Abstract

The thesis contains nine chapters focusing on the existence of solutions, exact controllability, and trajectory controllability of the different types of systems. The detailed layout of the thesis is as follows:

Chapter 1 deals with the introduction and historical background as well as mathematical preliminaries.

Chapter 2, contains the existence and uniqueness of the classical and mild solutions of the generalized impulsive evolution equation

$$\begin{aligned} x'(t) &= \mathcal{A}x(t) + \mathcal{F}_i(t, x(t), T_i x(t), S_i x(t)), \quad t \in [t_{i-1}, t_i), \quad t \neq t_i \\ x(0) &= x_0 \\ \Delta x(t_i) &= \mathcal{I}_i x(t_i), t = t_i, \quad for \quad i = 1, 2, 3, \ldots, N. \end{aligned} \tag{0.0.1}$$

over the finite interval $[0, T_0]$ in the Banach space $\mathbb{X}$. Here, $\mathcal{A} : \mathbb{X} \to \mathbb{X}$ is the linear part of the evolution equation, for all i, $\mathcal{F}_i : [0, T_0] \times \mathbb{X} \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$ are nonlinear functions operated over the interval $[t_{i-1}, t_i)$, operators $T_i, S_i : \mathbb{X} \times \mathbb{X}$ are operators operated over the interval $[t_{i-1}, t_i)$, and $\mathcal{I}_i : \mathbb{X} \to \mathbb{X}$ are the jumps at the time moments $t = t_i$.

Chapter 3 contains the Exact controllability of the system

$$\begin{aligned} x'(t) &= \mathcal{A}x(t) + \mathcal{F}_k(t, x(t), u(t)) + \mathcal{B}_k u(t) \quad t \in [t_{k-1}, t_k), \quad k = 1, 2, \cdots, \rho \\ x(0) &= x_0 \\ \Delta x(t_k) &= \mathcal{M}_k x(t_k) + \mathcal{N}_k u(t_k), \quad t = t_k, \quad k = 1, 2, \cdots, \rho \end{aligned} \tag{0.0.2}$$

over the interval $[0, T_0]$ using the concept of operator semigroup, linear and nonlinear functional analysis. In equation (1.3.2) the state $x(t)$ in the Hilbert space $\mathbb{X}$ for all $t \in J_0 = [0, T_0]$, $\mathcal{A}$, and $\mathcal{M}_k$ are linear operators on $\mathbb{X}$, $u \in L^2([0, T_0], \mathbb{U})$ $\mathbb{B}_k, \mathbb{N}_k$: $\mathbb{X} \times \mathbb{U}$ are bounded linear functions between Hilbert spaces $\mathbb{X}$ and $\mathbb{U}$, and $\mathcal{F}_k$: $[0, T_0] \times \mathbb{X} \times \mathbb{U} \to \mathbb{X}$ are nonlinear functions.

Chapter 4 discussed the trajectory controllability of first-order non-instantaneous

impulsive systems

$$
\begin{aligned}
x'(t) &= \mathcal{A}(t)x(t) + \mathcal{F}(t, x(t)) + \mathcal{W}(t), \quad t \in [s_k, t_k + 1),\ for\ all\ k = 0, 1, 2 \cdots, p \\
x(t) &= \mathcal{G}_k(t, x(t)) + \mathcal{W}_k(t), \quad t \in [t_k, s_k),\ for\ all\ k = 1, 2, \cdots, p,
\end{aligned} \tag{0.0.3}
$$

with local condition $x(0) = x_0$ and non-local condition $x(0) = x_0 - h(x)$ over the interval $[0, T]$ in the Banach space $\mathbb{X}$ where, $\mathcal{A} : \mathbb{X} \to \mathbb{X}$ is linear operator, $\mathcal{F}$, and $\mathcal{G}_k$ are nonlinear functions on $[0, T] \times \mathbb{X}$, and $\mathcal{W}, \mathcal{W}_k$ is trajectory controller.

Chapter 5 discussed the trajectory controllability of second-order systems

$$
\begin{cases} x''(t) = \mathcal{A}x + \mathcal{F}(t, x(t), x'(t)) + \mathcal{W}(t), \\ x(0) = x_{10}, \quad x'(0) = x_{20} \end{cases} \tag{0.0.4}
$$

by considering non-instantaneous impulses into account over the finite time interval $\Omega = [0, T_0]$. Here, at each time t, the state lies in $\mathbb{X}$, $\mathcal{A}$ is the linear on $\mathbb{X}$, $\mathcal{F} : \Omega \times \mathbb{X}^2 \to \mathbb{X}$ is a non-linear function, and $\mathcal{W}(t)$ is the trajectory controller of the system.

