Chapter 1

Introduction

The subject of particle physics is a universe in itself. Enormous number of particles have been identified using large number of experimental facilities. Alongside these observations, ample theoretical frameworks have also been developed over past few decades. Masses and life times of these particles vary over a very large range and it's indeed challenging to uncover the underlying dynamics they follow. Apart from these, the mechanism of interaction among these particles seems quite complex to apprehend. Out of the four presently known fundamental interactions, weak interaction is of utmost importance due to its remarkable relevance in violation of various conservation laws.

1.1 Weak Decays

In 1933, Enrico Fermi first coined the term *weak interaction* and provided the theory of for it [16]. In 1960, Sheldon Glashow, Abdus Salam and Steven Weinberg unified the electromagnetic and weak interactions, now known as electroweak interaction [17–19]. Leptons don't participate in strong interactions because they don't have color, neutrino on the other hand carry no electric charge and hence doesn't participate in electromagnetic interaction. However, both of them along with quarks interact through weak interaction.

Weak interaction is the only interaction which changes the flavor of the particles involved in the interaction i.e. it can change one type of quark into another by the exchange of intermediate vector bosons namely $W^{\pm}$, W^0 and Z. $W^{\pm}$, W^0 and Z are massive particles that take part in interaction between leptons and neutrinos. These particles were predicted in 1979 by Weinberg, Salam and Glashow and were detected and measured at CERN's proton anti-proton collider in 1983 [20]. For electroweak unification to exist, the masses of $W^{\pm}$ and Z were estimated to be 80 GeV and 90 GeV respectively. At CERN, the measured masses were 82 GeV and 93 GeV, respectively giving excellent conformation for electroweak unification [20]. Current values of masses for W and Z particles combining the experiments at Tevatron and at CERN's LEP electron-positron collider are $M_W = 80.41 \pm 0.18$ GeV and $M_Z = 91.1884 \pm 0.0022$ GeV [21–24]. Weak interactions can be classified in to three different categories namely "Leptonic", "Semi-Leptonic" and "Hadronic". These interactions are further classified into two categories (i) Charged Current Interaction (CCI) and (ii) Neutral Current Interaction (NCI). CCIs are always mediated by either absorption or emission of W^+ or W^- particles and these W bosons further absorb or emit neutrino along with an electrically charged particle. During these type of processes, W bosons can emit electrons or absorb positrons or can change the flavor of the quark as well as its electric charge as observed in β decay. CCIs also involve the transfer of weak isospin. On the other hand, Z boson is electrically neutral and doesn't affect the quantum numbers like electric charge, flavor, baryon number, lepton number etc of interacting particles.

The significance of flavour dynamics in elementary particle interactions is crucial. Determination of Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing elements is possible only with precise knowledge of the properties of the heavy flavor hadrons, particularly b hadrons [25]. CKM matrix being a unitary matrix can not be parameterized without three mixing angles and CP-violating KM phase [26]. Quark mixing is only seen in weak decays making them a valuable probe for determining the elements of CKM matrix. Testing of CKM sector of standard model relies heavily on the precise determination of $|V_{ub}|$ and $|V_{cb}|$, which in turn, complement the measurement of CP asymmetries in B decays. In 2020, $|V_{cb}|$ was determined for the first time from exclusive decay at LHCb collaboration. The average of inclusive and exclusive decays resulted in $|V_{cb}|$ value to be $(42.19 \pm 0.78) \times 10^{-3}$ [27] and $(39.25 \pm 0.56) \times 10^{-3}$ respectively [28]. Both results are approximately 3σ apart from each other which calls for the effective determination of form factors over entire q^2 range [27]. Several parametrizations for modelling the form factor have been proposed, out of which model derived by Caprini, Lellouch and Neubert (CLN) has been the reference model and that by Boyd, Grinstein and Lebed (BGL) is more general model which was proposed to overcome the limitation of the CLN model [29–34]. Despite these efforts, significant difference in the $|V_{cb}|$ value was not found and problem was still open. Calculating the form factor at non-zero recoil value is the "Gold Standard" for breaking free of parametrization dependence [35]. Novel method used by the authors of [27] has determined the result of $|V_{cb}|$ for the first time using B_s^0, which is quite close to the measurements of $|V_{cb}|$ using exclusive as well as inclusive decays [28]. Determination of $|V_{cb}|$ is used in calculations of different properties like branching ratios and branching fractions which are further used as a probe for lepton flavor universality or its violation. Many groups on the theoretical front currently focus on the calculation of the decay properties and parameters of semileptonic B_s^0 decays. Recently, form factor $f_{+,0}^{B_s \to D_s}$ at $q^2 = 0$ has been calculated and the branching fraction ratio $R(D_s)$ for $B_s \to D_s l \nu_l$, where $l = e, \mu$ and $B_s \to D_s \tau \nu_\tau$ is calculated using QCD light cone sum rule (LCSR) within the framework of heavy quark effective theory (HQET) [36]. Form factors $f^{B \to D^*}$ and $f^{B_s \to D_s^*}$ have been calculated at zero recoil using Lattice QCD (LQCD) calculations [37]. Motivated by LQCD, a new method was proposed by the authors of [38] to obtain $|V_{cb}|$ which might shed light on the major discrepancy between inclusive and exclusive determination of this element. They have also done the first prediction of the branching fraction ratio $R(D_s) = \mathcal{B}(B_s \to D_s \tau \nu_\tau)/\mathcal{B}(B_s \to D_s l \nu_l) = 0.301$, where l is either electron or muon, which however deviates about 4σ from the standard model (SM) predictions. Above results and discussion highlights the importance of semileptonic decays of B_s meson and advocates the attempt to study them in more depth.

