Abstract of Ph.D. Thesis

Study of rational Fourier series

for the award of the Degree of

Doctor of Philosophy

in

Mathematics

by

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February 2024

The Fourier series is often studied by replacing the trigonometric or exponential systems with other orthogonal systems like Walsh, Vilenkin and many more. Their properties are usually analyzed and compared with those of the classical Fourier series. One such orthogonal system is the Malmquist-Takenaka system, often referred to as a rational orthogonal system. This thesis focuses on the rational Fourier series, with its orthogonal system being a rational orthogonal system. The rational Fourier series reduces to the classical Fourier series under certain conditions on the poles of the rational orthogonal system. The order of magnitude of rational Fourier coefficients for functions belonging to different classes of generalized bounded variations is studied. These results are further extended to two- and multiple-variable functions, thus obtaining the order of magnitude of double and multiple rational Fourier coefficients for functions belonging to different classes of generalized bounded variations. The rate of convergence for rational, conjugate rational and double rational Fourier series of functions of generalized bounded variations is determined, hence obtaining the quantitative version of the Dirichlet-Jordan test for rational, conjugate rational and double rational Fourier series, respectively. Lastly, the convergence and integrability properties of rational and double rational trigonometric systems are examined, where the trigonometric system is replaced by rational orthogonal systems with real fixed poles.