$$A = \frac{\alpha}{a} \tag{1.26.21}$$

1.26.23 Material parameter

The dimensionless viscosity ratio K known as Material parameter is defined as

$$K = \frac{k}{\mu}. \tag{1.26.22}$$

1.27 Homotopy Analysis Method

Two continuous functions from one topological space to another are called homotopic if one can be continuously deformed into the other, such a deformation is called a homotopy between the two functions.

The basic idea of HAM method [120] is to produce a succession of approximate solutions that tend to exact solution of the problem. Presence of auxiliary parameters and functions in approximate solution, results in production of a family of approximate solutions, rather than a single solution produced by traditional perturbation methods.

The general approach used by HAM is to solve non-linear equation,

$$\mathcal{N}\left(u(t)\right) = 0, \quad t \ > \ 0, \tag{1.27.1}$$

where $\mathcal{N}$ is a nonlinear operator and $u(t)$ is unknown function of independent variable t.

1.27.1 Zero-order deformation equation

Let $u_0(t)$ denote an initial guess of exact solution of Equation (1.27.1), $\hbar \neq 0$ an auxiliary parameter, $H(t) \neq 0$ auxiliary function and $\mathcal{L}$ an auxiliary linear operator with property,

$$\mathcal{L}\left(f(t)\right) = 0 \;\text{ when }\; f(t) = 0. \tag{1.27.2}$$

The auxiliary parameter $\hbar$, auxiliary function $H(t)$, and auxiliary linear operator $\mathcal{L}$ play important roles within HAM to adjust and control convergence region of solution series. Liao [120] constructs, using $q \in [0, 1]$ as an embedding parameter, so-called zero-order deformation equation,

$$(1-q)\mathcal{L}\left[\Phi(t;\ q)-u_0(t)\right]=q\hbar H(t)\mathcal{N}\left[\Phi(t;\ q)\right], \tag{1.27.3}$$

where $\Phi(t;\ q)$ is solution which depends on $\hbar$, $H(t)$, $\mathcal{L}$, $u_0(t)$ and q. When $q = 0$, zero-order deformation Equation (1.27.3) becomes,

$$\Phi(t;\ 0)=u_0(t), \tag{1.27.4}$$

when $q=1$, since $\hbar \neq 0$ and $H(t)\neq 0$, then Equation (1.27.3) reduces to,

$$\mathcal{N}\left[\Phi(t;\ 1)\right]=0. \tag{1.27.5}$$

So, $\Phi(t;\ 1)$ is exactly solution of nonlinear Equation (1.27.1). Expanding $\Phi(t;\ q)$ in Taylor's series with respect to q, we have

$$\Phi(t;\ q)=u_0(t)+\sum_{m=1}^{\infty}q^m u_m(t), \tag{1.27.6}$$

where,

$$u_m(t)=\frac{1}{m!}\left.\frac{\partial^m\Phi(t;\ q)}{\partial q^m}\right|_{q=0}. \tag{1.27.7}$$

If power series (1.27.6) of $\Phi(t;\ q)$ converges at $q=1$, then we get following series solution,

$$u(t)=u_0(t)+\sum_{m=1}^{\infty}u_m(t), \tag{1.27.8}$$

where terms $u_m(t)$ can be determined by so-called high-order deformation equations which are described below.

1.27.2 High-order deformation equation

Define vector

$$\vec{u_n}=\left\{u_0(t),u_1(t),u_2(t),...,u_n(t)\right\}. \tag{1.27.9}$$

Differentiating Equation (1.27.3) m times with respect to embedding parameter q, then setting $q=0$ and dividing them by $m!$, we have so-called m^{th}-order deformation equation,

$$\mathcal{L}\left[u_m(t)-\chi_m u_{m-1}(t)\right]=\hbar H(t)\mathcal{R}_m\left(\vec{u_m},t\right), \tag{1.27.10}$$

where

$$\chi_m=\begin{cases}0, & m\leq 1,\\ 1, & otherwise\end{cases} \tag{1.27.11}$$

$$\mathcal{R}_m\left(\vec{u_{m-1}},\ t\right) = \frac{1}{(m-1)!}\frac{\partial^{m-1}\mathcal{N}\left[\Phi(t;\ q)\right]}{\partial q^{m-1}}\bigg|_{q=0}. \tag{1.27.12}$$

For any given nonlinear operator $\mathcal{N}$, term $\mathcal{R}_m\left(\vec{u_{m-1}}, t\right)$ can be easily expressed by Equation (1.27.12). Thus, we can gain $u_1(t),\ u_2(t)$... by means of solving linear high-order deformation Equation (1.27.10) one after other in order. m^{th}-order approximation of $u(t)$ is given by,

$$u(t) = \sum_{k=0}^{m} u_k(t). \tag{1.27.13}$$

Liao [120] points out that so-called generalized Taylor's series provides a way to control and adjust convergence region through an auxiliary parameter $\hbar$ such that homotopy analysis method is particularly suitable for problems with strong nonlinearity. Abbasbandy [111] gives meaning of auxiliary parameter $\hbar$, and hence uncovers essence of generalized Taylor's expansion as kernel of homotopy analysis method.