In Standard Model (SM), Flavor Changing Neutral Current (FCNC) processes are administered by one-loop graphs represented by Penguins or box diagrams. Apart from dominant $b \to s + X$ decay, where $X = g, \gamma, (l^+, l^-)$ and $\nu\bar{\nu}$, one would expect FCNC decays $b \to d + X$, where $X = \gamma, (l^+, l^-)$ and $\nu\bar{\nu}$. In SM, $b \to d$ decays are suppressed with compared to $b \to s$ by a factor of $|V_{td}/V_{ts}|^2 \approx 1/20$ [39, 40]. Using the form factor for $b \rightarrow d$ decays, we can learn a great deal about the renowned CKM mechanism in the SM [41], which will enable us to calculate branching fraction and ratios of branching fractions, providing hint towards the physics beyond SM by analysing violation of Lepton Flavor Universality. In 2012, LHCb collaboration had announced the discovery of rare $B^+ \rightarrow \pi^+\mu^+\mu^-$ decay and calculated branching fraction to be $(2.3 \pm 0.6(stat.) \pm 0.1(syst.)) \times 10^{-8}$ and the branching fraction ratio $R = \mathcal{B}(B^+ \rightarrow \pi^+\mu^+\mu^-)/\mathcal{B}(B^+ \rightarrow K^+\mu^+\mu^-) = 0.053 \pm 0.014(stat.) \pm 0.001(syst.)$ [42]. Belle collaboration has also updated the upper limit for $\mathcal{B}(B^+ \rightarrow \pi^+\mu^+\mu^-)$ to be $< 6.9 \times 10^{-8}$ [43]. Very recently, Belle collaboration has provided branching fraction for decay $B^+ \rightarrow \pi^+\pi^-l^+\nu_l$ in fully constructed events [44]. Several theoretical results are also available for rare $b \rightarrow d$ transition. Based on Effective Electroweak Hamiltonian approach, branching fraction of the decay $B^+ \rightarrow \pi^+\tau^+\tau^-$ has been calculated by [45]. Form factor and branching fraction for decay $B \rightarrow \pi l^+l^-$ and $B \rightarrow \rho l^+l^-$ has been calculated using Relativistic Quark Model (RQM) based on quasi potential approach [3], constituent quark model [6] and light front quark model [10,46]. Authors of [7,9,47–49] have calculated form factor for semileptonic $B \rightarrow \pi/P/V/Kl^+l^-$ decay, [50] has predicted the differential rate, direct CP-asymmetry and isospin asymmetry for $B \rightarrow \pi l^+l^-$ decay and Helicity form factor for $B \rightarrow \rho$ has been calculated by [51] in the framework of QCD Light-Cone Sum Rules (LCSR). Semileptonic decay $B \rightarrow \pi l \nu_l$ has been calculated using Lattice QCD framework in [52–54]. Rare $B^+ \rightarrow \pi^+l^+l^-$ is calculated in R-parity violating supersymmetric standard model, where R-parity violating effects of $B^+ \rightarrow \pi^+l^+l^-$ and the normalized Forward-Backward asymmetry $A_{FB}(B^+ \rightarrow \pi^+l^+l^-)$ is discussed. [5]. $b \rightarrow d$ decay serves as an important probe in search for CP violation and New Physics as they also follow the same flavor changing neutral current trend at quark level. Like $b \rightarrow d$ transitions, there is $b \rightarrow s$ transition which is also loop suppressed in SM. CKM matrix element associated with $b \rightarrow s$ transition is $|V_{bs}|$ for which CP violating phase $\beta \approx 0$, while the element for $b \rightarrow d$ transitions is $|V_{bd}|$ and $\beta \neq 0$, which indicates comparatively large probability of observing CP violations in $b \rightarrow d$ transitions [55]. Furthermore, decay rate $\Gamma(B \rightarrow \pi\mu^+\mu^-)$ is largely suppressed in comparison to $\Gamma(B \rightarrow K\mu^+\mu^-)$ hence making it easier to distinguish New Physics(NP) effects in $b \rightarrow d$ transitions. Above discussion provides inspiration and motivation for precise analysis of these decays.

