Department of Applied Mathematics

Mathematical Study of Compact Stars in General Relativity

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by

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0.1 Abstract

Chapter 1 contains an introduction to the general theory of relativity. It also contains the summary of each chapter of the thesis.

Chapter 2 describes a class of new solutions for Einstein's field equations under Karmarkar [3] conditions, by choosing the ansatz $e^{\lambda(r)} = \frac{1+k\frac{r^2}{R^2}}{1+\frac{r^2}{R^2}}$ in static spherically symmetric spacetime metric. The karmarkar [3] conditions provides a relation between Riemann curvature tensor R_{ijkl} in the form

$$R_{1414}R_{2323} = R_{1212}R_{3434} + R_{1224}R_{1334}. \tag{1}$$

This can be written in the form

$$\frac{\nu''}{\nu'} + \frac{\nu'}{2} = \frac{\lambda' e^{\lambda}}{2\left(e^{\lambda}-1\right)}, \tag{2}$$

The general solution of equation (2) is given by

$$e^{\nu} = \left[A + B\int\sqrt{(e^{\lambda(r)}-1)}dr\right]^2, \tag{3}$$

where A and B are constants of integration.
The pressure anisotropy takes the form

$$8\pi\sqrt{3}S = 8\pi p_r - 8\pi p_{\perp} = -\frac{\nu' e^{-\lambda}}{4}\left[\frac{2}{r} - \frac{\lambda'}{e^{\lambda}-1}\right]\left[\frac{\nu' e^{\nu}}{2rB^2} - 1\right]. \tag{4}$$

In the case of isotropic distribution of matter, we have $S = 0$ which leads to either $\frac{2}{r} - \frac{\lambda'}{e^{\lambda}-1} = 0$ or $\frac{\nu' e^{\nu}}{2rB^2} - 1 = 0$. The former case leads to Schwarzschild [6] exterior solution and the latter gives the solution given by Kohler and Chao [4]. It is found that some pulsars like 4U 1820-30, PSR J1903+327, 4U 1608-52, Vela X-1, PSR J1614-2230, Cen X-3 can be accommodated in this model. We have displayed the nature of physical parameters and energy conditions throughout the distribution using numerical and graphical methods for a particular pulsar 4U 1820-30 and found that the solution satisfies all physical requirements.

Chapter 3 deals with a new class of singularity-free interior solutions describing anisotropic matter distribution on static spherically symmetric spacetime metric. Das *et. al.* ([1], [2]) considered metric potential g_{rr} as $B_0^2(r) = \frac{1}{(1-\frac{r^2}{R^2})^4}$, and $B_0^2(r) = \frac{1}{(1-\frac{r^2}{R^2})^6}$, respectively and developed the models of relativistic stars. We have generalized the work of Das *et. al.* ([1], [2]) by considering metric potential g_{rr} as

$$B_0^2(r) = \frac{1}{(1-\frac{r^2}{R^2})^n}, \tag{5}$$

where $n > 2$ is a positive integer. Also, $B_0^2(r) = 1$ ensures that it is finite at the centre. It is regular at the centre since $(B_0^2(r))' = 0$, we obtained the models of relativistic stars and it is observed that all the physical quantities are well behaved up to n= 70. The various physical characteristics of the model are examined for the pulsar PSRJ1903+327. Analysis shows that all the physical acceptability conditions are satisfied.

Chapter 4, In this chapter the new exact solutions of Einstein-Maxwell system of equations for charged anisotropic models have been obtained by choosing ansatz $e^\lambda = 1 + \frac{r^2}{R^2}$, here we consider linear equation of state for radial pressure $p_r = A\rho - B$, where A and B are constants. The expression of charge is considered as

$$E^2 = \frac{\alpha \frac{r^2}{R^2}}{R^2(1 + \frac{r^2}{R^2})^2}\ , \tag{6}$$

The physical acceptability conditions of the model have been investigated, and it is shown that the model is compatible with several compact star candidates like 4U 1820-30, PSR J1903+327, EXO 1785-248, LMC X-4, SMC X-4, Cen X-3. A noteworthy feature of the model is that it satisfies all the conditions needed for a physically acceptable model. It is observed that when $\alpha = 0$. i.e. in the case of uncharged matter distribution the model reduces to the Thomas and Pandya [7].

Chapter 5, contains a new exact solution of Einsteins's field equations on Finch Skea spacetime. In the literature, we have seen the linear equation of state of the form $p_r = \alpha\rho - \beta$, where α and β are constants. We have considered linear equation of state of the form $p_r = \alpha\left(1 - \frac{r^2}{R^2}\right)\rho$, where $0 < \alpha < 1$. This choice of equation of state is valid as the terms of density are linear and p_r is less than ρ throughout distribution including origin as $0 < \alpha < 1$. The solution of field equations has been obtained and the expression of density, radial pressure, and tangential pressure have been calculated. The interior spacetime metric is matched with the Schwarzschild exterior spacetime metric

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{7}$$

and the values of constants of integration and mass have been obtained. It is observed that the total mass of stellar configuration is one-fourth of the radius. The bound on parameter α has been calculated and it is observed that all the physical plausibility conditions are satisfied for $0.06 < \alpha < 0.17$. In particular the pulsar 4U 18020 30 has been considered to demonstrate that all physically viable conditions are satisfied.

Chapter 6, In this chapter, we have reported that Nasheeha *et. al.* [5] studied that models of steller configuration by considering metric potential $g_{rr} = \frac{1+ar^2}{1+(a-b)r^2}$ and

equation of state

$$p_r = \tau\rho^{(1+\frac{1}{p})} + \eta\rho - \omega, \tag{8}$$

where τ, η, ω and p are real constants. It is noted that the metric potential g_{tt} and many physical entities are not well-behaved in the case of a = b. We consider metric potential $g_{rr} = 1 + ar^2$ which is particular case of $g_{rr} = \frac{1+ar^2}{1+(a-b)r^2}$ when a = b. If p = 1 in equation (8), then it becomes a quadratic equation of state. If $\tau = 0$ in equation (8), then it becomes a linear equation of state. If $\eta = 0$, in equation (8), then it becomes polytrope with polytropic index p. If $p = \frac{-1}{2}$, $\omega = 0$ and $\tau = -\alpha$, in equation (8), then it becomes chaplygin equation of state. If $p = -2$, then it becomes a color-flavor-locked (CFL) equation of state. The physical viability of models is tested for strange star candidate 4U 1820 - 30 having mass $M = 1.58M_{\odot}$ and radius R = 9.1 km. All the models are found to be physically plausible. The stability of our model with various equations of state has been compared with the work of Nasheeha *et. al.* [5].

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