Chapter 6 derive a set of sufficient conditions for the existence of a mild solution for the generalized fractional order impulsive system

$$
\begin{aligned}
{}^cD^\lambda x(t) &= \mathcal{A}x(x) + \mathcal{F}_k\left(t, x(t), \int_0^t a_k(t, \tau, x(\tau))d\tau\right),\ t \in [s_{k-1}, t_k),\ k = 1, 2, \cdots, p \\
x(t) &= \mathcal{G}_k(t, x(t)),\ t \in [t_k, s_k)
\end{aligned}
$$

with local condition $x(0) = x_0$ and non-local condition $x(0) = x_0 + h(x)$ over the interval $[0, T]$ in a Banach space $\mathbb{X}$. Here $A : \mathbb{X} \to \mathbb{X}$ is linear operator, $P_k x = \int_0^t a_k(t, \tau, x(t))d\tau$ are nonlinear Volterra integral operator on $\mathbb{X}$, $\mathcal{F}_k : [0, T] \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$ are nonlinear functions applied in the intervals $[s_{k-1}, t_k)$ and $\mathcal{G}_k : [0, T] \times \mathbb{X}$ are set of nonlinear functions applied in the interval $[t_k, s_k)$ for all $k = 1, 2, \cdots, p$.

Chapter 7 developed the necessary criteria for a mild solution and classical solution of the impulsive fractional evolution problem,

$$
\begin{aligned}
{}^cD^\alpha x(t) &= \mathcal{A}x(t) + \mathcal{F}(t, x(t)) \quad t \neq t_k,\ k = 1, 2, \cdots, p \\
\Delta x(t_k) &= \mathcal{I}_k(x(t_k)), \quad t = t_k,\ k = 1, 2, \cdots, p \\
x(t_0) &= u_0
\end{aligned} \tag{0.0.5}
$$

over the interval $[0, T_0]$ on a Banach space $\mathbb{X}$. Here, ${}^cD^\alpha$ denotes Caputo fractional differential operator of order $0 < \alpha \leq 1$, $\mathcal{A} : \mathbb{X} \to \mathbb{X}$ is linear operator and $\mathcal{F} : [0, T_0] \times \mathbb{X} \to$ is nonlinear function. $I_k : \mathbb{X} \to \curvearrowleft$ are impulse operator at time $t = t_k$, fro $k = 1, 2, \cdots, p$ and their existence and uniqueness. We also developed conditions under which classical and mild solutions of (1.3.5) coincide.

Chepter 8 considered non-instantaneous impulsive integro-differential fractional order ($0 < \lambda \leq 1$ and $0 \leq \mu \leq 1$) evolution system of Hilfer type

$$\begin{aligned} D_{0+}^{\lambda,\mu}x(\zeta) &= -\mathcal{A}x(t) \\ &\quad + \mathcal{F}\left(t, u(t), \int_0^t a(t,\tau,x(\tau))d\tau\right),\ t \in \left[\cup [s_i, t_{i+1})\right] \cup [s_p, T_0] \\ u(t) &= \mathcal{G}_k(t, x(t)),\ t \in [t_1, s_1) \cup [t_2, s_2) \cup \cdots \cup [t_p, s_p), \end{aligned}$$

and discussed the existence of solutions with local condition $\mathcal{I}_{0+}^{(1-\lambda)(1-\mu)}x(0) = x_0$ and non-local $\mathcal{I}_{0+}^{(1-\lambda)(1-\mu)}[x(0) - h(x)] = x_0$ initial conditions over the finite interval $[0, T_0]$ in a Banach space $\mathbb{X}$. $D^{\lambda,\mu}$ differential operators of Hilfer type, $\mathcal{A} : \mathbb{X} \to \mathbb{X}$ is a linear part of the integrodifferential evolution equation, $Kx = \int_0^t a(t,\tau,x(\tau))d\tau$ is nonlinear Volterra integral operator on $\mathbb{X}$, $\mathcal{F} : [0, T_0] \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$ is nonlinear function and $\mathcal{G}_k : [0, T_0] \times \mathbb{X}$ are set of non-linear functions applied in the interval $[t_k, s_k)$ for all $i = 1, 2, \cdots, p$.

Chapter 9 discussed the Trajectory controllability of infinite dimensional Hilfer fractional control systems

$$\mathcal{D}_{0+}^{\lambda,\mu}x(t) + Ax(t) = \mathcal{F}(t, x(t), \int_0^t a(t,\tau,x(\tau))d\tau) + \mathcal{W}(t)$$

over the interval $[0, T_0]$ with classical condition $\mathcal{I}_{0+}^{(1-\lambda)(1-\mu)}x(0) = x_0$ and non-local conditions $\mathcal{I}_{0+}^{(1-\lambda)(1-\mu)}[x(0) - h(x)] = x_0$ in the Banach space $\mathbb{X}$ Where, $\mathcal{A} : \mathbb{X} \to \mathbb{X}$ is a linear part of the integrodifferential evolution equation, $Kx = \int_0^t a(t,\tau,x(\tau))d\tau$ is nonlinear Volterra integral operator on $\mathbb{X}$, $\mathcal{F} : [0, T_0] \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$ is nonlinear function.