In some processes, weak decays are hard to detect due to the shadow of very short life spanned strong decays which further makes the measurement of weak decays very uncertain. Weak decay is the only process which involves Parity (P) as well as Charge Conjugation-Parity (CP) violations. CP violation was first observed in the decay of neutral Kaons [56–58] and theoretical framework about CP violation was given by Cabibbo-Kobayashi-Maskawa (CKM) matrix which is a unitary matrix and explains the mixing of different flavor of quarks in weak interaction [59]. Elements of CKM matrix are measured experimentally and are crucial for the prediction of rates for weak decays involving quarks. The Standard Model of the particle physics which narrates the behaviour of fundamental particles and their interactions is strongly based on the weak interaction and its associated decay processes. Any deviation from the measured value of the decay parameters could strongly signal towards the physics beyond the Standard Model called New Physics (NP). In addition to these, weak decays are also crucial towards the understanding of formation of stars and astronomical objects and thus in a way weak decays play a major role in understanding of fundamental laws of physics at small scales as well the behaviour of the universe at large scales.

1.2 Experimental Facilities at Glance

Experimental verification of theoretical results are of utmost importance for the discovery of new phenomenon. Same goes true with Particle Physics as well. With the advancement of technology, large numbers of sophisticated experimental facilities are being built and existing facilities are being upgraded to provide high precision results. Following is the brief glance of few selected experimental establishments.

At CERN

ATLAS and CMS detectors are aimed at studying the Standard Model (SM), Higgs Boson, search for extra dimensions and particles that make up the dark matter. ATLAS also searches for CP violation in B and D meson decay. LHCb is specifically designed to work on CP violation and FCNC decays for B meson [60–62].

ALICE detector is designed to study phase of the matter called Quark-Gluon-Plasma

(QGP) and predicts how the particles that create the observable universe are formed after Big-Bang [63]. Fixed-Target experiments like COMPASS that studies the structure of hadrons and strong force between the quarks is investigated at experiment called DIRAC [64–66].

CERN has also contributed some phenomenal discoveries in the area of electroweak physics through stunning experiments [67–70].
First direct evidence of Weak Neutral Current was announced by the Gargamelle bubble chamber experiment in 1973 [71]. Then in 1983, UA1 and UA2 detectors discovered the W boson in January and Z boson in June [72, 73].

At FermiLab

Tevatron accelerator has two detectors, namely CDF and D0. Both have worked towards analysing the structure of the universe and in 1995 both of them announced the discovery of top quark [74, 75]. Experiments like SELEX and FOCUS focus on search for charmed mesons, baryons and their spectroscopy. They also search for exotic states and some rare and forbidden decays [76, 77].

At FAIR

The CBM experimental set up tries to answer fundamental questions like *what is the equation of high density state of nuclear matter?* and *what is the dynamics of neutron star merger?* [78, 79] PANDA studies the hadron structure and exotic hadrons [80]. NUSTAR collaboration is devoted to the study of astrophysics, nuclear structure and reaction governing astrophysics [81].

Apart from these facilities, BABAR at SLAC (USA) [82], Belle at KEK (Japan) [83], actively search for CP violation in B meson and provide CKM measurements, BESIII at BEPC (China) searches for CP violation in $D_{(s)}$ decays [84], production and decays of light hadron [85] and also search for Physics beyond SM [86]. Humongous amount of data is available from these experimental facilities that provides excellent motivation to work in the area of theoretical particle physics so as to uncover the governing dynamics at hadron level [87–90].