1.27.3 Convergence analysis

One of chief aims of HAM method is to produce solutions that will converge in a much larger region than solutions obtained with traditional perturbation methods. Solutions obtained using this method depend on our choice of linear operator $\mathcal{L}$, auxiliary function $H(t)$, initial approximation $u_0(t)$ and value of auxiliary parameter $\hbar$.

Choice of base functions influence convergence of solution series significantly. For example, solution may be expressed as a polynomial or as a sum of exponential functions. It is expected that, base functions that more closely mimic behavior of actual solution should provide much better results than base functions whose behavior differs greatly from behavior of actual solution. Choice of a linear operator, auxiliary function, and initial approximation often determines base functions present in solution. Having selected a linear operator, auxiliary function, and an initial approximation, deformation equations can be developed and solved in series solution. Solution obtained in this way, still contains auxiliary parameter $\hbar$. This solution should be valid for a range of values of $\hbar$. In order to determine optimum value of $\hbar$, $\hbar$ curves of solution are plotted. These curves are obtained by plotting partial sums $u_m(t)$ or their first few derivatives evaluated at a particular value of t against parameter $\hbar$. As long as equation (1.27.1) with given initial or boundary conditions has a unique solution, partial sums and their derivatives will converge to correct solution for all values of $\hbar$ for which solution converges. Which means that $\hbar$ curves will be essentially horizontal over range of $\hbar$ for which solution converges. As long as, $\hbar$ is chosen in this horizontal region, solution must converge to actual solution of equation (1.27.1).

1.28 Review of relevent literature

The study of MHD flow of Newtonian fluid and Non-Newtonian fluid is important phenomena in science and technology fields. In this thesis, study of two dimensional MHD flow of different types of Newtonian fluid and Non-Newtonian fluids likes, Carreau fluid, Micropolar fluid and Hybrid Williamson fluid with heat and mass transfer are discussed. The governing equations are convert in system of Non-linear partial differential equations. So, HAM has been applied for solving governing equations. All the problems (related to the thesis) are briefly reviewed here.

Fluid dynamics is main branch of science which is used to solve many natural phenomena such as flying of birds, swimming of fishes and the development of weather conditions to be studied technically [54]. The study of charge particle in motion, the forces created by electric and magnetic field, and the relationship between them give rise to the subject Electrodynamics. The collective effects of these three significant branches of science namely, Fluid dynamics, Thermodynamics and electrodynamics give rise to the topic Magneto-fluid dynamics (MFD) which in the form of definition read as The science of motion of electrically conducting fluid in the presence of a magnetic field. The study of Magnetohydrodynemics (MHD) flow of Non-Newtonian fluid has various application in science and engineering fields. The set of equations that describe MHD are a combination of the Navier Stokes and Maxwell's equations. Research works in the magneto hydrodynamics have been advanced significantly during the last few decades in natural sciences and engineering disciplines after the pioneer work of Hartmann [49] in liquid metal duct flows under the strong external magnetic field. Recently, the study of MHD flow done by Kumar and Gupta [97], Borrelli et al. [6] and Kataria et al. [44].

The diverse applications of non-Newtonian fluids in engineering and manufacturing processes have recently drawn researchers' attention. These fluids have the characteristic that the connection between stress and deformation rate is nonlinear. Molten polymers, pulps, and Chyme are examples of this type of fluid. Owing to the fact that it has numerous industrial uses, such as the extrusion of polymer sheets, emulsion-coated sheets like photographic films, melts and solutions of high molecular weight polymers, etc. Williamson [110] initially introduced Williamson fluid model in his groundbreaking study on the flow of pseudo-plastic materials. He created a model equation to describe the movement of pseudo-plastic fluids, and an experiment to test this theory.

Carreau fluid model is another category of non-Newtonian fluids. Such a model has applications in manufacturing processes such as aqueous, and melts. The shear thickening and shear thinning properties of many non-Newtonian fluids are also described by this model. Many scholars have dedicated their effort to explore the properties of such models due to the wide range of applications of the Carreau model in technological processes. The behavior of polymer suspensions in many flow issues is compatible with the Carreau fluid. It is an example of a pure viscous fluid whose viscosity varies with the rate of deformation. The fluid viscosity is based on the shear rate in a model created by Carreau et al. [102]. Carreau fluid flow with convective condition addressed by Madhu et al. [78]. Fluids having microstructure and an asymmetrical stress tensor are known as micropolar fluids. In terms of physical representation, they are fluids made up of randomly oriented particles suspended in a viscous medium. These fluids are used to study the movement of colloidal suspensions, brain fluid, lubricants, and liquid crystals. Eringen [9, 10] created the hypothesis of micropolar fluids. Chaudhary and Jha [106] examined MHD Micropolar fluid flow past a vertical plate.