1.3 Contemporary Theoretical Approaches

Various theoretical approaches enable us to predict the existence of new particles or states way before they are detected experimentally. For example, Higgs Boson was predicted back in 1964 within the Standard Model and it was experimentally confirmed in 2012 at CERN [91]. Without theoretical framework, it will be almost impossible to understand the underlying physics behind the large amount of data generated by experiments. Theoretical framework can be a guiding tool for experimentalists to design experiments by forecasting the anticipated outcomes of different scenario that can help researchers to direct their efforts and resources in a focused way. Essentially theoretical frameworks are required to develop our understanding of particle physics. Without that, the progress in particle physics would be slow and incoherent. Over the years, large number of theoretical models and frameworks have been developed and modified that can be categorised according to (i) first principles based (ii) QCD sum rules and (iii) effective field theories and potential models.

1.3.1 Lattice Quantum Chromodynamics (LQCD)

LQCD is a first principle based approach to QCD. It is a non-perturbative lattice gauge theory of quarks and gluons formulated on a grid of points in space and time. With the help of quark and gluon degrees of freedom, one can construct correlation function between hadronic states. Results obtained using LQCD are generally the closest to the available experimental data. LQCD has provided much precise results in heavy flavor sector. LQCD calculations require enormous amount of computational power which could be quite resource hungry and simulations can take even months to complete. On top of that discretization of space and time leads to errors and uncertainties in the calculations.

Despite of all odds, LQCD has now been the indispensable tool to compute weak decay parameters like decay constants, form factors, branching fractions etc. Leptonic decay constants for $D_{(s)}$ [92–95], $B_{(s)}$ [95–99] and B_c [100, 101] mesons have been calculated using lattice correlation function having 2-point at zero momentum with high credibility result and precision. Semileptonic form factors for rare $B \to Kl^+l^-$ decay have also been carried out using LQCD [102,103]. Branching fraction for ($B_s \to \mu^+\mu^-$) and ($B_d \to \mu^+\mu^-$) is calculated using LQCD [104].

Apart from this, Quantum Field Theory (QFT) and Path Integral Formulation (PIF) are among other approaches that are considered to be the theories based on first principles. QFT combines Quantum Mechanics with Special Theory of Relativity and serves as a mathematical tool to for the explanation of particle fields and their interactions. PIF is a mathematical framework adopted in QFT to calculate probability amplitudes for particle interactions.

1.3.2 Quantum Chromodynamics sum rules

In QCD, behaviour of quark-gluon is related to the properties of hadrons through a correlation function of quark-gluon field for specific hadron. Correlation function is then analyzed using Operator Product Expansion (OPE) technique, which decomposes the function into series of terms representing the contributions from different operators of increasing dimension, coefficients of these operators uncover the properties of hadron. OPE terms are then equated with dispersion relation to obtain sum rules. QCD sum rule has been the most successful theory after LQCD in providing the insights into the weak decays of hadrons. Form factor and branching fractions for $\bar{B}_s^0 \to \phi l^+l^-$ have been calculated [105] and total and differential decay widths have been calculated for $B_c \to J/\phi\ (\eta_c)\ l\nu_l$ using QCD sum rule [106].

1.3.3 Potential Models

Potential models provide simplified description of complex phenomena of hadronic interactions and help to build a bridge between calculated and experimental data. Potential models are extensively used to analyze very specific cases like quark confinement in particle physics. Potential models assume the potential energy function based on the interaction among quarks and gluons and Schrödinger equation is solved considering that potential. Potential models are famous for their ease of calculation over more complex first principle based theories. Potential energy functions are ultimately derived from first principle based methods like LQCD. Potential model is highly pronounced in calculation of mass spectra and decay properties of heavy mesons. The authors of [107] have computed mass spectra and decay properties of heavy quarkonia ($b\bar{b}$ and $c\bar{c}$) and results of these calculations have been applied to calculate mass spectra and lifetime of B_c meson without adding any additional parameter in potential. Several decay properties like leptonic decay constants, digamma, digluon, dilepton, three gamma, three gluon, gamma-digluon, charge radii as well as electromagnetic transition rates are computed using Cornell Potential [108]. Few of the famous confinement potentials which are widely used and proven to be consistent with experimental data in terms of accuracy and precision are briefly discussed below.

Cornell Potential

Cornell potential is a combination of Coulomb and linear confinement potential which is the most widely used to obtain hadronic properties for heavy flavor sector. The potential is given by [109].

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + Kr \tag{1.1}$$

Here α_s is the strong running coupling constant and K represents the spring constant, sometimes also called confinement strength, r is the separation between interacting quarks. Potential incorporates both asymptotic freedom at small distances where One Gluon Exchange (OGE) dominates which is represented by Coulomb term and at large distances, the potential increases linearly which is represented by confinement.

Martin (Power law) Potential

Potential form for this potential is given by [110]

$$V(r) = C\left[\frac{r}{1GeV^{-1}}\right]^{\eta} + D \tag{1.2}$$

here η, C and D are model parameters and are fixed by fitting to the experimental data. This potential is also one of the well-known potential used to describe heavy quarkonia systems. Martin potential has also been included into effective field theories like Non Relativistic QCD (NRQCD) and potential-Non Relativistic QCD (pNRQCD).

Relativistic Harmonic Confinement Model (RHM)

Here the confinement is described by the potential of harmonic nature. The non-relativistic reduction of Dirac equation is done and binding energy is obtained. The Lorentz scalar plus vector harmonic confinement potential is given by [111]

$$V(r) = \frac{1}{2}(1+\gamma_0)A^2r^2 + B \tag{1.3}$$

here RHM parameters are given by A and B [112] and γ^0 is the Dirac Matrix. This model is equally applicable to both heavy and light hadrons.

There are plenty of potential models available in literature having variety of applications, one differing from the other. The main idea behind the invention of potential model was to have framework which can explain both mass spectroscopy and decay properties of hadrons, which is still not achieved in its full capacity.

1.3.4 Effective Field Theories (EFT)

EFTs provide powerful theoretical framework for analyzing system at specific energy scales while ignoring the high energy or short distance physics which provide the specific description of the system of interest. EFTs are organized in terms of the ratio of characteristic energy scale of interest to a higher energy scale associated with the underlying theory which provides a controlled and systematic approximation scheme where higher order terms can be included to improve the accuracy of the result or description. Following is the list of few popular and successful EFTs:

Chiral Perturbation Theory (ChPT) is the effective field theory based on the underlying theory of Quantum Chromodynamics (QCD), which studies systems that interact strongly but at the energies far below hadron masses. Here quarks and gluons are not considered as degrees of freedom and only degree of freedom is that of hadron. ChPT basically is a model independent approach which is dependent on an expansion of amplitudes in terms of light quark masses and its momenta [113].

Heavy Quark Effective Theroy (HQET) is the EFT that describes the interaction among heavy quarks such as bottom and charm quarks. It's a different approach to solve QCD problems which involves heavy quarks. Some symmetry properties which are not apparent in QCD appear prominently in HQET. HQET is mainly used to describe the relations between the form factors of heavy quarks. It is also helpful in simplifying lattice simulations and sum rule analysis of heavy hadrons. In HQET, the heavy quark is assumed to be a static source of gluon field in the leading approximation and $1/m$ corrections can be effectively included perturbatively [114].

Non Relativistic Quantum Chromodynamics (NRQCD) deals with the interaction between heavy quarks in nonrelativistic realm. It is mainly used to describe the systems like quarkonia (Meson consisting of quark and its own anti quark ($q\bar{q}$)).

Electroweak Chiral Lagrangian Theory (EChLT) combines electroweak interaction with the principles of chiral symmetry breaking in QCD and is mainly used to provide framework to study the interaction among pions, photons and weak gauge bosons.

1.4 Inclination Towards Covariant Confined Quark Model

Covariant Confined Quark Model (CCQM) is also one of the QFT inspired EFT approach to study hadronic systems. CCQM is highly pronounced and effective model to study heavy light system dynamics. Major advantage of this model is that it allows one to calculate form factor on entire q^2 range. This EFT approach has built-in property of infrared confinement [115–117] which allows one to take care of IR divergences. The results obtained for weak decays using CCQM has very good match with other theoretical approaches like LQCD and LCSR as well as available experimental data. In spite of the huge potential of CCQM to compute system dynamics of wide range of systems, it requires very few model parameters which is one of the desirable qualities of a good model. These properties provide strong motivation to use CCQM to study dynamics of different hadronic systems. The detailed explanation about CCQM is given in the next chapter.