Numerous researchers have acknowledged the importance of studying magnetohydrodynamic (MHD) Natural Convection Flow with Synchronized Heat and Mass Transfer Due to a Stretching Sheet due to its frequent occurrence in Geophysical and Energy Transfer Problems, which include both Polymer and Metal Sheets. The effect of a uniform transverse magnetic field on the natural convection flow of an electrically conducting fluid past a vertical plate have been discussed by Raptis and Singh [18]. Hossain [70] investigated MHD natural convection fluid flow in the presence of viscous dissipation and Joule heating effects. Öztop et al. [46] scrutinized a numerical analysis on Natural convection flow of entropy optimized MHD fluid with local heat source. Due to the widespread use of magnetohydrodynamic (MHD) flow mixed convection heat transfer of fluid within the boundary layer in industrial technology and geothermal applications, as well as the MHD power production systems, this technology is of great interest. Selimefendigil and Öztop [33] studied numerical study of mixed convection of non-Newtonian power law fluids under the influence of an inclined magnetic field. Chamkha [7] examined effects of chemical reaction in MHD mixed convective flow along a vertical stretching sheet. Later, Akinshilo [3] found mixed convective heat transfer analysis of MHD fluid flow considering radiation effect through vertical porous medium. Also, Cho [25] discovered effect of inclined magnetic field on MHD Mixed convection heat transfer and entropy generation of Cu-water fluid.

Ohmic heating is another name for joule heating. It is a process through which heat is generated when an electric current flows through a conductor. Electric stoves, electric heaters, incandescent light bulbs, electric fuses, electronic cigarettes, thermistors, food processing, and many more industrial and technological activities employ joule heating in various ways. The viscosity of the fluid will absorb energy from the motion of the fluid and convert it into internal energy of the fluid in a viscous fluid flow. It entails warming the fluid. Dissipation, often known as viscous dissipation, is the term used to describe this largely irreversible process. Several applications, including those where considerable temperature increases are seen in polymer manufacturing flows like injection moulding or extrusion at high rates, are of interest for viscous dissipation. Swain et al. [24] examined Viscous dissipation and joule heating effects on MHD flow past a stretching sheet. Daniel et al. [140] scrutinized double stratification effects on unsteady MHD mixed convection flow of fluid with viscous dissipation and Joule heating effects. Characteristics of Joule heating and viscous dissipation on three-dimensional flow of Oldroyd B nanofluid with thermal radiation have been studied by Kumar et al. [55]. Das et al. [116] explored effects of Joule heating and viscous dissipation on MHD mixed convective slip flow over an inclined porous plate.

MHD flow past a heated surface have applications in manufacturing processes such as the cooling of the metallic plate, nuclear reactor, extrusion of polymers etc. Ali et al. [131] studied effects of thermal radiation and heat generation/absorption in fluid flow regime. Rehman et al. [59] discussed effects of heat generation/absorption on Carreau fluid flow in a thermally stratified medium. Reddy et al. [103] explored radiation and heat generation/absorption on MHD nanofluid flow with heat and mass transfer over an inclined vertical porous plate. Patel et al. [38] discovered MHD Micropolar Nanofluid flow over a Stretching/Shrinking Sheet. Jena et al. [119] studied effect of chemical reaction and heat source/sink on MHD viscoelastic fluid flow over a vertical stretching sheet. Daniel et al. [139] examined the effects of thermal radiation on MHD nanofluid flow over nonlinear stretching sheet. Ramzan et al. [83] found effects of radiative and joule heating effects on the MHD micropolar fluid flow with partial slip and convective boundary condition. Soomro et al. [32] examined effetcs of velocity slip on MHD mixed convection Williamson nanofluid flow along a vertical surface. Kayalvizhi et al. [73] explored effects velocity slip on heat and mass fluxes of MHD flow over a stretching sheet. Ibrahim [137] examined MHD fluid flow past a stretching sheet with convective boundary condition. Nayak et al. [81] studied MHD nanofluid flow over an stretching sheet with convective boundary conditions. Lin et al. [141] studied about the effects of suction or injection on laminar boundary layer flow of power law fluids past a flat surface with magnetic field. Sharma et al. [61] explored Soret and Dufour effetcs on MHD Flow Considering chemical Reaction. Imtiaz et al. [72] scrutinized the flow of viscous fluid by a curved stretching surface with Soret and Dufour effects.

Numerous logical strategies like differential transformation method, least square method, HAM are seen in the writing for tackling the physical and designing issues. For a portion of the previously mentioned issues, mathematical methods have been created to acquire the precise answer for quite a long time. In any case, because of certain limitations, researchers have thought about scientific methodologies as another option. Among the most popular techniques in this area, which is broadly applied in science and designing, is bother procedure. It should be noticed that bother procedure can't be applied to emphatically nonlinear issues, as it unequivocally relies on small/large parameters. There have been developed approaches like Adomian deterioration strategy and variational emphasis technique that do not rely on small/large parameters. The significant hindrance of these techniques is that they can't guarantee the assembly of series arrangement. Then again, HAM proposed by Liao [121] is an overall insightful way to deal with getting series solution for unequivocally nonlinear conditions, which can give us a basic method for guaranteeing the assembly of arrangement series.